# Distributed TDOA-based indoor source localisation

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# Outline

- Background
- Problem Formulation
- Pruning out erroneous TOA measurements
- TDOA-based localization
- Experiments
- Conclusion

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# Background

- Localisation techniques
  - In the hospital
  - In museums

- Outdoor environment: GPS

- Indoor environment:

  TOA, TDOA, RSS, AOA

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# Problem Formulation

- Time difference of arrival techniques
- $M$ receivers with known location

$$\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_M$$

- $N$ transmitters whose locations are to be estimated

$$\boldsymbol{s}_1, \boldsymbol{s}_2, \ldots, \boldsymbol{s}_N$$

- The time of arrival (TOA) information

$$t_{ij} = c^{-1}\|\boldsymbol{r}_i - \boldsymbol{s}_j\| + n_{ij}$$

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# Pruning out erroneous TOA measurements

- Erroneous TOA measurements
- TOA measurements

$$t_{ij} = \|\boldsymbol{r}_i - \boldsymbol{s}_j\|$$

↓

$$-(\boldsymbol{r}_i - \boldsymbol{r}_1)^T(\boldsymbol{s}_j - \boldsymbol{s}_1) = 0.5(t_{ij}^2 - t_{1j}^2 - t_{i1}^2 + t_{11}^2)$$

↓

$$\boldsymbol{R}^T\boldsymbol{S} = T$$

# Pruning out erroneous TOA measurements

- The set of all $N-1$ unique combinations of the set of TOA measurements

$$U_{N-1} = \binom{S_N}{N-1}$$

- For a specific combination, construct $\boldsymbol{T}_u$ , and compute $e_u$

$$e_u = \|\boldsymbol{T}_u\|_F^2 = \sum_{i=1}^{N_r} \sigma_i^2(\boldsymbol{T}_u)$$

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# Pruning out erroneous TOA measurements

**Algorithm 1** Pruning incorrect TOA measurements

1. For $n = 0, 1, N - N_{min}$
2. Generate the set of all possible combinations of the set $S_{N-n}$

$$U_{N-n+1} = \binom{S_{N-n}}{N-n+1}$$

3. For each $u \in U_{N-n+1}$, construct $\boldsymbol{T}_u$ and compute $e_u$.
4. Update the best TOA sets,

$$S_{N-n+1} = \begin{cases} \arg\min_s e_u & \text{if } \min e_u / \max e_u < \alpha \\ S_{N-n} & \text{otherwise} \end{cases}$$

5. End if $S_{N-n+1} = S_{N-n}$

# TDOA-based localization
## —— Centralised localisation 1

- TDOA measurements

$$\Delta t_{1j} = t_1 - t_j = \frac{\|\boldsymbol{s} - \boldsymbol{r}_1\|}{c} - \frac{\|\boldsymbol{s} - \boldsymbol{r}_j\|}{c} + n_{1j}$$

- Define

$$d_j^2 = (x_j - x_0)^2 + (y_j - y_0)^2$$

$$K_j^2 = x_j^2 + y_j^2$$

- For receiver *j*:

$$-x_j x_0 - y_j y_0 = d_{j1} d_1 + \frac{1}{2}(d_{j1}^2 - K_j^2)$$

# TDOA-based localization
## —— Centralised localisation 1

- Vector form:

$$\boldsymbol{A}_1\boldsymbol{s} = d_1\boldsymbol{b}_1 + \boldsymbol{c}_1$$

- Least-square problem:

$$\min_{s} ||\boldsymbol{A}_1\boldsymbol{s} - (d_1\boldsymbol{b}_1 + \boldsymbol{c}_1)||_2^2$$

- Solution:

$$\hat{\boldsymbol{s}} = (\boldsymbol{A}_1^T\boldsymbol{A}_1)^{-1}\boldsymbol{A}_1^T(d_1\boldsymbol{b}_1 + \boldsymbol{c}_1)$$

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# TDOA-based localization
# —— Centralised localisation 2

- Least-square problem:

$$\min_{\boldsymbol{s}_k} ||\boldsymbol{A}_k \boldsymbol{s}_k - (d_k \boldsymbol{b}_k + \boldsymbol{c}_k)||_2^2$$

- Solution:

$$\hat{\boldsymbol{s}}_k = (\boldsymbol{A}_k^T \boldsymbol{A}_k)^{-1} \boldsymbol{A}_k^T (d_k \boldsymbol{b}_k + \boldsymbol{c}_k)$$

- Average location estimation:

$$\hat{\boldsymbol{s}} = \frac{1}{M} \sum_{k=1}^{M} \hat{\boldsymbol{s}}_k$$

# TDOA-based localization
# —— Distributed localisation

- Distributed localisation:

$$\min_{\boldsymbol{s}_k} \sum_{k=1}^{M_k} ||\boldsymbol{A}_k \boldsymbol{s}_k - (d_k \boldsymbol{b}_k + \boldsymbol{c}_k)||_2^2,$$

$$\text{subject to } \boldsymbol{s}_k = \boldsymbol{s}_j, \forall (k, j) \in E$$

- Solution: standard solvers.

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# Experiments

## —— Experimental Setup

- Room: $6 \times 5 \times 4$ m
- $8$ receivers, $30$ transmitters
- A hit: an erroneous TOA measurement being detected
- A false alarm: a correct TOA measurement being classified as erroneous

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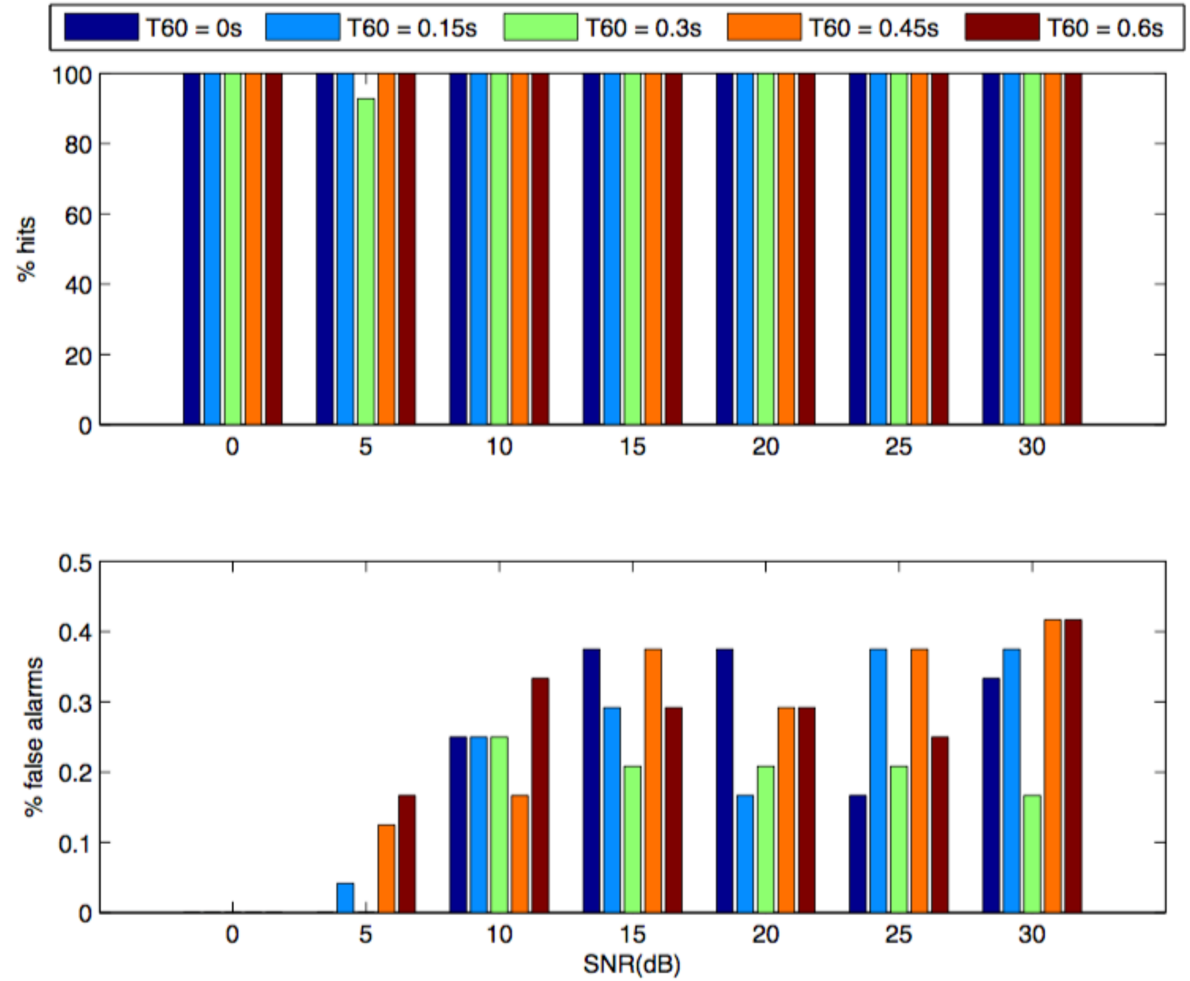
# Experiments

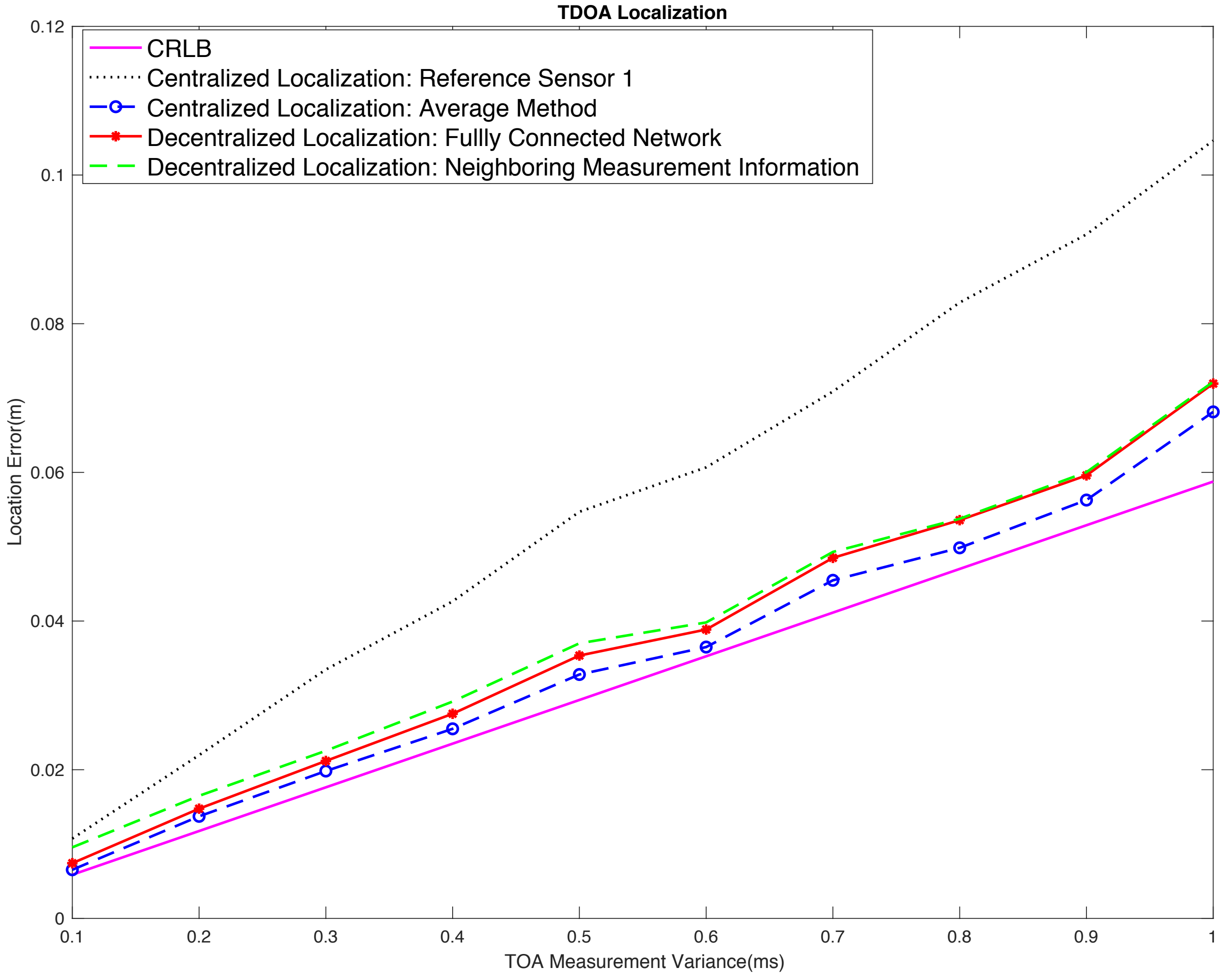


# Experiments

## —— Experimental Setup

- Room: $10 \times 10$ m
- $8$ receivers uniformly placed on the boundary
- Two receivers can communicate when the distance is less than $10$ m
- Propagation speed: $c = 340$ m/s

# Experiments



# Conclusions

- An algorithm to prune out erroneous TOA measurements.
- TDOA-based localisation
  - Centralised localisation
  - Distributed localisation
- Accessing to neighbouring information doesn’t decrease the accuracy.

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