COOPERATIVE TRACKING USING MARGINAL DIFFUSION PARTICLE FILTERS

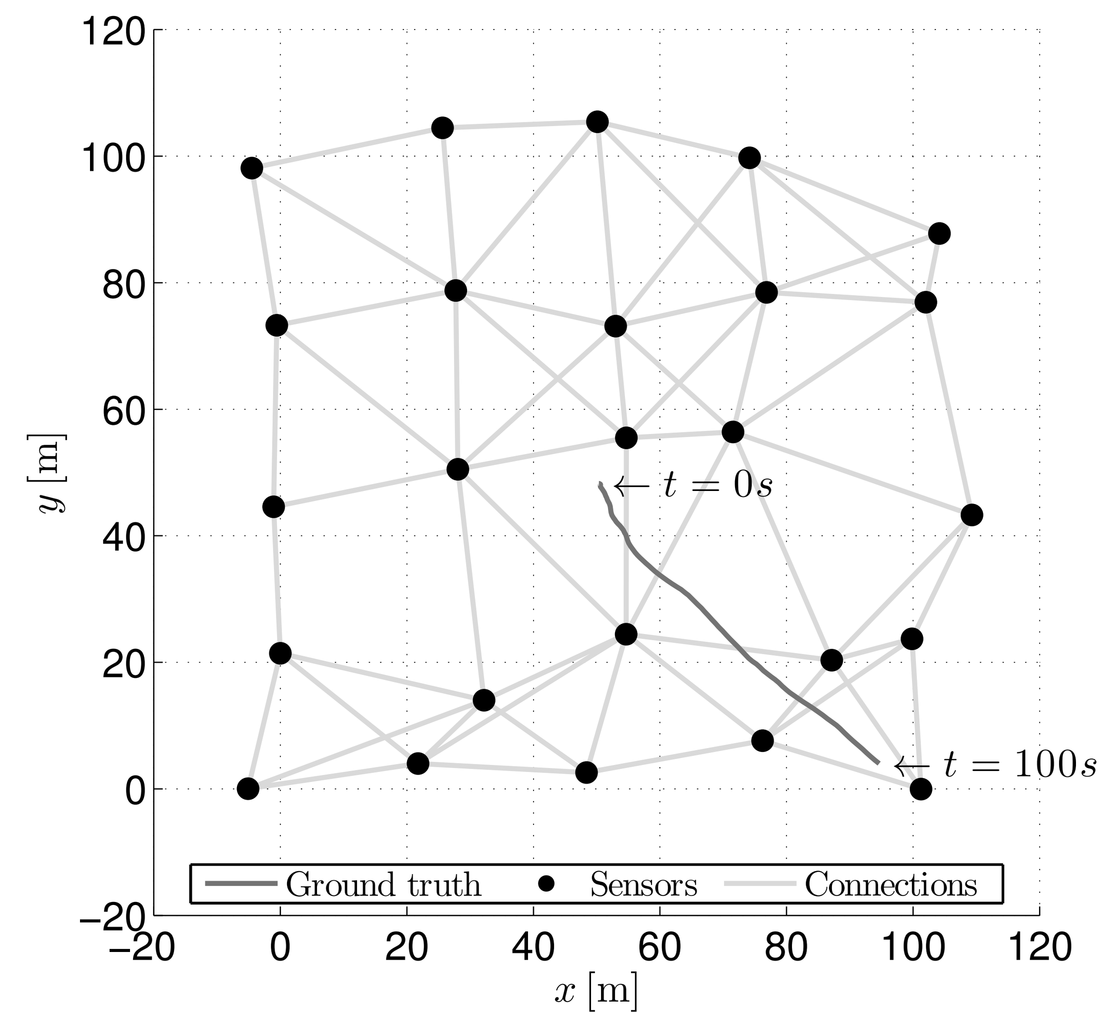
Marcelo G. S. Bruno[1] (bruno@ita.br) and Stiven S. Dias[2] (stiven.dias@embraer.com.br)

[1]Instituto Tecnológico de Aeronáutica, São José dos Campos, Brazil [2]Embraer S.A., São José dos Campos, Brazil

PROBLEM FORMULATION

▶ Evaluated scenario

- R sensors dispersed over a partially connected network;
- A single target is moving within the sensors field.



▶ The target state transition equation is given by

$$\boxed{\mathbf{x}_{n+1} = \mathbf{f}_n(\mathbf{x}_n) + \mathbf{G}_n \mathbf{u}_n}$$

where $\mathbf{u}_n \sim N(\mathbf{0}, \mathbf{Q}_n)$ and $\mathbf{f}_n(\cdot)$ is an arbitrary (possibly nonlinear) function known for all $n \geq 0$.

▶ The measurement available at network location $r \in \mathcal{V} \triangleq \{1, 2, \ldots, R\}$ is defined as

$$\boxed{\mathbf{y}_{n,r} = \mathbf{h}_{n,r}(\mathbf{x}_n) + \mathbf{v}_{n,r}}$$

where $\mathbf{v}_{n,r} \sim N(\mathbf{0}, \mathbf{R}_{n,r})$ and $\mathbf{h}_{n,r}(\cdot)$ is an arbitrary function known for all $n \geq 0$ and all indexes $r \in \mathcal{V}$.

▶ Cooperative/distributed tracking

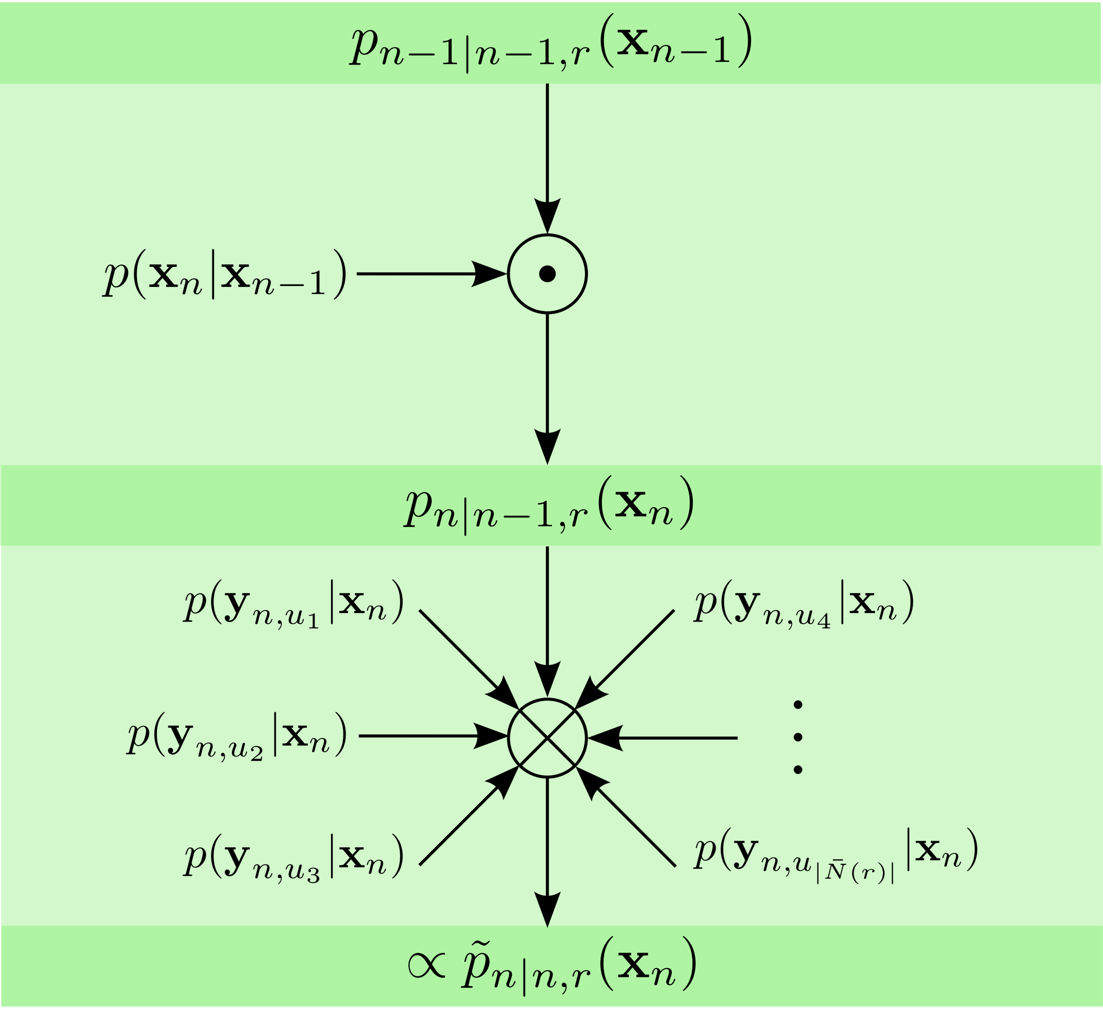
- Nodes cooperate with each other via message passing to track the kinematic state sequence $\{\mathbf{x}_n\}$, $n \geq 1$, in a fully distributed fashion;
- Each node r can only access observations $\{\mathbf{y}_{n,\ell}\}$, $\ell \in \bar{\mathbf{N}}(r)$, in its *closed neighborhood* $\bar{\mathbf{N}}(r)$.

Goal

Derive fully distributed algorithms that recursively estimate $\mathbf{x}_n$ at instant n given $\mathbf{y}_{0:n,1:R}$ defined as the vector that collects the observations $\mathbf{y}_{k,r}$.

ATC DIFFUSION FILTER

▶ Adapt Step



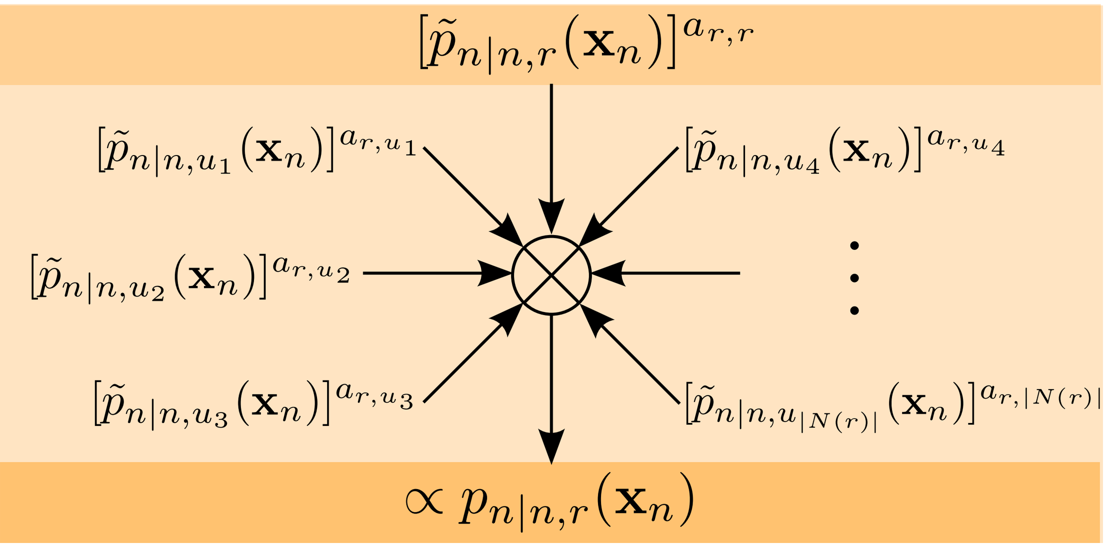
- Prediction:

$$\boxed{p_{n|n-1,r}(\mathbf{x}_n) = \int_{\Re^N} p(\mathbf{x}_n|\mathbf{x}_{n-1})\, p_{n-1|n-1,r}(\mathbf{x}_{n-1})\, d\mathbf{x}_{n-1}}$$

- Update:

$$\boxed{\tilde{p}_{n|n,r}(\mathbf{x}_n) \propto \left[\prod_{u \in \bar{N}(r)} p(\mathbf{y}_{n,u}|\mathbf{x}_n)\right] p_{n|n-1,r}(\mathbf{x}_n)}$$

▶ Combine Step



- Kullback-Leibler fusion rule:

$$\boxed{p_{n|n,r}(\mathbf{x}_n) \propto \prod_{u \in \bar{N}(r)} [\tilde{p}_{n|n,u}(\mathbf{x}_n)]^{a_{r,u}}}$$

which minimizes

$$\sum_{u \in \bar{N}(r)} a_{r,u} D_{KL}(p^\star || \tilde{p}_{n|n,u})$$

over all possible p.d.f.'s $p^\star$, where $\sum_{u \in \bar{N}(r)} a_{r,u} = 1$, $\forall r \in \mathcal{V}$.

ATC MARGINAL DIFFUSION PF

▶ Marginal Particle Filter Adapt Step

Assume that the posterior p.d.f. $p_{n-1|n-1,r}$ at node r at instant $n-1$ is approximated in a Monte Carlo sense by

$$p_{n-1|n-1,r}(\mathbf{x}_{n-1}) \approx \sum_{j=1}^{J} w_{n-1,r}^{(j)}\, \delta(\mathbf{x}_{n-1} - \mathbf{x}_{n-1,r}^{(j)}).$$

Then, the new set of weighted samples representing the posterior p.d.f. $p_{n|n,r}$ at instant n is given by

$$\boxed{\tilde{\mathbf{x}}_{n,r}^{(j)} \sim \pi_{n|n-1,r}(\mathbf{x}_n)}$$

$$\boxed{\tilde{w}_{n,r}^{(j)} \propto \left[\prod_{u \in \bar{N}(r)} p(\mathbf{y}_{n,u}|\tilde{\mathbf{x}}_{n,r}^{(j)})\right] \frac{\sum_{l=1}^{J} w_{n-1,r}^{(l)}\, p(\tilde{\mathbf{x}}_{n,r}^{(j)}|\mathbf{x}_{n-1,r}^{(l)})}{\pi_{n|n-1,r}(\tilde{\mathbf{x}}_{n,r}^{(j)})}}$$

for $j \in \{1, \ldots, J\}$, where the proposal p.d.f.

$$\boxed{\pi_{n|n-1,r}(\mathbf{x}_n) = \mathcal{N}(\mathbf{x}_n|\ \mathbf{m}_{n|n-1,r}, \boldsymbol{\Sigma}_{n|n-1,r})}$$

is based on an EKF Gaussian posterior such that

$$\mathbf{m}_{n|n-1,r} = \mathbf{f}_{n-1}(\hat{\mathbf{x}}_{n-1|n-1,r})$$
$$\boldsymbol{\Sigma}_{n|n-1,r} = \tilde{\mathbf{F}}_{n-1}\mathbf{P}_{n-1|n-1,r}\tilde{\mathbf{F}}_{n-1}^T + \mathbf{G}_{n-1}\mathbf{Q}_{n-1}\mathbf{G}_{n-1}^T.$$

▶ Combine Step

We create first a Gaussian approximation at each node $r \in \mathcal{V}$

$$\boxed{\tilde{p}_{n|n,r}(\mathbf{x}_n) \approx \mathcal{N}(\mathbf{x}_n|\ \tilde{\mathbf{x}}_{n|n,r}, \tilde{\mathbf{P}}_{n|n,r})}$$

with

$$\tilde{\mathbf{x}}_{n|n,r} = \sum_{l=1}^{J} \tilde{w}_{n,r}^{(l)}\, \tilde{\mathbf{x}}_{n|n,r}^{(l)}$$

$$\tilde{\mathbf{P}}_{n|n,r} = \sum_{l=1}^{J} \tilde{w}_{n,r}^{(l)} (\mathbf{x}_{n|n,r}^{(l)} - \tilde{\mathbf{x}}_{n|n,r})(\mathbf{x}_{n|n,r}^{(l)} - \tilde{\mathbf{x}}_{n|n,r})^T.$$

Thus, the merged p.d.f. $p_{n|n,r}$ at node r is also Gaussian

$$\boxed{p_{n|n,r}(\mathbf{x}_n) \approx \mathcal{N}(\mathbf{x}_n|\ \hat{\mathbf{x}}_{n|n,r}, \mathbf{P}_{n|n,r})}$$

with fused covariance matrix and mean vector given by

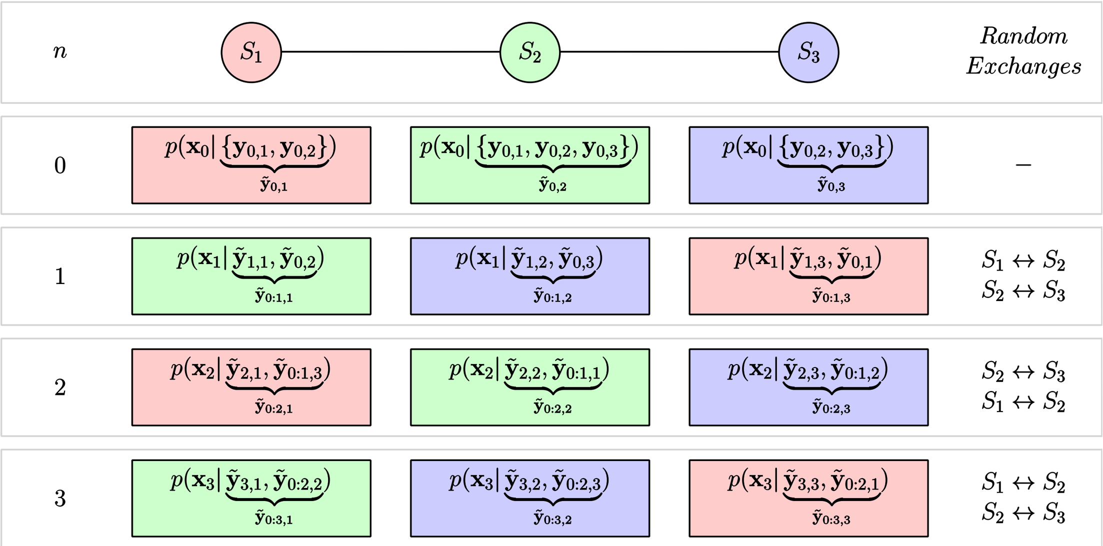
$$(\mathbf{P}_{n|n,r})^{-1} = \sum_{u \in \bar{N}(r)} a_{r,u} (\tilde{\mathbf{P}}_{n|n,u})^{-1}$$

$$\hat{\mathbf{x}}_{n|n,r} = \mathbf{P}_{n|n,r} \left[\sum_{u \in \bar{N}(r)} a_{r,u} (\tilde{\mathbf{P}}_{n|n,u})^{-1}\, \tilde{\mathbf{x}}_{n|n,u}\right].$$

Finally, node r resamples $\mathbf{x}_{n,r}^{(j)} \sim \mathcal{N}(\mathbf{x}_n|\ \hat{\mathbf{x}}_{n|n,r}, \mathbf{P}_{n|n,r})$ and resets $w_{n,r}^{(j)} = 1/J$ for $j \in \{1, \ldots, J\}$.

MARGINAL RNDEX DIFFUSION PF

▶ Random Exchange Diffusion Step

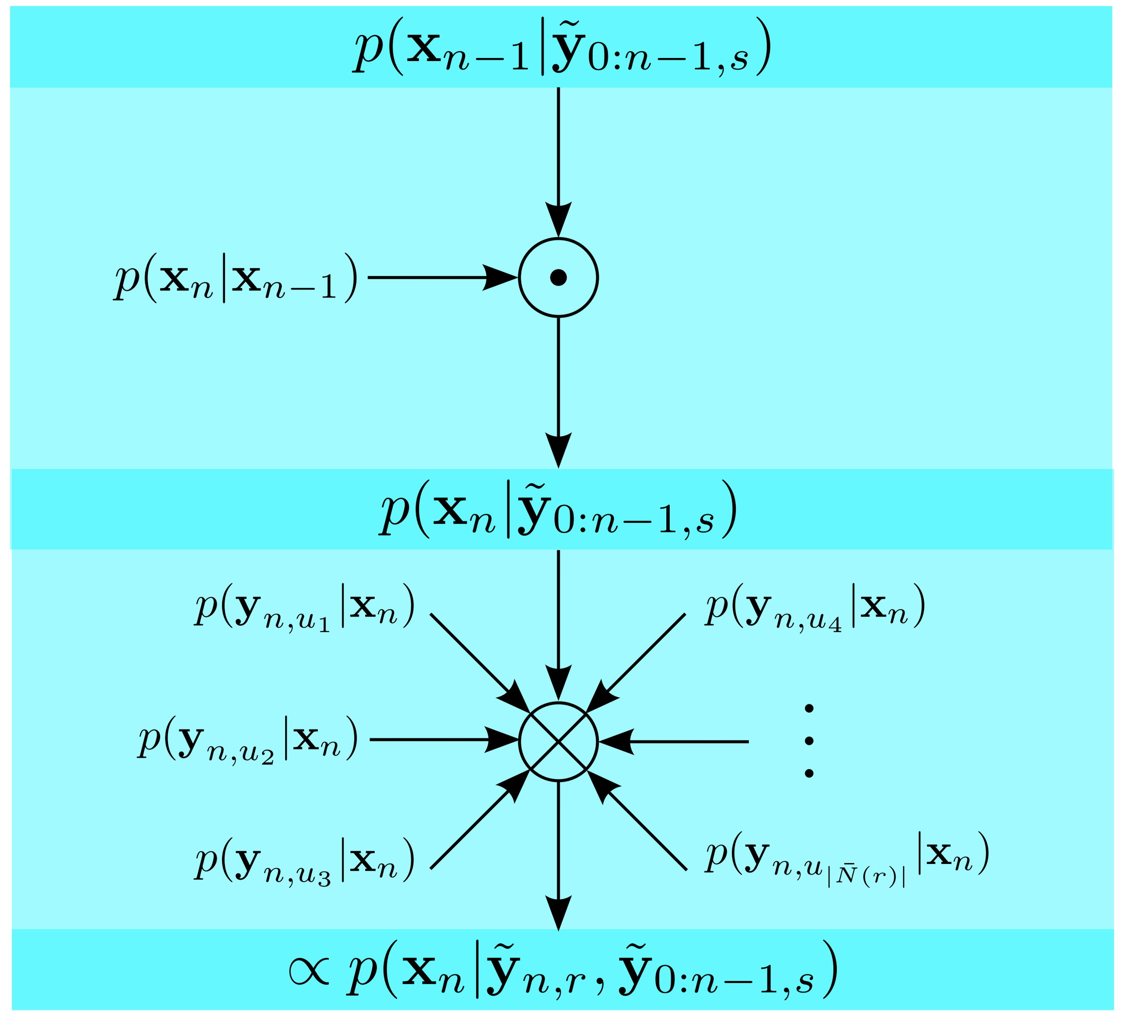


- Node r receives from node s a Gaussian approximation at instant $n-1$ of the posterior p.d.f. $p(\mathbf{x}_{n-1}|\tilde{\mathbf{y}}_{0:n-1,s})$;
- Then, node r resamples $\mathbf{x}_{n-1,s}^{(j)} \sim \mathcal{N}(\mathbf{x}_{n-1}|\ \hat{\mathbf{x}}_{n-1|n-1,s}, \mathbf{P}_{n-1|n-1,s})$ and set $w_{n-1,s}^{(j)} = 1/J$ for $j \in \{1, \ldots, J\}$.

Remark

At the end of the random node path $\{l_0, l_1, \ldots, l_n\}$, node l_n has a Monte Carlo representation of $p(\mathbf{x}_n|\tilde{\mathbf{y}}_{0,l_0}\tilde{\mathbf{y}}_{1,l_1} \ldots \tilde{\mathbf{y}}_{n,l_n})$.

▶ Local Data Fusion Step



$$\boxed{\mathbf{x}_{n,r}^{(j)} \sim \pi_{n|n-1,s}(\mathbf{x}_n) \triangleq \mathcal{N}(\mathbf{x}_n|\ \mathbf{m}_{n|n-1,s}, \boldsymbol{\Sigma}_{n|n-1,s})}$$

$$\boxed{w_{n,r}^{(j)} \propto \left[\prod_{u \in \bar{N}(r)} p(\mathbf{y}_{n,u}|\mathbf{x}_{n,r}^{(j)})\right] \frac{\sum_{l=1}^{J} w_{n-1,s}^{(l)}\, p(\mathbf{x}_{n,r}^{(j)}|\mathbf{x}_{n-1,s}^{(l)})}{\pi_{n|n-1,s}(\mathbf{x}_{n,r}^{(j)})}}$$

SIMULATION EXAMPLE

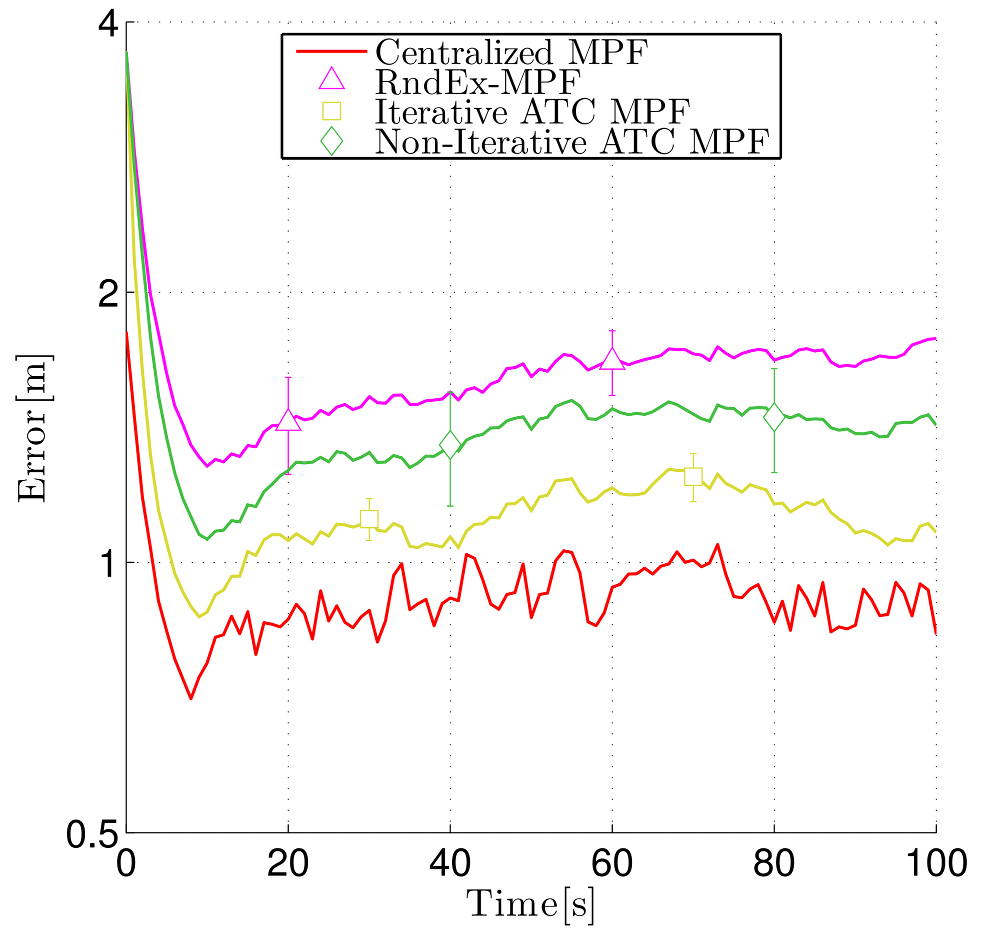
▶ The random sequence $\{\mathbf{x}_n\}$ evolves in time according to the linear, white-noise acceleration model

$$\boxed{\mathbf{x}_{n+1} = \mathbf{F}\,\mathbf{x}_n + \mathbf{u}_n.}$$

▶ At each instant n, $R = 25$ RSS sensors record dBm measurements $\{y_{n,r}\}$ at each network location r such that

$$\boxed{y_{n,r} = \underbrace{P_0 - 10\zeta_r \log_{10}\left(\frac{||\mathbf{H}\,\mathbf{x}_n - \mathbf{x}_r||}{d_0}\right)}_{h_{n,r}(\mathbf{x}_n)} + v_{n,r}.}$$

▶ Empirical RMS position estimate error



▶ Communication and processing performances

Evaluated Algorithm	RX Rate	TX Rate	Duty Cycle
RndEx-MPF	148 B/s	132 B/s	2.7 %
Iterative ATC MPF	2.9 KB/s	604 B/s	2.8 %
Non-Iterative ATC MPF	317 B/s	64 B/s	2.4 %

REFERENCES

- Bruno, M. G. S. and Dias, S. S., "**A Bayesian Interpretation of Distributed Diffusion Filtering Algorithms**," *IEEE Signal Processing Magazine*, 2018 (accepted).
- Dias, S. S. and Bruno, M. G. S., "**Performance Bounds for Cooperative RSS Emitter Tracking Using Diffusion Particle Filters**," *25th European Signal Processing Conference*, 2017.
- Dias, S. S. and Bruno, M. G. S., "**Distributed Bernoulli Filters for Joint Detection and Tracking in Sensor Networks**," *IEEE Transactions on Signal and Information Processing over Networks*, vol. 2, no. 3, pp. 260–275, September 2016.