

# ANTENNA SELECTION FOR LARGE-SCALE MIMO RECEIVERS WITH LOW-RESOLUTION ADCs

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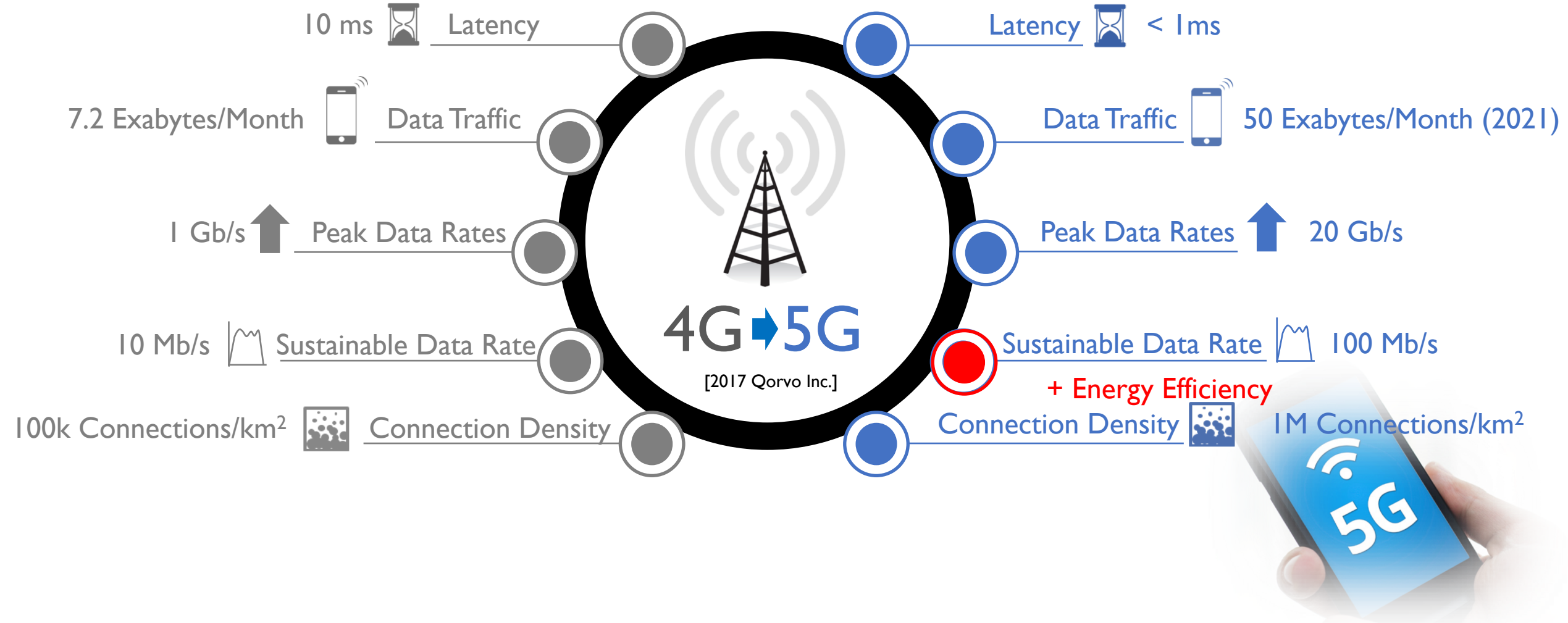
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WHAT STARTS HERE CHANGES THE WORLD



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Communications Group



# VISION FOR 5G COMMUNICATIONS



[<https://www.qorvo.com/design-hub/blog/getting-to-5g-comparing-4g-and-5g-system-requirements>]

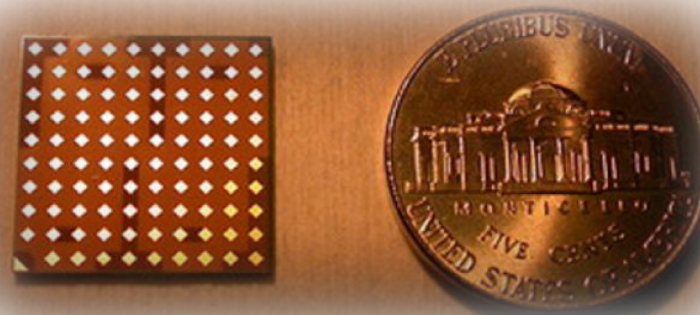
# LARGE-SCALE ANTENNA SYSTEMS FOR 5G

[Andrews14]

## Millimeter wave communications

[Pi&Khan11]

- High frequency: 30 – 300 GHz
- Large bandwidth: 100MHz – 1GHz
- Large pathloss / blockage



[IBM mmWave antennas]

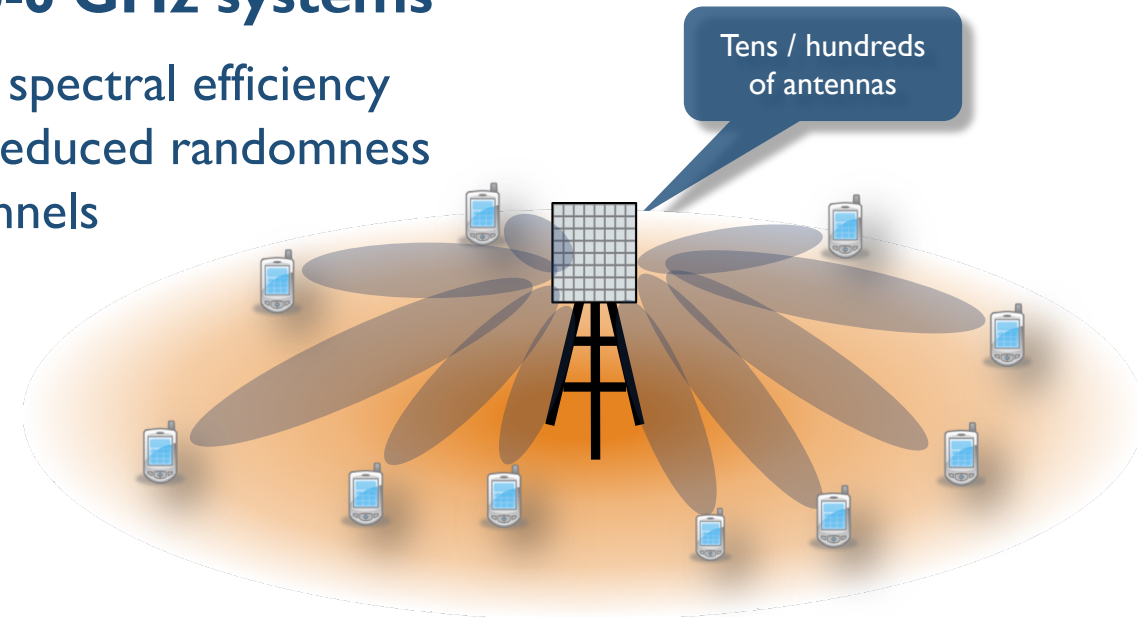
## Massive MIMO in sub-6 GHz systems

[Marzetta10]

- Substantial increase in spectral efficiency
- Channel hardening – reduced randomness
- Quasi-orthogonal channels



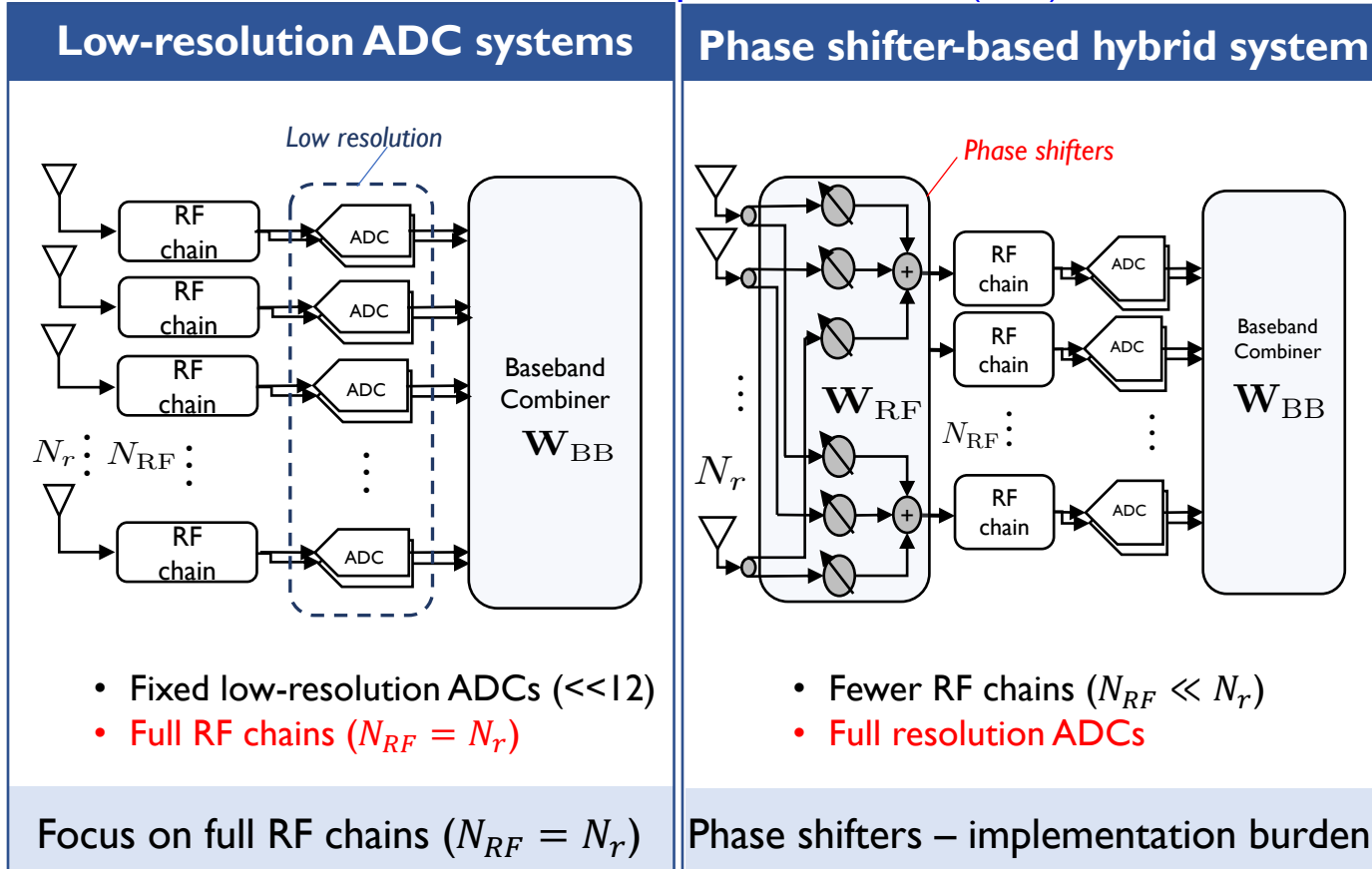
[Bristol and Lund universities]



# MOTIVATION

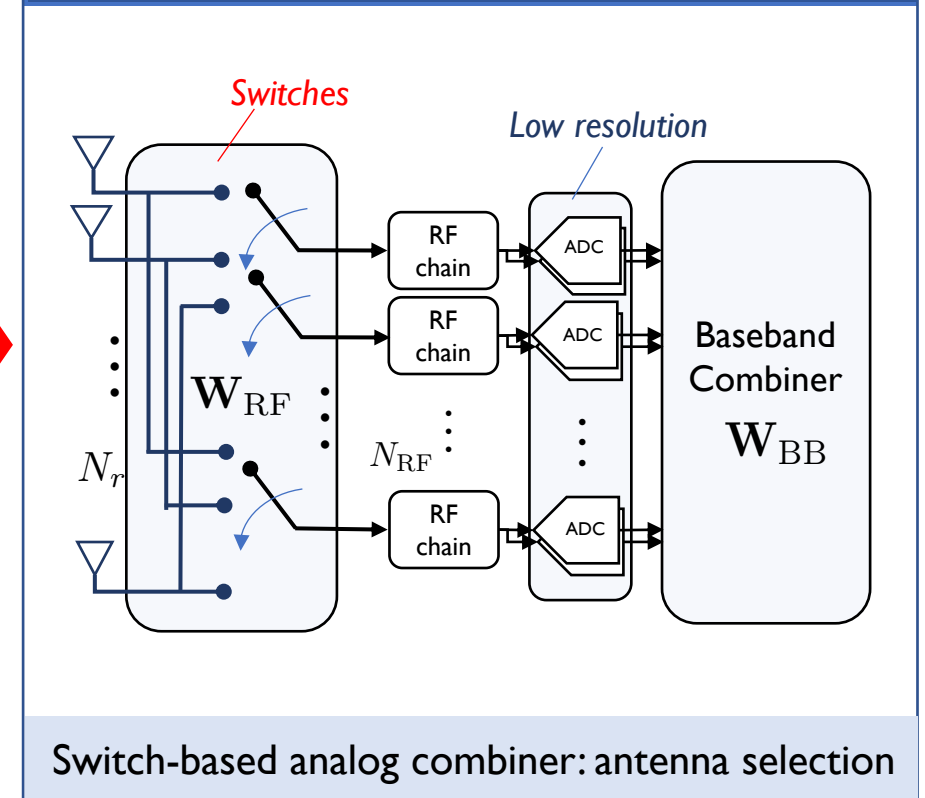
## □ New energy-efficient receiver architecture

Conventional low-power solutions (A, B)



New low-power solution

Switch-based hybrid low-resolution ADC system

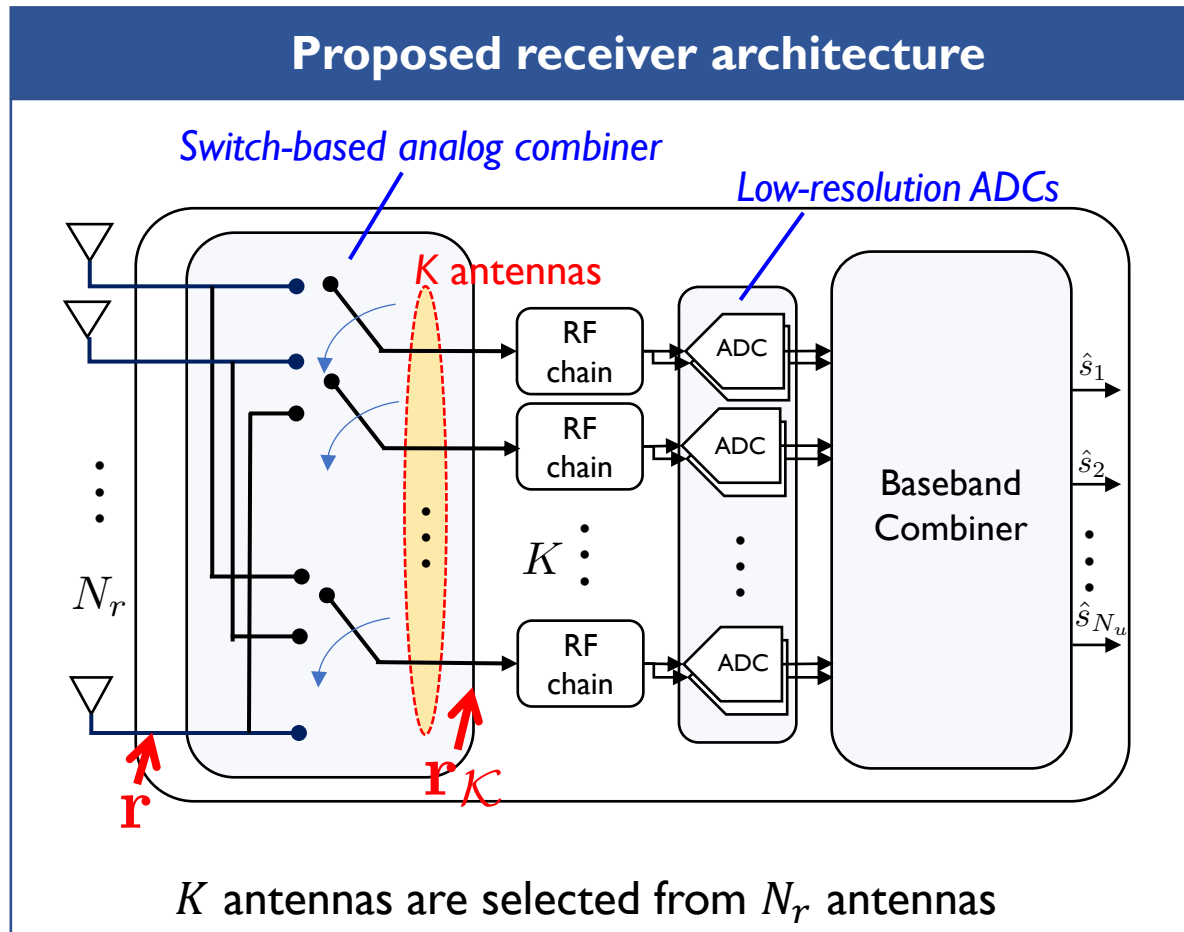


- Previous antenna selection algorithms – ignore quantization error ex. **\*FAS** [Gharavi-Alkhansari04], **\*\*ZF-TAS** [Lin&Tsai12] [Amadori15]

Develop antenna selection algorithm generalized for different quantization levels



# SYSTEM MODEL



## Multi-user MIMO uplink system

- Single cell environment
- $N_r$  Rx antennas
- Select  $K$  antennas
- Serve  $N_u \leq K$  users
- Single-antenna user

## Rx analog baseband signal vector

- Narrowband channel model

$$\mathbf{r} = \sqrt{\rho} \mathbf{H} \mathbf{s} + \mathbf{n}$$

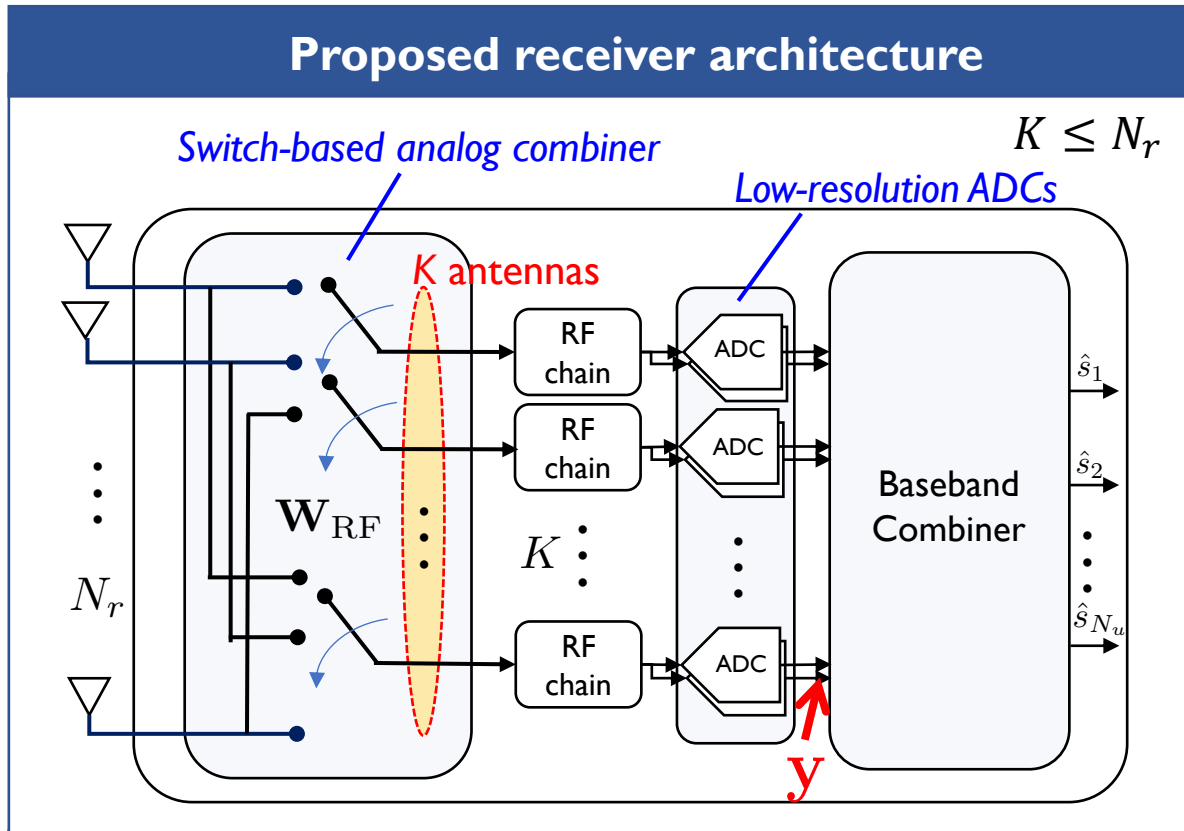
$\sqrt{\rho}$ : Average Tx power  
 $\mathbf{H}$ : Channel matrix  
 $\mathbf{s}$ : User symbols  
 $\mathbf{n}$ : AWGN vector

## Rx signal vector after antenna selection

$$\mathbf{r}_{\mathcal{K}} = \sqrt{\rho} \mathbf{H}_{\mathcal{K}} \mathbf{s} + \mathbf{n}_{\mathcal{K}}$$

$\mathcal{K}$ : Index set of selected antennas

# SYSTEM MODEL



**Mutual Information**

$$C(\mathbf{H}_K) = \log_2 \left| \mathbf{I}_K + p_u \alpha^2 (\alpha^2 \mathbf{I}_K + \mathbf{R}_{\mathbf{q}\mathbf{q}})^{-1} \mathbf{H}_K \mathbf{H}_K^H \right|$$

*Quantization noise variance (assumes Gaussian  $\mathbf{q}$ )*

## Quantized signal vector

- Linear approximation by using AQNM\*

$$\mathbf{y} = \mathcal{Q}(\text{Re}\{\mathbf{y}\}) + j\mathcal{Q}(\text{Im}\{\mathbf{y}\})$$

$$= \alpha \sqrt{p_u} \mathbf{H}_K \mathbf{s} + \alpha \mathbf{n}_K + \mathbf{q}$$

$r_K \rightarrow \triangleleft \alpha \rightarrow \oplus \rightarrow y$   
 $\uparrow$   
 $q$

*Linear quantization gain*      *Additive quantization noise*

- Linear quantization gain

$$\alpha = 1 - \beta < 1$$

$$\beta = \frac{\mathbb{E}[|y - y_q|^2]}{\mathbb{E}[|y|^2]} \approx \frac{\pi\sqrt{3}}{2} 2^{-2b}$$

- Quantization noise covariance

$$\mathbf{R}_{\mathbf{q}\mathbf{q}} = \alpha(1 - \alpha) \text{diag}(\rho \mathbf{H}_K \mathbf{H}_K^H + \mathbf{I})$$

*Diagonal matrix*

\*Additive quantization noise model

# PROBLEM FORMULATION

## □ Mutual information maximizing antenna selection

- Select antennas to maximize mutual information

$$C(\mathbf{H}_{\mathcal{K}^*}) = \max_{\mathcal{K} \subset \{1, \dots, N_r\} : |\mathcal{K}| = K} C(\mathbf{H}_{\mathcal{K}})$$

- Approaches



- Mutual information under coarse quantization

$$C(\mathbf{H}_{\mathcal{K}}) = \log_2 \left| \mathbf{I}_K + p_u \alpha^2 (\alpha^2 \mathbf{I}_K + \mathbf{R}_{\text{qq}})^{-1} \mathbf{H}_{\mathcal{K}} \mathbf{H}_{\mathcal{K}}^H \right|$$

**Quantization noise variance**  
 $\mathbf{R}_{\text{qq}} = \alpha(1 - \alpha) \text{diag}(p_u \mathbf{H}_{\mathcal{K}} \mathbf{H}_{\mathcal{K}}^H + \mathbf{I}_K)$

Antenna selection measure needs to consider quantization error

# GENERALIZED GREEDY SELECTION MEASURE

## Greedy mutual information maximization solution

- At  $(n+1)$ th antenna selection

$$\mathbf{D}_{n+1} = \text{diag}(1 + p_u(1 - \alpha)\|\mathbf{f}_{\mathcal{K}(i)}\|^2) \quad \mathbf{f}_i^H: i\text{th row of } \mathbf{H}$$

$$C(\mathbf{H}_{n+1}) = \log_2 \left| \mathbf{I}_K + p_u \alpha \mathbf{D}_{n+1}^{-1} \mathbf{H}_{n+1} \mathbf{H}_{n+1}^H \right|$$

$$= C(\mathbf{H}_n) + \log_2 \left( 1 + \frac{p_u \alpha}{d_{\mathcal{K}(n+1)}} c_{\mathcal{K}(n+1),n} \right)$$

Matrix determinant lemma  
 $|\mathbf{A} + \mathbf{u}\mathbf{v}^H| = |\mathbf{A}|(1 + \mathbf{v}^H \mathbf{A}^{-1} \mathbf{u})$

Function of previous antennas

Functions of  $(n+1)$ th antenna

Mutual information gain
$c_{\mathcal{K}(n+1),n} = \mathbf{f}_{\mathcal{K}(n+1)}^H \left( \mathbf{I} + p_u \alpha \mathbf{H}_n^H \mathbf{D}_n^{-1} \mathbf{H}_n \right)^{-1} \mathbf{f}_{\mathcal{K}(n+1)}$
Mutual information increase from selecting $(n+1)$ th antenna

vs.

Quantization error penalty
$d_{\mathcal{K}(n+1)} = 1 + p_u(1 - \alpha)\ \mathbf{f}_{\mathcal{K}(n+1)}\ ^2$
Quantization error penalty for selecting $(n+1)$ th antenna



Selection
$J = \arg \max_j \frac{c_{j,n}}{d_j}$
Generalized FAS <small>[Gharavi-Alkhansari04]</small>
Gain vs. Penalty

# QUANTIZATION-AWARE FAST ANTENNA SELECTION

## Complexity reduction

- Matrix inversion is involved in MI gain

$$c_{j,n} = \mathbf{f}_j^H \left( \mathbf{I} + p_u \alpha \mathbf{H}_n^H \mathbf{D}_n^{-1} \mathbf{H}_n \right)^{-1} \mathbf{f}_j$$

Matrix inversion requires high complexity

- Simplifying MI gain update

$$\begin{aligned} \mathbf{Q}_{n+1} &= \left( \mathbf{I} + p_u \alpha \mathbf{H}_{n+1}^H \mathbf{D}_{n+1}^{-1} \mathbf{H}_{n+1} \right)^{-1} \\ &= \mathbf{Q}_n - \mathbf{a} \mathbf{a}^H \end{aligned}$$

Matrix inversion lemma

where  $\mathbf{a} = \left( c_{J,n} + \frac{d_J}{p_u \alpha} \right)^{-1/2} \mathbf{Q}_n \mathbf{f}_J$  and  $\mathbf{Q}_0 = \mathbf{I}_{N_u}$

$$\begin{aligned} c_{j,n+1} &= \mathbf{f}_j^H \mathbf{Q}_{n+1} \mathbf{f}_j = \mathbf{f}_j^H (\mathbf{Q}_n - \mathbf{a} \mathbf{a}^H)^H \mathbf{f}_j \\ &= c_{j,n} - |\mathbf{f}_j^H \mathbf{a}|^2 \end{aligned}$$

Matrix inversion lemma

$$(A + UCV)^{-1} = A^{-1} - A^{-1}U(C^{-1} + VA^{-1}U)^{-1}VA^{-1}$$

## Proposed algorithm

### Quantization-Aware Fast Antenna Selection (QAFAS)

- 1) Initialize:  $\mathcal{T} = \{1, \dots, N_r\}$  and  $\mathbf{Q} = \mathbf{I}$ .
- 2) Initialize antenna gain and compute penalty:  
 $c_j = \|\mathbf{f}_j\|^2$  and  $d_j = 1 + \rho(1 - \alpha)\|\mathbf{f}_j\|^2$  for  $j \in \mathcal{T}$ .
- 3) Select antenna  $J = \operatorname{argmax}_{j \in \mathcal{T}} c_j/d_j$
- 4) Update candidate set:  $\mathcal{T} = \mathcal{T} \setminus \{J\}$ .
- 5) Compute:  $\mathbf{a} = \left( c_J + \frac{d_J}{\rho\alpha} \right)^{-1/2} \mathbf{Q} \mathbf{f}_J$  and  $\mathbf{Q} = \mathbf{Q} - \mathbf{a} \mathbf{a}^H$ .
- 6) Update  $c_j = c_j - |\mathbf{f}_j^H \mathbf{a}|^2$  for  $j \in \mathcal{T}$ .
- 7) Go to step 3 and repeat until select  $K$  antennas.

Complexity

$\mathcal{O}(K N_r N_u)$

# Rate-Maximization Antenna Selection

## □ Asymptotic complexity analysis

- Complexity for step 5:  $O(K N_u^2)$ 
  - $K$  iterations x Inner product  $\mathbf{Q}\mathbf{f}_J$
- Complexity for step 6:  $O(K N_r N_u)$ 
  - $K$  iterations
  - x  $N_r$  updates
  - x Inner product  $\mathbf{f}_j^H \mathbf{a}$
- Large antenna arrays ( $N_r \gg N_u$ )

Overall complexity becomes  $O(K N_r N_u)$

## □ Proposed algorithm

### Quantization-Aware Fast Antenna Selection (QAFAS)

- 1) Initialize:  $\mathcal{T} = \{1, \dots, N_r\}$  and  $\mathbf{Q} = \mathbf{I}$ .
- 2) Initialize antenna gain and compute penalty:  
 $c_j = \|\mathbf{f}_j\|^2$  and  $d_j = 1 + \rho(1 - \alpha)\|\mathbf{f}_j\|^2$  for  $j \in \mathcal{T}$ .
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- 4) Update candidate set:  $\mathcal{T} = \mathcal{T} \setminus \{J\}$ .
- 5) Compute:  $\mathbf{a} = (c_J + \frac{d_J}{\rho\alpha})^{-\frac{1}{2}} \mathbf{Q}\mathbf{f}_J$  and  $\mathbf{Q} = \mathbf{Q} - \mathbf{a}\mathbf{a}^H$ .
- 6) Update  $c_j = c_j - |\mathbf{f}_j^H \mathbf{a}|^2$  for  $j \in \mathcal{T}$ .
- 7) Go to step 3 and repeat until select  $K$  antennas.

Complexity

$O(K N_r N_u)$

same as FAS [Gharavi-Alkhansari04]



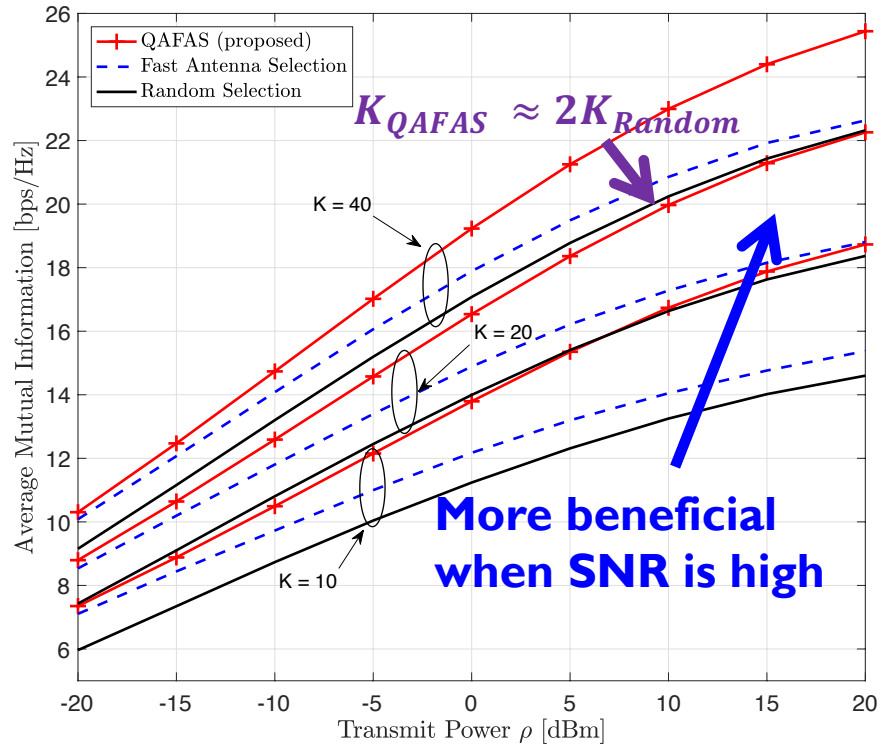
# SIMULATION – SUB-6GHz SYSTEM

$K$ : # of selected antennas

## Avg. mutual information vs. Transmit power

Figure 1

(3 quantization bits)



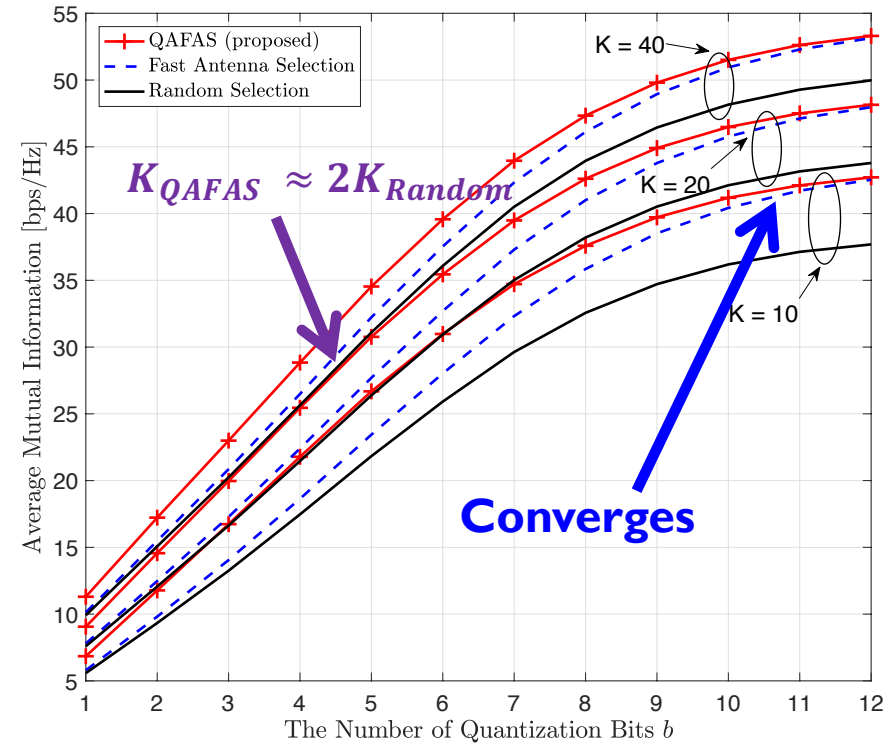
Why?

Quantization error dominates thermal noise : QAFAS is effective under quantization error

## Avg. mutual information vs. # of ADC bits

Figure 2

(5 dBm Tx power)



Why?

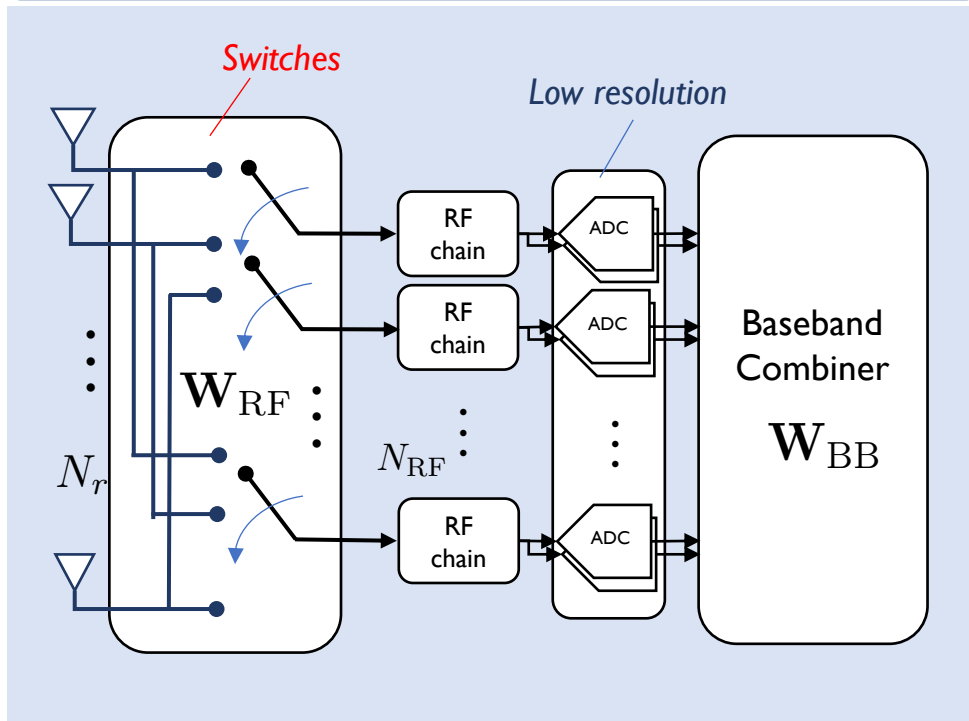
Quantization error becomes negligible : generalization of FAS algorithm

Settings

128 antennas, 2.4 GHz carrier frequency, 10 MHz bandwidth, 10 users, 2km cell radius, 100m min. user distance to BS

# SUMMARY OF CONTRIBUTIONS

## Switch-based hybrid low-res. ADC system



## Generalized selection measure

$$J = \arg \max_j \frac{c_{j,n}}{d_j}$$

Tradeoff between **mutual information gain** and **quantization penalty**

## Quantization-aware FAS

- 1) Initialize:  $\mathcal{T} = \{1, \dots, N_r\}$  and  $\mathbf{Q} = \mathbf{I}_{N_u}$ .
- 2) Initialize antenna gain and compute penalty:  
 $c_j = \|\mathbf{f}_j\|^2$  and  $d_j = 1 + \rho(1 - \alpha)\|\mathbf{f}_j\|^2$  for  $j \in \mathcal{T}$ .
- 3) Select antenna :  $J = \operatorname{argmax}_{j \in \mathcal{T}} c_j/d_j$ .
- 4) Update candidate set:  $\mathcal{T} = \mathcal{T} \setminus \{J\}$ .
- 5) Compute:  $\mathbf{a} = (c_J + \frac{d_J}{\rho\alpha})^{-\frac{1}{2}} \mathbf{Q}\mathbf{f}_J$  and  $\mathbf{Q} = \mathbf{Q} - \mathbf{a}\mathbf{a}^H$ .
- 6) Update  $c_j = c_j - |\mathbf{f}_j^H \mathbf{a}|^2$  for  $j \in \mathcal{T}$ .
- 7) Go to step 3 and repeat until select  $K$  antennas.

**BACK UP SLIDES**

# GREEDY MI-MAXIMIZING ANTENNA SELECTION

## □ Mutual information expression

quantization noise covariance

$$\mathbf{R}_{\text{qq}} = \alpha(1 - \alpha) \text{diag}(\rho \mathbf{H}_{\mathcal{K}} \mathbf{H}_{\mathcal{K}}^H + \mathbf{I})$$

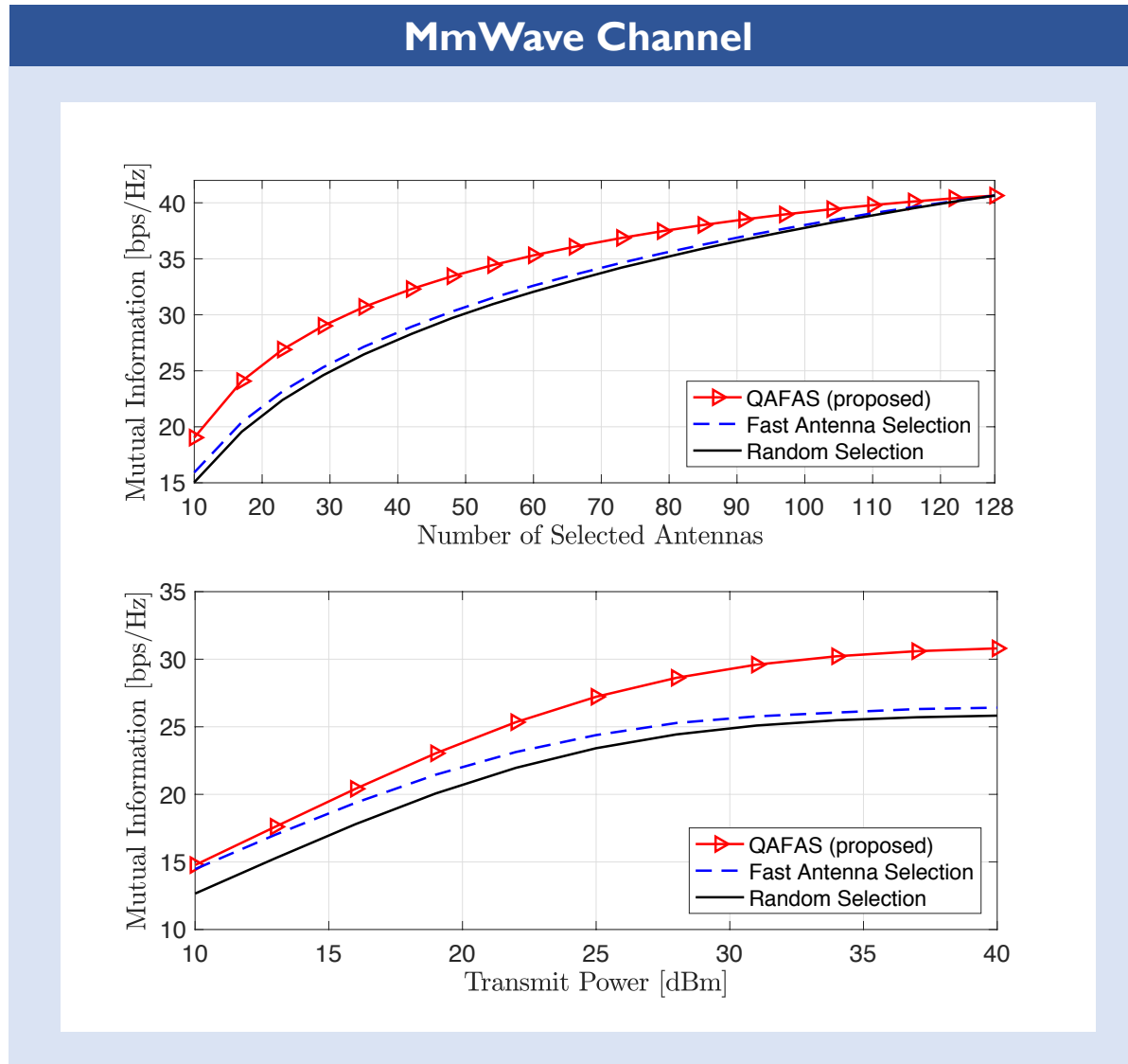
$$\begin{aligned} \blacksquare C(\mathbf{H}_{\mathcal{K}}) &= \log_2 \left| \mathbf{I} + \rho \alpha^2 (\alpha^2 \mathbf{I} + \mathbf{R}_{\text{qq}})^{-1} \mathbf{H}_{\mathcal{K}} \mathbf{H}_{\mathcal{K}}^H \right| \\ &= \log_2 \left| \mathbf{I} + \rho \alpha \mathbf{D}_{\mathcal{K}}^{-1} \mathbf{H}_{\mathcal{K}} \mathbf{H}_{\mathcal{K}}^H \right| \quad \text{where } \mathbf{D}_{\mathcal{K}} = \text{diag}(1 + \rho(1 - \alpha) \|\mathbf{f}_{\mathcal{K}(i)}\|^2) \end{aligned}$$

## ■ At (n+1)th selection stage

$$\begin{aligned} C(\mathbf{H}_{n+1}) &= \log_2 \left| \mathbf{I} + \rho \alpha \mathbf{D}_{n+1}^{-1} \mathbf{H}_{n+1} \mathbf{H}_{n+1}^H \right| \\ &= \log_2 \left| \mathbf{I} + \rho \alpha \mathbf{H}_{n+1}^H \mathbf{D}_{n+1}^{-1} \mathbf{H}_{n+1} \right| \\ &= \log_2 \left| \mathbf{I} + \rho \alpha \left( \mathbf{H}_n^H \mathbf{D}_n^{-1} \mathbf{H}_n + \frac{1}{d_{\mathcal{K}(n+1)}} \mathbf{f}_{\mathcal{K}(n+1)} \mathbf{f}_{\mathcal{K}(n+1)}^H \right) \right| \\ &\stackrel{(a)}{=} C(\mathbf{H}_n) + \log_2 \left( 1 + \frac{\rho \alpha}{d_{\mathcal{K}(n+1)}} c_{\mathcal{K}(n+1),n} \right) \quad \text{where } c_{\mathcal{K}(n+1),n} = \mathbf{f}_{\mathcal{K}(n+1)}^H \left( \mathbf{I} + \rho \alpha \mathbf{H}_n^H \mathbf{D}_n^{-1} \mathbf{H}_n \right)^{-1} \mathbf{f}_{\mathcal{K}(n+1)} \end{aligned}$$

where (a) comes from **matrix determinant lemma**  $|\mathbf{A} + \mathbf{u}\mathbf{v}^H| = |\mathbf{A}|(1 + \mathbf{v}^H \mathbf{A}^{-1} \mathbf{u})$

# SIMULATION – MILLIMETER WAVE SYSTEM



- ### Settings
- 256 antennas
  - 52 selected antennas
  - 3 channel paths
  - 8 users
  - 28 GHz carrier freq.
  - 200 MHz BW
  - 200m cell radius,
  - 20m min. user distance
  - 3 ADC bits