

CLUSTERING-GUIDED GP-UCB FOR BAYESIAN OPTIMIZATION

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Introduction and Motivation

- Bayesian optimization is a powerful method for finding extrema of an objective function [1, 3].
- One of acquisition functions, GP-upper confidence bound (GP-UCB) determines where next to sample from the true function, balancing exploration and exploitation.
- We first present a geometric interpretation of GP-UCB.
- We develop GP-UCB to clustering-guided method, called as clustering-guided GP-UCB (CG-GPUCB).

Background

- Bayesian optimization [1]
- ✓ Global optimization for black-box function.

Algorithm 1 Bayesian Optimization with GP regression

Require: Initial data $\mathcal{D}_{1:T} = \{(\mathbf{x}_1, y_1), \dots, (\mathbf{x}_T, y_T)\}$, and $T \in \mathbb{N} > 0$

- 1: **for** $t = 1, 2, \dots, T$ **do**
- 2: Find \mathbf{x}_{I+t} that maximizes the acquisition function over the current GP: $\mathbf{x}_{I+t} = \arg \max_{\mathbf{x}} a(\mathbf{x} | \mathcal{D}_{1:I+t-1})$.
- 3: Sample the objective function: $y_{I+t} = f(\mathbf{x}_{I+t}) + \epsilon_{I+t}$.
- 4: Augment the data: $\mathcal{D}_{1:I+t} = \{\mathcal{D}_{1:I+t-1}, (\mathbf{x}_{I+t}, y_{I+t})\}$.
- 5: Update the GP, computing $\mu_{I+t}(\mathbf{x}), \sigma_{I+t}^2(\mathbf{x})$:
- 6: **end for**
- 7: **return** $\mathbf{x}^* = \arg \max_{\mathbf{x} \in \{\mathbf{x}_1, \dots, \mathbf{x}_{I+T}\}} \mu_{I+T}(\mathbf{x})$

- GP-UCB [2]
- ✓ Linear combination of posterior mean and standard deviation function.
- ✓ Trade-off hyperparameter controls tightness of confidence bound.

$$a(\mathbf{x} | \mathcal{D}_{1:t}) = -\mu(\mathbf{x}) + \kappa \sigma(\mathbf{x})$$

Selected References

- [1] E. Brochu, V. M. Cora, and N. de Freitas, "A tutorial on Bayesian optimization of expensive cost functions, with application to active user modeling and hierarchical reinforcement learning," 2010, arXiv preprint arXiv:1012.2599.
- [2] N. Srinivas, A. Krause, S. Kakade, and M. Seeger, "Gaussian process optimization in the bandit setting: No regret and experimental design," in Proceedings of the International Conference on Machine Learning (ICML), Haifa, Israel, 2010, pp. 1015–1022.
- [3] J. Snoek, H. Larochelle, and R. P. Adams, "Practical Bayesian optimization of machine learning algorithms," in Advances in Neural Information Processing Systems (NIPS), Lake Tahoe, NV, USA, 2012, vol. 25, pp. 2951–2959.

Experimental Results

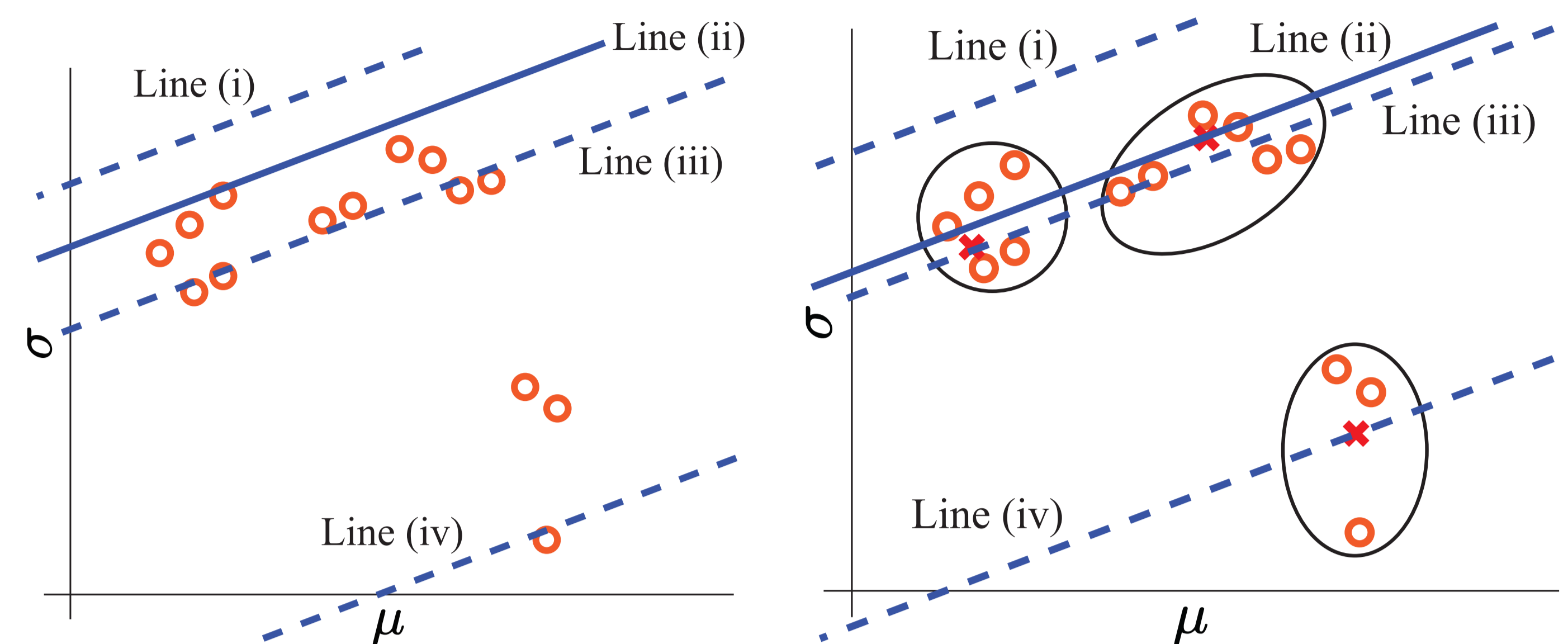
- Our method was conducted on synthetic function, benchmarks, and hyperparameter optimization (HPO).
- We showed the effect of hyperparameters for CG-GPUCB-NN and CG-GPUCB².
- HPO for logistic regression (LR) and deep convolutional networks (DCN) was tested.

Conclusion

- We presented our own geometric interpretation of GP-UCB and new acquisition function, CG-GPUCB.
- Our method outperformed in non-smooth function with sharp peaks and valleys.

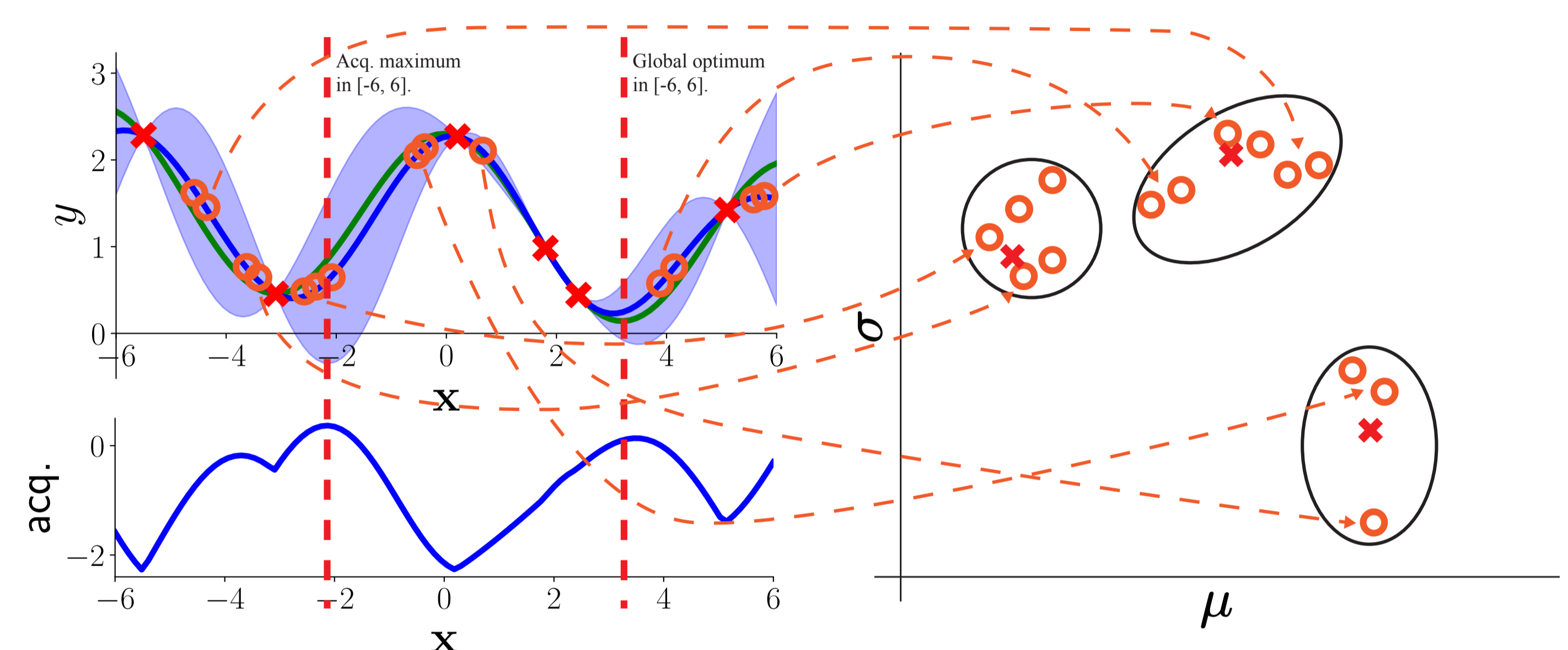
Clustering-Guided GP-UCB

- Geometric view of GP-UCB



$$\sigma(\mathbf{x}) = -\frac{1}{\kappa} (\mu(\mathbf{x}) + a(\mathbf{x} | \mathcal{D}_{1:t}))$$

- Proposed method: Clustering-guided GP-UCB



- ✓ The points on posterior mean-standard deviation space are grouped, and a query point is selected on the chosen cluster.
- ✓ We select a query point using one of two criteria, CG-GPUCB-NN and CG-GPUCB².

$$\mathbf{x}_{t+1} = \arg \min_{\mathbf{x} \in \mathcal{C}_{i^*}} \|\mu(\mathbf{x}), \sigma(\mathbf{x})\|^T - \mathbf{c}_{i^*}\|_2^2$$

$$\mathbf{x}_{t+1} = \arg \max_{\mathbf{x} \in \mathcal{C}_{i^*}} a(\mathbf{x} | \mathcal{D}_{1:t})$$

Algorithm 2 Bayesian Optimization with CG-GPUCB

Require: Initial data $\mathcal{D}_{1:T} = \{(\mathbf{x}_1, y_1), \dots, (\mathbf{x}_T, y_T)\}$, $T \in \mathbb{N} > 0$, and $K \in \mathbb{N} > 0$ (number of clusters)

- 1: **for** $t = 1, 2, \dots, T$ **do**
- 2: Calculate centers \mathbf{c}_i of K clusters determined in the μ - σ space, given the current GP.
- 3: Find the best cluster \mathcal{C}_{i^*} via (9).
- 4: Find \mathbf{x}_{I+t} by (10) or (11).
- 5: Sample the objective function: $y_{I+t} = f(\mathbf{x}_{I+t}) + \epsilon_{I+t}$.
- 6: Augment the data: $\mathcal{D}_{1:I+t} = \{\mathcal{D}_{1:I+t-1}, (\mathbf{x}_{I+t}, y_{I+t})\}$.
- 7: Update the GP via (4) & (5).
- 8: **end for**

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