ROBUST SEQUENTIAL TESTING OF MULTIPLE HYPOTHESES IN DISTRIBUTED SENSOR NETWORKS

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Introduction

- Sequential detectors can significantly reduce the average number of samples compared to fixed sample size tests with the same reliability.
- Sequential detectors for multiple hypotheses can solve non-binary decision problems.
- Fully distributed methods exploit the inherent scalability, fault-tolerance, and absence of a single point of failure in sensor networks.
- Distributional uncertainties in real-world applications call for robust solutions.

Contributions

- We propose the Consensus + Innovations Matrix Sequential Probability Ratio Test ($\mathcal{CI}$MSPRT) as a sequential multiple hypothesis test for distributed sensor networks.
- We provide an accurate prediction of the average stopping time of the $\mathcal{CI}$MSPRT.
- We robustify the $\mathcal{CI}$MSPRT using least favorable densities (LFDs).
- We validate the performance of the $\mathcal{CI}$MSPRT and the robust LFD-$\mathcal{CI}$MSPRT in a shift-in-variance test.
- We analyze the impact of network size and connectivity on the performance.

Problem Formulation

- Detect the presence of one out of M signals $x_m(t)$ with different variances σ_m^2 in a non-Gaussian environment with a distributed sensor network.

⇒ Shift-in-variance test between M hypotheses under ε-contamination:

$$\mathcal{H}_m : x_m(t) \sim \mathcal{N}(0, \sigma_m^2), \qquad p_m^{\text{cont}} = (1-\varepsilon)p_m^0 + \varepsilon h_m$$

h_m : probability density function of the contaminating noise under $\mathcal{H}_m$
$\mathcal{N}(0,\sigma^2)$: zero-mean normal distribution with variance σ^2
$p_m, p_m^0, p_m^{\text{cont}}$: true, nominal and contaminated probability density function under $\mathcal{H}_m$
ε : contamination factor

The Consensus + Innovations Matrix SPRT ($\mathcal{CI}$MSPRT)

- Calculation of the log-likelihood ratio of node k at time instant t and the corresponding test statistic for the hypothesis pair $\mathcal{H}_m, \mathcal{H}_n$:

$$\eta_{mn}^k(t) = \log\left(\frac{p_m(y_k(t))}{p_n(y_k(t))}\right) \qquad S_{mn}^k(t) = \sum_{l\in\mathcal{N}_k} w_{kl} S_{mn}^l(t-1) + \sum_{l\in\mathcal{N}_k} w_{kl}\eta_{mn}^l(t) \qquad (1)$$

$\mathcal{N}_k$: open neighborhood of node k
w_{kl} : coefficient for weighting the information of node l at node k
$y_k(t)$: measurement of node k at time instant t

- Decision thresholds for the hypothesis pair $\mathcal{H}_m, \mathcal{H}_n$ [1, 2, 3] :

$$\gamma_{mn}^u \geq \frac{4(Nr^2+1)\sigma_{\eta,m}^2}{7N\mu_{\eta,m}}\left[\log\left(\frac{\alpha}{2}\right) + \log\left(1 - e^{-\frac{N}{2(Nr^2+1)}\frac{\mu_{\eta,m}^2}{\sigma_{\eta,m}^2}}\right)\right]$$
$$\gamma_{mn}^l \leq \frac{4(Nr^2+1)\sigma_{\eta,n}^2}{7N\mu_{\eta,n}}\left[\log\left(\frac{\beta}{2}\right) + \log\left(1 - e^{-\frac{N}{2(Nr^2+1)}\frac{\mu_{\eta,n}^2}{\sigma_{\eta,n}^2}}\right)\right] \qquad (2)$$

r : rate of information flow in the network
α, β : required probabilities of false alarm and misdetection
$\mu_{\eta,m}, \sigma_{\eta,m}^2$: mean and variance of the log-likelihood ratio under $\mathcal{H}_m$

- Acceptance test at each node with decision rule:

if $\exists\, m \in \{1,\dots,M\}$ such that
$S_{mn}^k(t) \geq \gamma_{mn}^u \ \forall\, n \in \{1,\dots,M\}\setminus\{m\}$: accept $\mathcal{H}_m$
else : continue sampling,

The test is stopped and $\mathcal{H}_m$ is accepted as soon as all corresponding pairwise test statistics have crossed the threshold.

- Expected stopping time of the $\mathcal{CI}$MSPRT: slowest one-sided pairwise test

$$\mathbb{E}_m[T] \approx \max_{\substack{n=1,\dots,M\\ n\neq m}} \frac{\log(\gamma_{mn}^u)}{D(p_m|p_n)}. \qquad (3)$$

$D(p_m|p_n)$: Kullback-Leibler divergence between p_m and p_n

The Least-Favorable-Density-$\mathcal{CI}$MSPRT (LFD-$\mathcal{CI}$MSPRT)

Least Favorable Densities (LFDs)

- LFDs of Huber's clipped likelihood ratio test for some $c_m, c_n > 0$ [4]

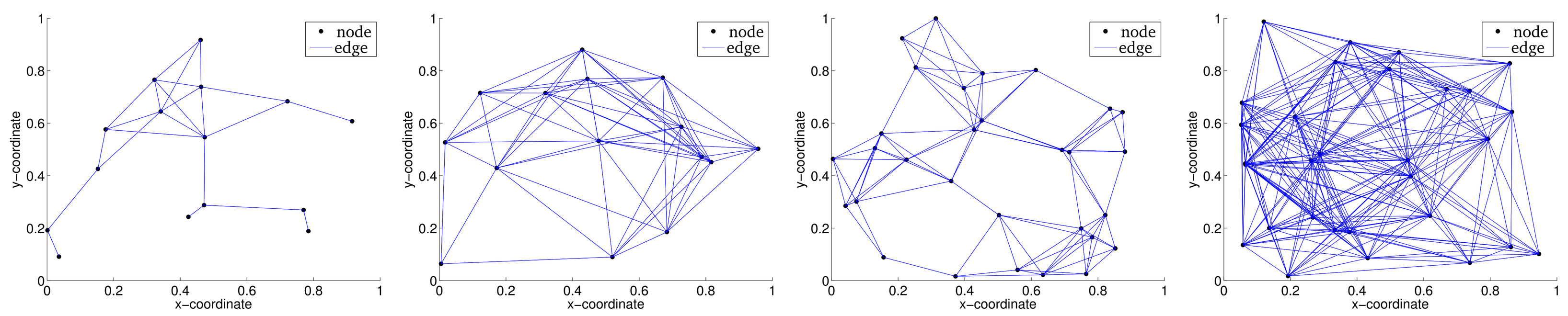
$$q_m = \max\left\{c_m p_n^0, (1-\varepsilon)p_m^0\right\}$$
$$q_n = \max\left\{c_n p_m^0, (1-\varepsilon)p_n^0\right\} \qquad (4)$$

Robustifying the $\mathcal{CI}$MSPRT

- We replace the log-likelihood ratio in (1) by the clipped log-likelihood ratios of the LFDs to obtain a robust test statistic:

$$\eta_{mn}^{k,\text{clipped}}(t) = \log\left(\frac{q_m(y_k(t))}{q_n(y_k(t))}\right) \qquad (5)$$

Sample networks: randomly generated simple, connected and undirected graphs



Results: Detecting the presence of one out of M signals

Setup

- four networks of different size and connectivity
- $M = 4$ signals with variances $\sigma_m \in \{1, 2, 4, 16\}$
- measurement noise: $h_m = \mathcal{N}(0, 81)$, $\varepsilon \in [0, 0.3]$
- probability of false alarm: $\alpha_{mn} = 0.01$
- 1 000 Monte-Carlo runs

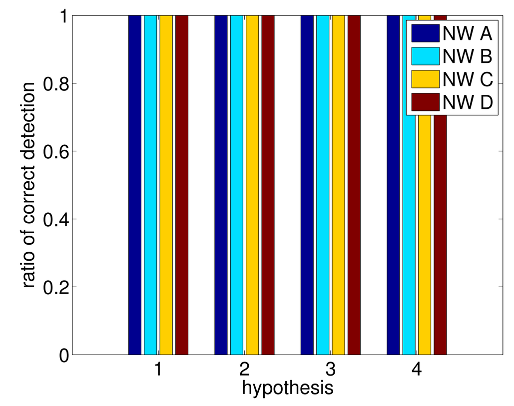
$\mathcal{CI}$MSPRT

- accurate prediction of the average stopping time
- higher variance leads to shorter testing time
- higher network connectivity drastically reduces average stopping time ⇒ good scalability
- network size only marginally effects performance

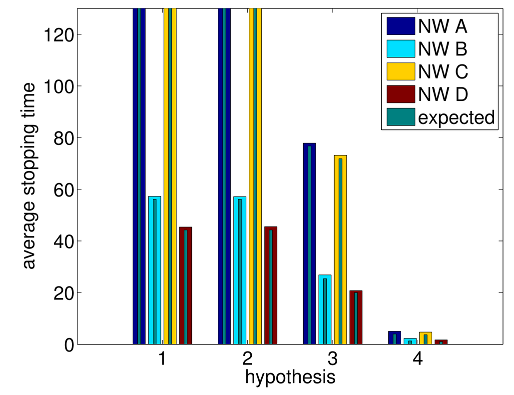
LFD-$\mathcal{CI}$MSPRT

- accurate detection results up to 10 % contamination (20 % under $\mathcal{H}_4$) irrespective of network size and connectivity
- network connectivity impacts average stopping time

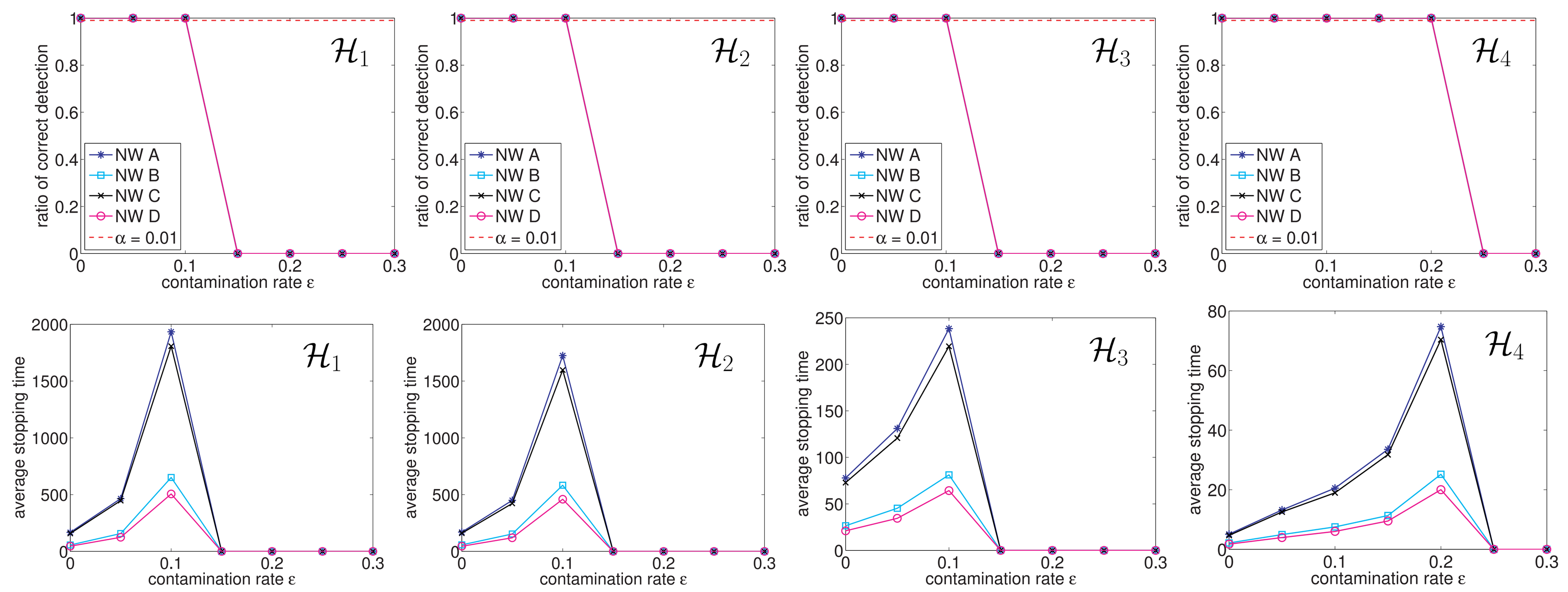
$\mathcal{CI}$MSPRT: Ratio of correct detection



$\mathcal{CI}$MSPRT: Average stopping time



LFD-$\mathcal{CI}$MSPRT: Ratio of correct detection and average stopping time



Related Work

[1] M. R. Leonard and A. M. Zoubir, "Robust distributed sequential hypothesis testing for detecting a random signal in non-Gaussian noise," in *Proc. 25th European Signal Processing Conference (EUSIPCO)*, Aug 2017.

[2] W. Hou, M. R. Leonard, and A. M. Zoubir, "Robust distributed sequential detection via robust estimation," in *Proc. 25th European Signal Processing Conference (EUSIPCO)*, Aug 2017.

[3] M. R. Leonard and A. M. Zoubir, "Robust sequential detection in distributed sensor networks," *IEEE Trans. Signal Proc.*, Feb 2018, submitted. [Online]. Available: https://arxiv.org/abs/1802.00263

[4] M. Fauß and A. M. Zoubir, "Old bands, new tracks – Revisiting the band model for robust hypothesis testing," *IEEE Trans. Signal Proc.*, vol. 64, no. 22, pp. 5875–5886, Nov 2016.