FRFT angle θ

ADAPTIVE STFT WITH CHIRP-MODULATED GAUSSIAN WINDOW

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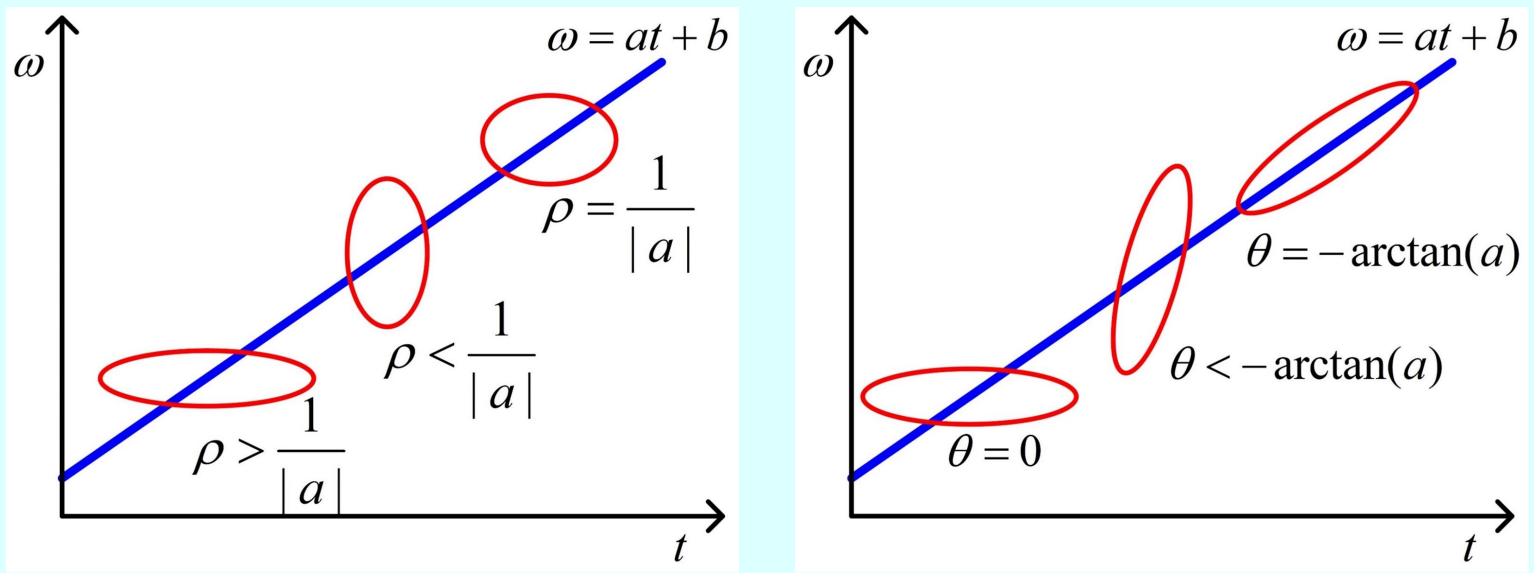
Problem Statement



Line: instantaneous frequency of the input signal

Ellipse: 3 dB contour of WVD of window function

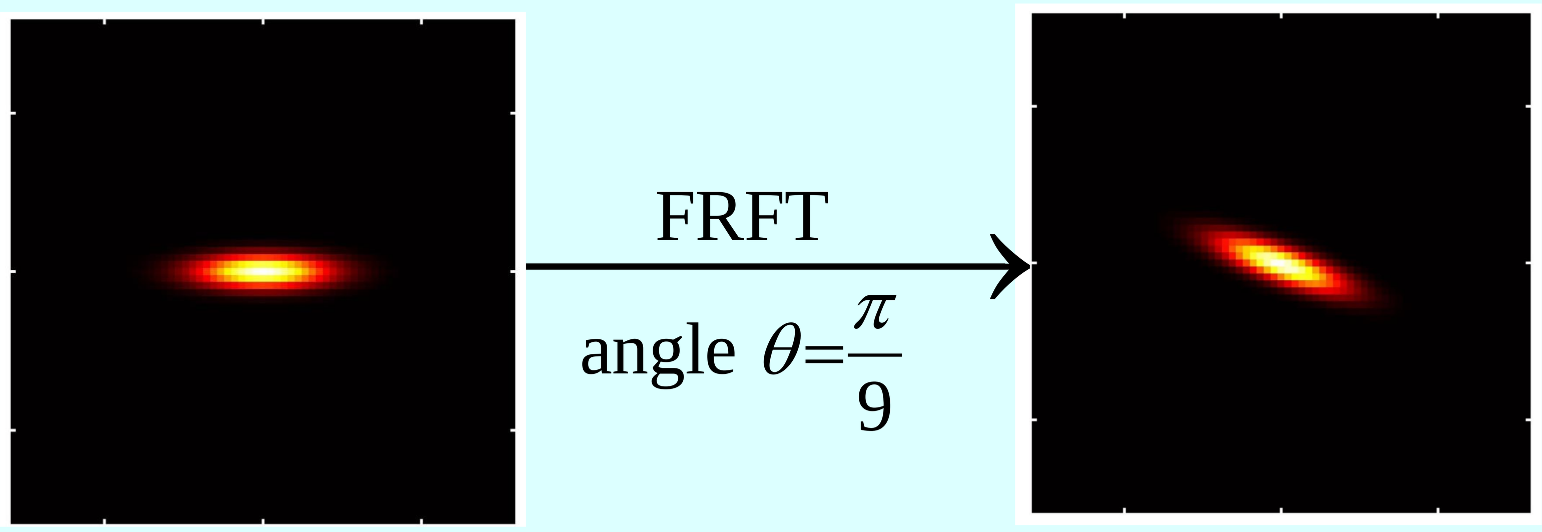
- Gaussian window with optimal variance $\rho = 1/|a|$
  - Resolution is still limited by the Heisenberg uncertainty principle
  - How to further increase resolution?

    Rotating the Gaussian window function
- What are the optimal values of
  - variance ρ (shape)
  - rotation angle θ

  for the highest resolution (energy concentration)?

Chirp-Modulated Gaussian Window

- Fractional Fourier transform (FRFT) can produce rotation in time-frequency (TF) plane



- FRFT of a Gaussian is another Gaussian multiplied by a chirp function.

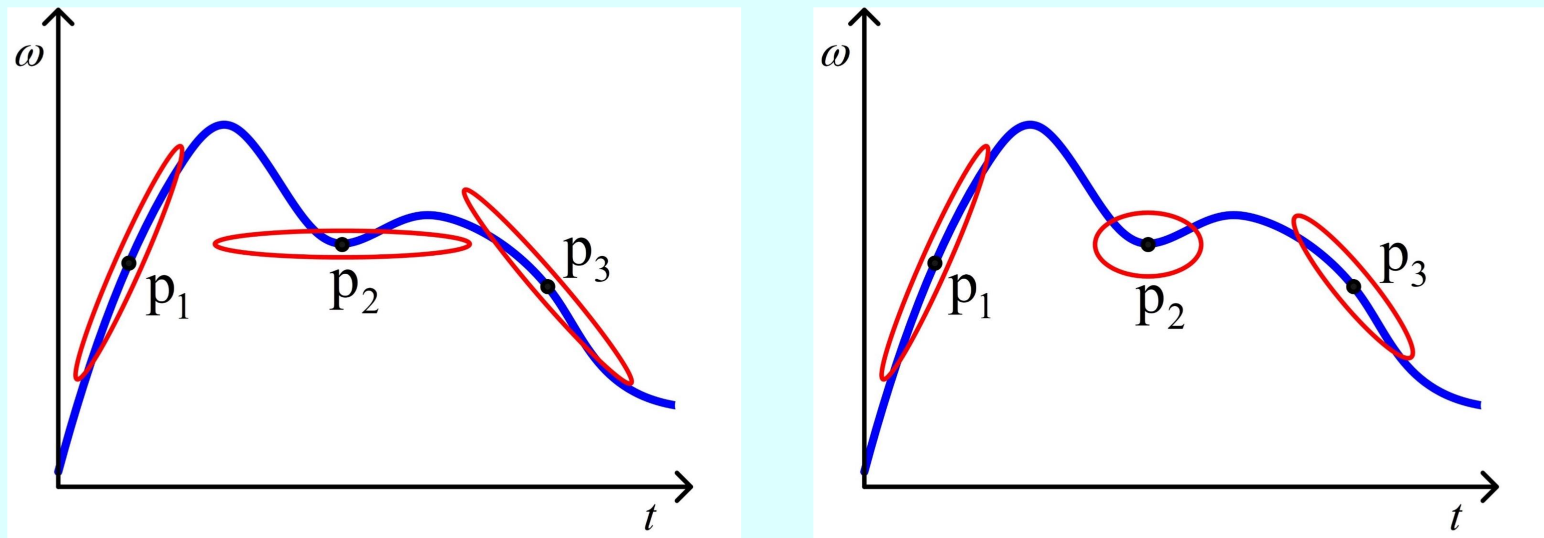
Optimal FRFT Angle

- As shown in the left figure, the optimal FRFT angle depends on the slope of the instantaneous frequency, i.e. chirp rate

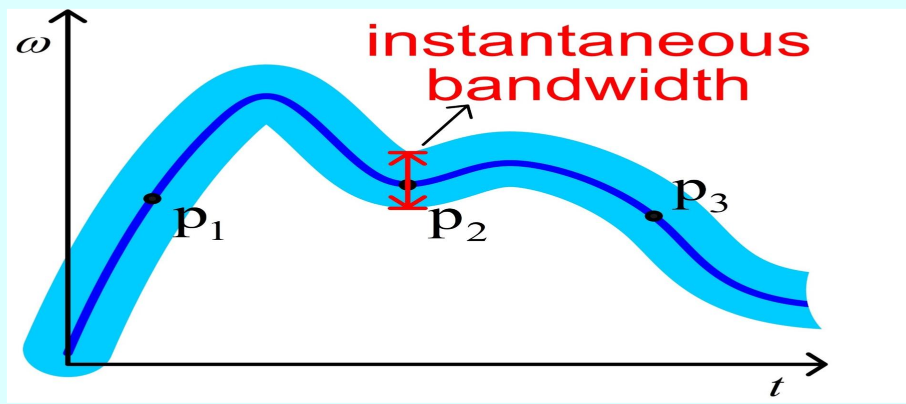
$$\theta_{\mathrm{opt}}(t) = -\arctan\left(\omega'_{\mathrm{inst}}(t)\right) = -\arctan\left(\varphi''(t)\right)$$

Optimal Variance

- Consider a more complicated signal



  - Large variance is a good choice for TF point p_1 but not for TF point p_2
  - The optimal variance depends on the chirp rate and its derivatives
- Instantaneous bandwidth is a measure of the blurring caused by window function

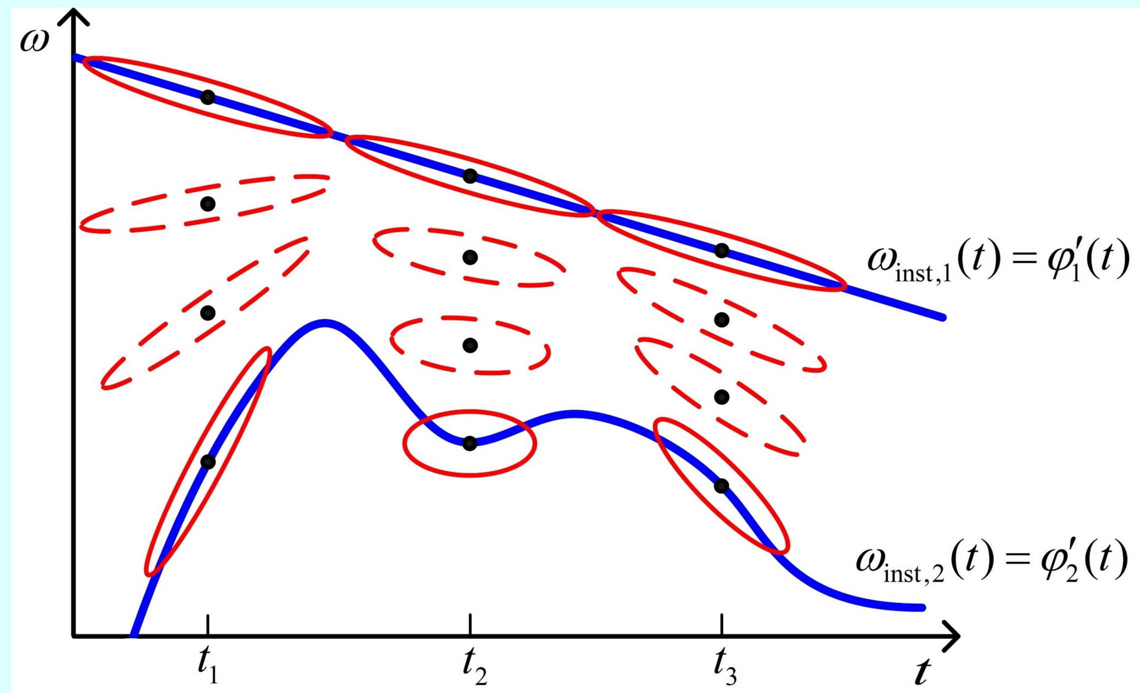


  - The optimal variance can be obtained by minimizing the instantaneous bandwidth

$$\rho_{\mathrm{opt}} \approx S + \sqrt{-S^2 - q/(2S)}$$

where
$$q = -(1+\varphi''^2)^3/\varphi'''^2,\ S = \sqrt{Q/12 + r/Q},$$
$$r = -\varphi''^4,\ Q = \sqrt[3]{6\left(9q^2 + \sqrt{81q^4 - 48r^3}\right)}$$

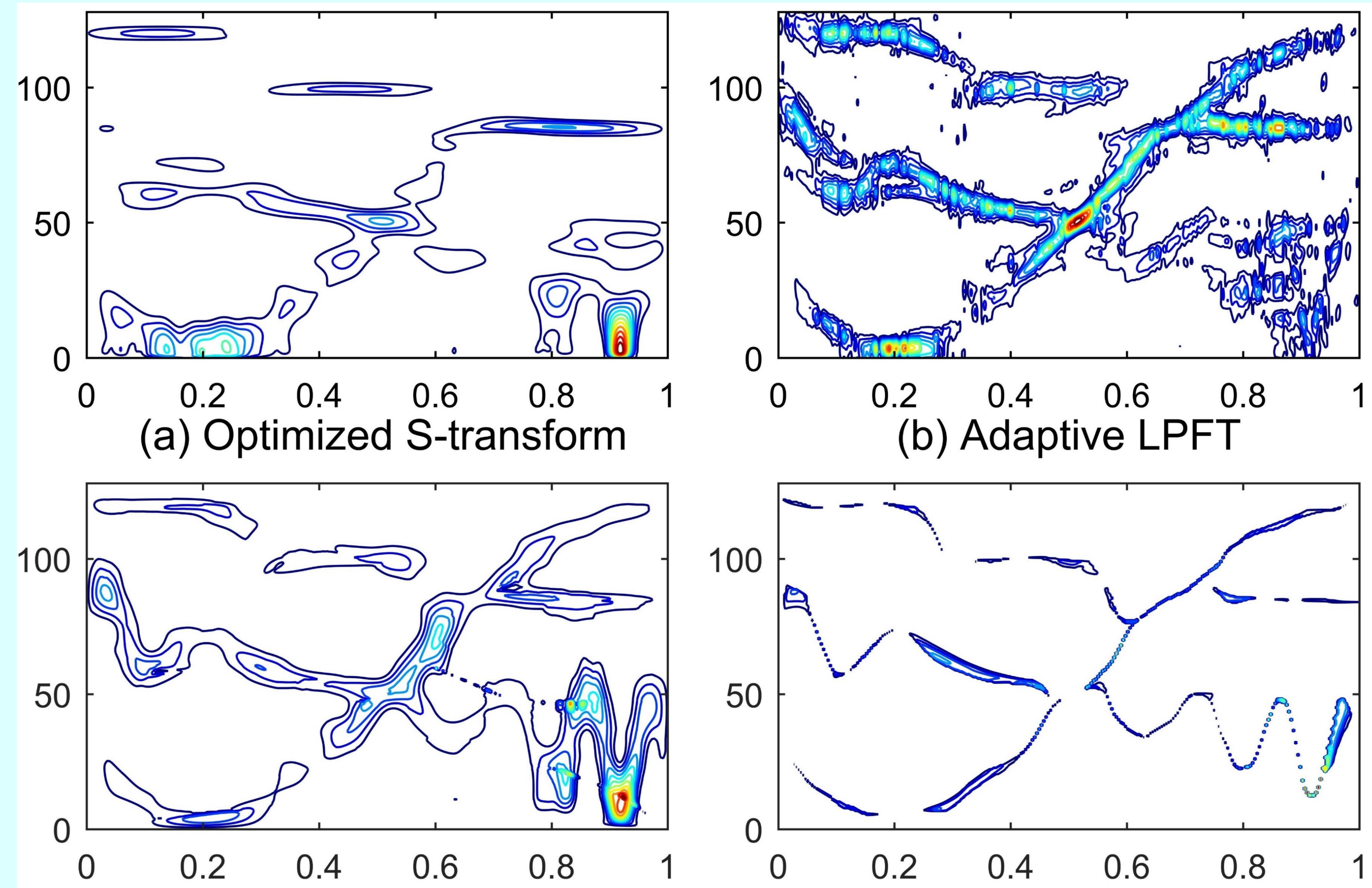
Time-Frequency-Varying Chirp-Modulated Gaussian Window



- TF points on each components (solid ellipses)
  - Calculate $\theta_{\mathrm{opt},i}(t)$, $\rho_{\mathrm{opt},i}(t)$ from $\omega'_{\mathrm{inst},i}(t)$
- TF points between components (dashed ellipses)
  - Optimal FRFT angle is between $\theta_{\mathrm{opt},i}(t)$'s
  - Optimal variance is between $\rho_{\mathrm{opt},i}(t)$'s
  - Find approximate values by interpolation for low complexity

Simulation

- Compared with three other linear adaptive TFRs



(a) Optimized S-transform

(b) Adaptive LPFT

(c) ASTFT with original Gaussian window

(d) Proposed ASTFT