OPTIMIZED SPARSE ARRAY DESIGN BASED ON THE SUM CO-ARRAY

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Introduction

Motivation

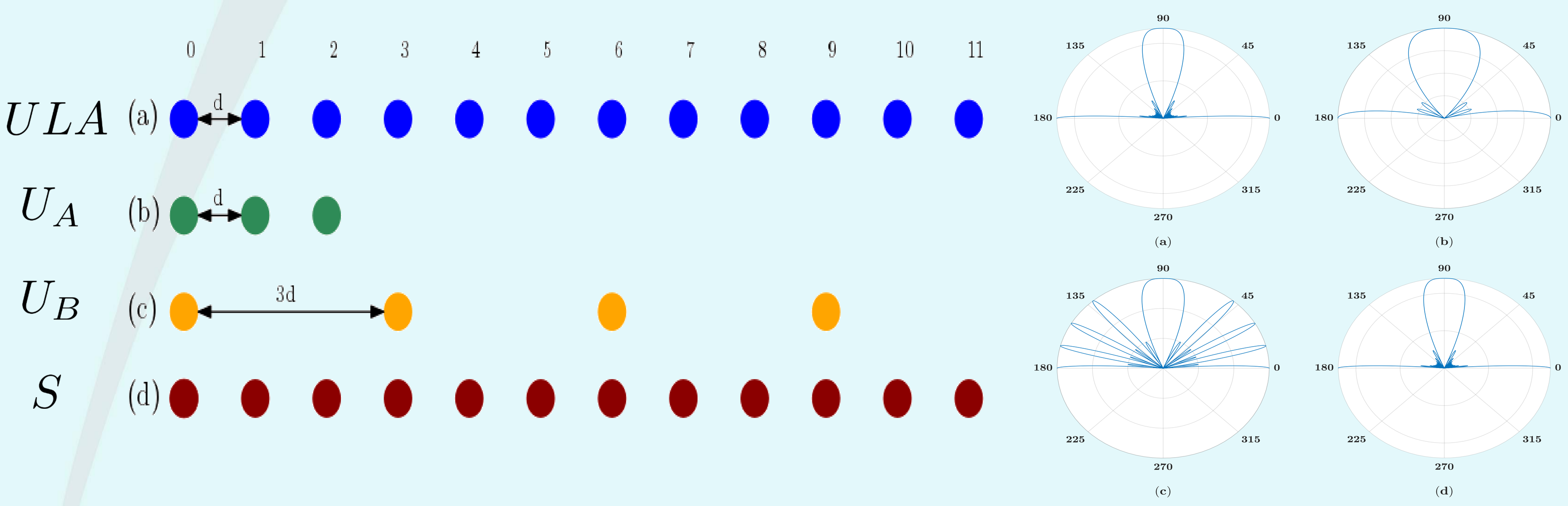
The number of elements of a uniform linear array (ULA) is the main bottleneck in many applications such as radar, communication and ultrasound imaging.

Main Goal

Reducing the number of elements while preserving the beam pattern, which determines the image quality or detection performance.

Solution

We propose the use of multiplicative beamforming along with sparse array design based on the sum co-array. We employ multiple sub-arrays, which the product of their beam patterns yields an effective beam pattern, similar to that of the full array. This allows for significant element reduction without compromising the performance.



Sparse Multiplicative Beamforming

Beam pattern of standard delay and sum (DAS) beamforming:

$$H_{\text{DAS}}(\theta) = \sum_{n=0}^{N-1} exp(2\pi j \frac{d \sin\theta}{\lambda} n).$$

Proposed Sparse Arrays

$$U_A = \{0, ..., A-1\}, \quad U_B = \{nA : n = 0, ..., B-1\}$$

where $N = AB$. **Sum co-array property:** $S = U_A + U_B = \{0, ..., N-1\}$.

Multiplicative Beamforming

1. Apply DAS on each sub-array U_A and U_B separately.
2. Multiply the two results.

Effective Beam Pattern

$$H(\theta) = \left(\sum_{n=0}^{A-1} exp(2\pi j \frac{d\sin\theta}{\lambda} n)\right)\left(\sum_{m=0}^{B-1} exp(2\pi j \frac{d\sin\theta}{\lambda} mA)\right) = H_{\text{DAS}}(\theta).$$

Minimal Number of Elements

$$\min_{A,B} A + B \quad s.t. \; N = AB \rightarrow A = \sqrt{N}, B = \sqrt{N}$$

Extension Beyond Two Sub-Arrays

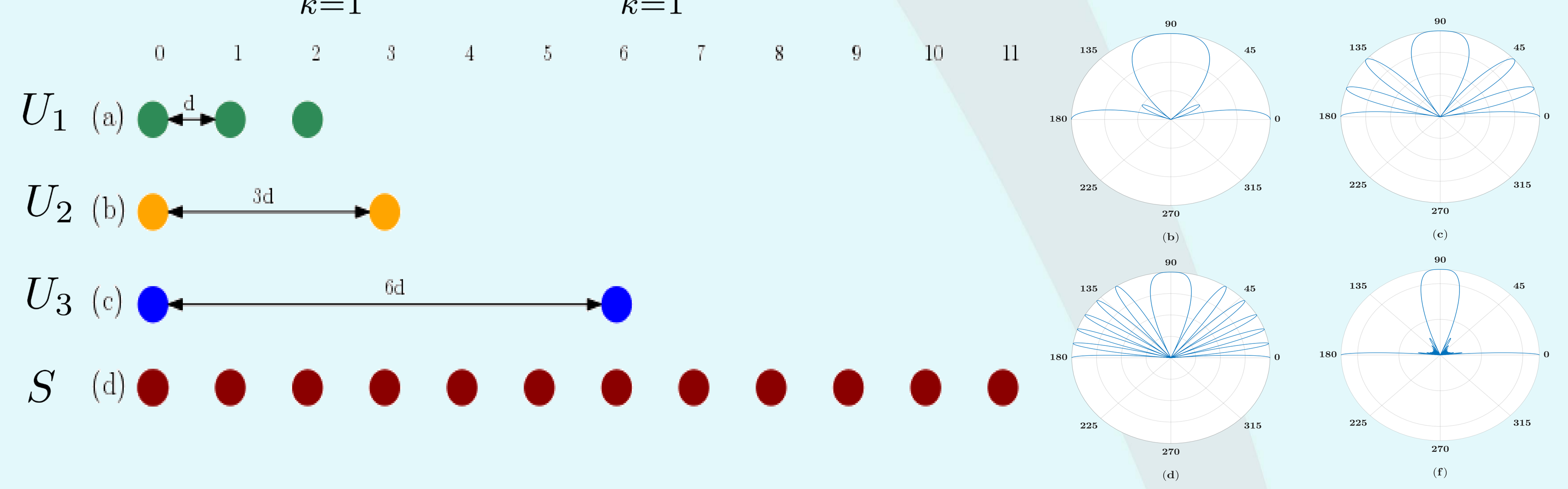
$$U_1 = \{0, ..., A_1 - 1\} \quad U_k = \{n \prod_{i=1}^{k-1} A_i : n = 0, ..., A_k - 1, \; k = 2, ..., K\}.$$

$$\rightarrow S = \sum_{k=1}^{K} U_i = \{0, ..., N-1\}$$

Effective Beam Pattern $H(\theta) = \prod_{k=0}^{K}\left(\sum_{n=0}^{A-1} exp\left(2\pi j \frac{d\sin\theta}{\lambda} n \prod_{i=1}^{k-1} A_i\right)\right) = H_{\text{DAS}}(\theta)$

Minimal Number of Elements

$$\min_{K, A_1, ..., A_K} \sum_{k=1}^{K} A_k \;\; s.t. \;\; N = \prod_{k=1}^{K} A_k \rightarrow K = \log_2 N, A_k = 2 \quad k = 1, ..., K.$$



Summary

Sparse Multiplicative Beamforming

- Performing multiplicative beamforming is equivalent to apply standard DAS on the sum co-array.
- We outline a sparse array design composed of two sub-arrays which their sum co-array is full ULA. This allows to obtain the beam pattern of the full array, while using fewer elements on the order of $2\sqrt{N}$.
- Extending this approach to numerous sub-arrays enables element reduction where the number of sensors is as low as $\log_2 N$.

Results

- We show in the simulations above that the beam pattern of the full array can be realized using several sub-arrays with fewer elements.
- The proposed method was applied for ultrasound imaging using *in-vivo* cardiac data. The results shown below, prove that the proposed approach is valid and suitable for clinical use.