# INSENSE: INCOHERENT SENSOR SELECTION FOR SPARSE SIGNALS

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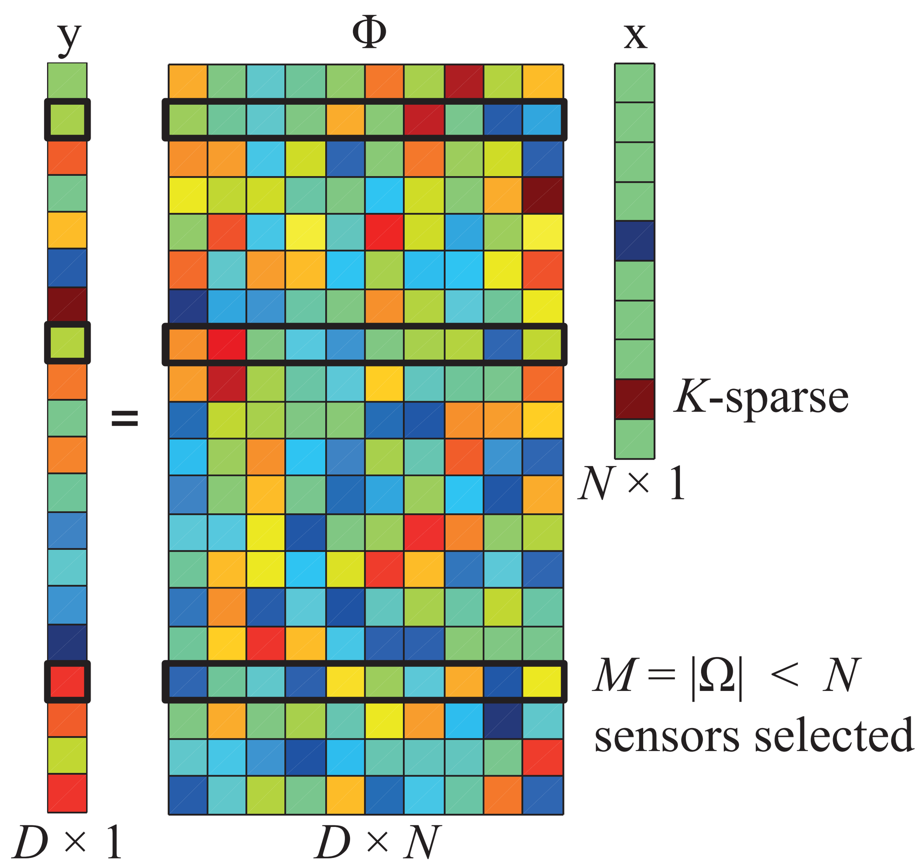
## Abstract


- Sensor selection: Intelligently selecting a **small subset** of a collection of available sensors
- The majority of sensor selection algorithms find the subset of sensors that best recovers an **arbitrary signal** from a number of linear measurements that is **larger than the dimension** of the signal
- We develop a **new sensor selection algorithm** for **sparse signals** that finds a subset of sensors that best recovers such signals from a number of measurements that is **much smaller than the dimension** of the signal
- Existing sensor selection algorithms **cannot be applied** in such situations
- Our proposed **Incoherent Sensor Selection** (Insense) algorithm minimizes a **coherence-based cost function** from signal processing


## Sensor Selection Problem

- $D$ available sensors obtain linear measurements of a signal $x \in \mathbb{R}^N$ according to $y = \Phi x$
- The sensor selection problem is of finding a subset $\Omega$ of sensors (rows of $\Phi$) of size $|\Omega| = M$ such that the signal $x$ can be recovered from its $M$ linear measurements




- **Classical Sensor Selection**
  - Signal $x$ is arbitrary (dense or sparse)
  - Overdetermined regime $M > N$
  - Closed form solution (least squares problem)
- **Sparse Sensor Selection**
  - Signal $x$ is sparse
  - Underdetermined regime $M \ll N$
  - No closed form solution

## Sensor Selection Formulation

- We propose to find the set of sensors that minimizes the average column coherence of $\Phi_\Omega$ :

$$\mu^2_{\text{avg}}(\Phi_\Omega) = \frac{1}{\binom{N}{2}} \sum_{1 \le i < j \le N} \mu^2_{ij}(\Phi_\Omega)$$

- We reformulate minimizing the coherence objective as this optimization problem:

$$\min_{z \in \{0,1\}^D} \sum_{1 \le i < j \le N} \frac{{G_{ij}}^2}{G_{ii}\, G_{jj}}, \text{subject to}\, G = \Phi^T Z \Phi,\ \mathbf{1}^T z = M$$

- Then, we relax this boolean optimization problem using a box constraint:

$$\min_{z \in [0,1]^D} \sum_{1 \le i < j \le N} \frac{{G_{ij}}^2}{G_{ii}\, G_{jj}}, \text{s.t.}\, G = \Phi^T Z \Phi,\ \mathbf{1}^T z = M$$

- The optimization problem supports an efficient gradient–projection algorithm to find an approximate solution.
- In order to make the optimization well-defined we add two small constants to the objective as follows:

$$f_\epsilon(z) = \sum_{1 \le i < j \le N} \frac{{G_{ij}}^2 + \epsilon_1}{G_{ii}\, G_{jj} + \epsilon_2} \quad \text{where } G = \Phi^T Z \Phi$$

## The Insense Algorithm

- The objective function above is **smooth** and **differentiable** but non-convex
- The box constraints on z are **linear**
- We minimize the objective using the following **iterative gradient-projection** algorithm

**Algorithm 1:** Insense

**Input**: $\Phi$
**Output**: $Z = \text{diag}(z)$
**Initialization**:
$z \leftarrow z_0$;
$G \leftarrow \Phi^T Z \Phi$;
**while** *stoppage criterion = false* **do**
1. $k \leftarrow k + 1$;
2. update $\nabla_z f(z^k)$ based on equation (7);
3. $\gamma_k \leftarrow$ line search$(f, \nabla_z f(z^k), z^k)$;
4. $z^k \leftarrow z^k - \gamma^k \nabla_z f(z^k)$ {gradient step};
5. $z^{k+1} \leftarrow P_{\text{SBS}}(z^k)$ {SBS projection step};

**end**

- $P_{\text{SBS}}$ denotes the projection onto the convex set defined by the scaled boxed-simplex (SBS) constraints $\mathbf{1}^T z = M$ and $z = [0,1]^D$

$$\min_z \frac{1}{2}\|z - y\|_2^2, \text{s.t.} \sum_i z_i = M, z_i \in [0,1]_{\forall i = 1,\dots,D}$$

- We develop a method to efficiently perform this projection step

## The SBS Projection

- Using the KKT condition and some manipulations
- The solution to the SBS projection is given by

$$z_i = \max(\min(y_i - \lambda, 1), 0)$$

where,

$$\lambda = (M - K_1 - \sum_{i \in \zeta} y_i)/|\zeta|$$

and the set $\zeta$ can be found with $\mathcal{O}(D \log D)$

## Results

### Structured Sensing Matrices

Identity/Gaussian sensing matrix

| Algorithms | $\mu_{\text{avg}}(\Phi_\Omega)$ | FP($\Phi_\Omega$) | CN($\Phi_\Omega$) | BP accuracy % |
|---|---|---|---|---|
| Insense | **0.3061±0.0047** | 1019 ±313 | 1.93±0.19 | **92.27±1.42** |
| EigenMaps | – | **0.00±0.00** | **1.00±0.00** | 4.00±0.00 |
| MSE-G | 0.3872±0.0305 | 1155±374 | 11.51±0.93 | 57.91±1.09 |
| FrameSense | – | **0.00±0.00** | **1.00±0.00** | 4.00±0.00 |
| MI-G | – | **0.00±0.00** | **1.00±0.00** | 4.00±0.00 |
| Entropy-G | – | **0.00±0.00** | **1.00±0.00** | 4.00±0.00 |
| Determinant-G | – | **0.00±0.00** | **1.00±0.00** | 4.00±0.00 |
| Greedy SS | – | **0.00±0.00** | **1.00±0.00** | 4.00±0.00 |
| Convex SS | 0.3137±0.0075 | 2279±470 | 2.22±0.25 | 88.64±3.64 |

Uniform/Gaussian sensing matrix

| Algorithms | $\mu_{\text{avg}}(\Phi_\Omega)$ | FP($\Phi_\Omega$) | CN($\Phi_\Omega$) | Gaussian % | BP accuracy % |
|---|---|---|---|---|---|
| Insense | **0.3165±0.0023** | 9320 ±3292 | **1.46±0.07** | **100±0** | **58.55±2.64** |
| EigenMaps | 0.3215±0.0021 | 7230±2319 | 2.07±0.12 | 90±0 | 57.60±3.72 |
| MSE-G | 0.5805±0.0440 | 78530±12450 | 5.99±0.31 | 17±4 | 49.90±3.54 |
| FrameSense | 0.3273±0.0059 | **6095±1708** | 3.19±0.92 | 84±5 | 58.15±2.26 |
| MI-G | 0.6814±0.0556 | 93260±109250 | 6.26±0.77 | 7±4 | 51.60±5.21 |
| Entropy-G | 0.7007±0.0804 | 98950±16216 | 6.61±0.48 | 5±7 | 53.70±5.21 |
| Determinant-G | 0.7303±0.0545 | 105700±11228 | 6.57±0.31 | 3±4 | 55.50±4.50 |
| Greedy SS | 0.7303±0.0545 | 105700±11228 | 5.57±0.31 | 3±4 | 55.50±4.50 |
| Convex SS | 0.5788±0.1140 | 75270±27383 | 5.97±0.77 | 20±15 | 54.40±4.20 |

### DNA Sensing Dataset

- Objective: Select **DNA probes** to detect bacteria
- Sensing matrix: Hybridization affinity of $D = 100$ random DNA probes to $N = 42$ bacterial species

| | BP accuracy in detecting organisms % | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Number of organisms | $K=2$ | | | $K=3$ | | | $K=5$ | | |
| Number of probes | 8 | 12 | 15 | 12 | 15 | 20 | 15 | 20 | 25 |
| Insense | **68.33** | **94.78** | **99.65** | **71.74** | **93.95** | **99.53** | **51.95** | **92.71** | **99.10** |
| EigenMaps | 49.65 | 84.69 | 94.66 | 54.68 | 78.09 | 96.25 | 27.47 | 72.13 | 95.30 |
| MSE-G | 60.79 | 91.53 | 97.91 | 67.16 | 89.15 | 98.40 | 43.26 | 83.52 | 97.40 |
| FrameSense | 61.83 | 88.40 | 95.71 | 62.32 | 82.29 | 98.36 | 35.16 | 81.92 | 96.50 |
| MI-G | 59.98 | 89.68 | 96.40 | 65.69 | 84.10 | 97.39 | 37.96 | 79.72 | 96.00 |
| Entropy-G | 61.25 | 91.53 | 98.61 | 66.35 | 88.96 | 99.19 | 42.86 | 89.61 | 97.50 |
| Determinant-G | 46.75 | 82.13 | 94.55 | 48.97 | 76.13 | 96.03 | 24.48 | 72.73 | 92.81 |
| Greedy SS | 57.54 | 87.70 | 96.87 | 59.65 | 84.64 | 97.34 | 36.16 | 80.22 | 94.11 |
| Convex SS | 53.36 | 87.94 | 98.94 | 57.58 | 87.59 | 98.89 | 38.46 | 83.52 | 98.40 |
| Random | 61.53 | 88.79 | 96.66 | 62.29 | 86.15 | 97.72 | 38.88 | 82.94 | 86.44 |

- Insense requires **significantly smaller number of probes** to achieve the same accuracy

## Summary

- Incoherent sensor selection (Insense) algorithm for the underdetermined sensor selection
- Optimizes the **average squared coherence** of the columns of the selected sensors (rows)
- Interesting **future direction**:
  - **Large-scale** sensors selection
  - Sensor selection in **classification** and **clustering**

A. Aghazadeh, R. G. Baraniuk et al. “Insense: Incoherent sensor selection for sparse signals,” Acoustics, Speech and Signal Processing (ICASSP-18).

A. Aghazadeh, R. G. Baraniuk et al. "Universal microbial diagnostics using random DNA probes." Science advances 2.9 (2016): e1600025.