Spatial Array Thinning for Interference Cancellation under Connectivity Constraints

Hamed Nosrati, Elias Aboutanios and David B. Smith

University of New South Wales | Faculty of Engineering | School of Electrical Engineering and Telecommunications

Introduction

- Many applications such as communications, radar systems, radio telescopes, and automotive sensing stand to benefit from large phased arrays
- With the increasing the size of phased arrays, the hardware, connection, and computational costs increases significantly
- Antenna selection has been proven to yield hardware and processing savings as well as enabling improved interference mitigation by thinning the array. This is possible due to the redundancy inherent to a ULA.
- Achieving full array reconfigurability, requires that every front-end be connected via switches to every antenna in the array
- The complexity of connectivity, routing, and RF multiplexing makes this approach impractical
- We propose placing constraints on the antenna elements available for each front-end to simplify the connections, routing, and RF circuity. We solve the array thinning problem using graph optimization
- We revisit antenna selection in the case of single interference cancellation defining it as a minimum *k*-clique problem in graph optimization. We then propose a *k*-clique version of a generalized minimum clique (GMCP) problem to address connectivity constraints
- The connectivity constraints are formulated as a given set of clusters of antenna elements
- After relaxing and lifting the problem by a semidefinite method and finding the lower bound for this formulation, we proceed to obtain the optimum clusters
- We show by simulations that the performance of the proposed optimum clustering technique is very close to unconstrained antenna selection despite the added connectivity constraints. Furthermore, in addition to the dimensionality reduction provided by antenna selection, thinning under connectivity constraints is more computationally efficient in the optimization phase as it operates over a smaller subspace of possible solutions

Spatial Array Thinning

- The signal-to-interference-plus noise ratio (SINR) at the output of the optimum beamformer for a phased array containing *N* elements is given by

$$\begin{aligned}\mathrm{SINR}_{out} &= P_s\mathbf{v}_s^H\mathbf{R}_n^{-1}\mathbf{v}_s\\ &\approx N\mathrm{SNR}\left(1-|\alpha_{js}|^2\right)\end{aligned}$$

- α_{js} is the spatial separability between the signal of interest and interference measured by the spatial correlation coefficient (SCC)

$$\begin{aligned}\alpha_{js} &= \frac{\mathbf{v}_j^H\mathbf{v}_s}{\|\mathbf{v}_j\|\,\|\mathbf{v}_s\|}\\ &= \frac{\mathbf{v}_j^H\mathbf{v}_s}{\sqrt{\mathbf{v}_j^H\mathbf{v}_j}\sqrt{\mathbf{v}_s^H\mathbf{v}_s}} = \frac{\mathbf{v}_j^H\mathbf{v}_s}{N^2}\end{aligned}$$

- A smaller correlation between the signal and interference translates to a higher SINR at the output
- Through antenna selection we cast the problem as the combinatorial optimization

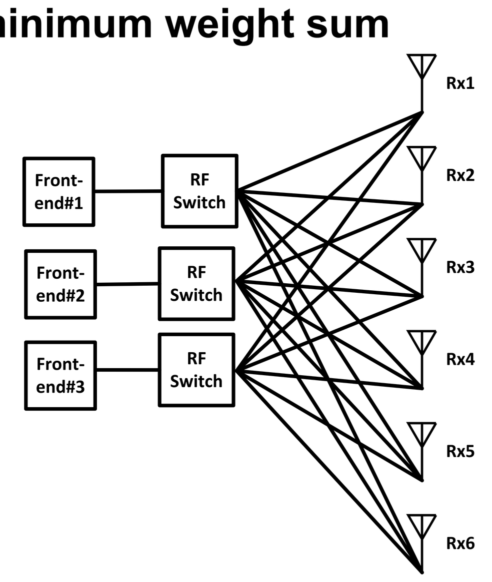
$$\begin{aligned}&\min_{\mathbf{c}}\ f(\mathbf{c})\\ &\text{s.t. } c_i(c_i-1)=0 \quad i=1...N\\ &\text{and } \mathbf{c}^T\mathbf{c}=k,\end{aligned}$$

$$f(\mathbf{c}) = |\alpha_{js}|^2 = \frac{\|\mathbf{c}\|^2_{\mathbf{W}_r}}{k^2} = \frac{\mathbf{c}^T\mathbf{W}_r\mathbf{c}}{k^2}.$$

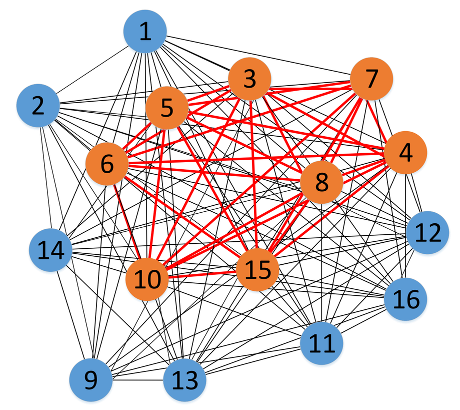
$$\mathbf{v}_{js} = \mathbf{v}_s \odot \mathbf{v}_j^*. \qquad \mathbf{W}_r = \mathrm{real}(\mathbf{v}_{js}\mathbf{v}_{js}^H),$$

- This optimization problem is a non-convex quadratic programming with quadratic constraints
- Due to the binary constraints, it is intractable and no exact analytical solution is available
- The solution can be approximated via heuristic methods e.g. correlation measurements, difference of two convex sets (DCS), and randomized semidefinite programming

- Noting that W_r is a similarity matrix, the array can be modelled as a weighted complete graph
- The optimization is then recast as finding a *k*-clique of minimum weight sum



Antenna selection in a reconfigurable antenna array



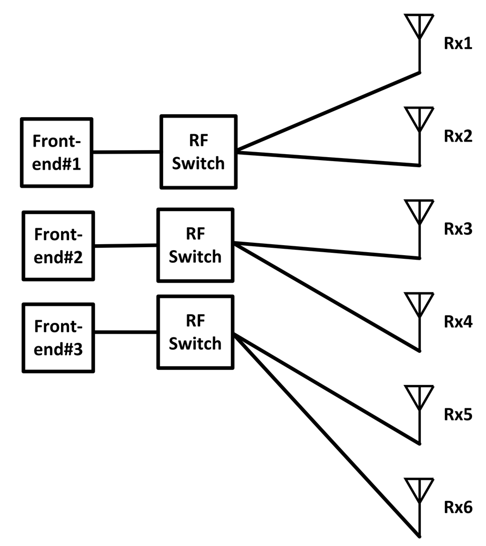
The minimum *k*-clique problem

Proposed Method

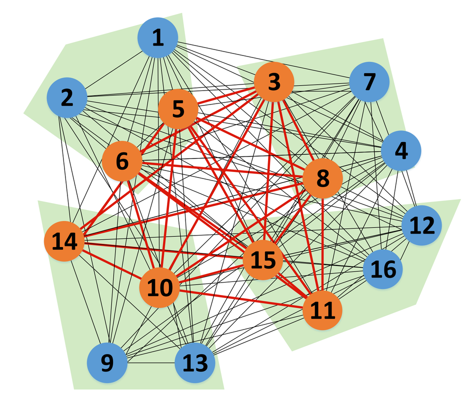
- Let *G = (V;E;W$_r$)* where *V* is the set of vertices and *E* the edges be a similarity graph representing the full array associated with the correlation steering vector v_{sj} for a specific scenario
- Assume that there are *M* clusters as the characteristic vectors of disjoint subsets of *V*. Letting *c* be the characteristic vector of the final *k*-clique, the GMCP for antenna selection under the connectivity constraints is

$$\begin{aligned}&\min_{\boldsymbol{c}}\ \frac{1}{k^2}\mathbf{c}^T\mathbf{W}_r\mathbf{c}\\ &\text{s.t. } c_i(c_i-1)=0 \quad i=1...N,\\ &\quad\ \mathbf{c}^T\mathbf{V}_i\mathbf{c}=k_i \quad i=1...M,\end{aligned}$$

- To satisfy the connectivity criteria, connectivity constraints are added to optimization to limit the number of antenna elements that each front-end is permitted to serve
- The clique problem becomes a *k*-clique version of the generalized minimum clique problem
- The complete graph is divided into some clusters and a *k*-clique GMCP is devised to find a clique containing exactly *k* vertices from each cluster such that the cost of the induced subgraph is minimized



Antenna selection in a reconfigurable antenna array with connectivity constraints



The generalized minimum *k*-clique problem (GMCP)

- Given the *M* clusters, the optimum antenna selection was formulated
- We aim to determine the optimum clustering configuration itself for a particular number of cluster *M*

$$\begin{aligned}&\max_{\mathcal{P}_M}\ \mathcal{D}(\mathcal{P}_M)\\ &\text{s.t.}\quad [\mathbf{X}_i]_j \in \{0,\frac{1}{\|\mathbf{v}_j\|}\}^N, \quad j=1,...,M\\ &\qquad \mathrm{rank}(\mathbf{X})=M,\\ &\qquad \mathbf{X}\mathbf{1}_M=\mathbf{1}_N,\\ &\qquad \mathbf{X}^T\mathbf{1}_N=\mathbf{1}_M.\end{aligned}$$

$$\mathbf{X} = \left(\frac{\mathbf{v}_1}{\|\mathbf{v}_1\|}, \frac{\mathbf{v}_2}{\|\mathbf{v}_2\|}, ..., \frac{\mathbf{v}_M}{\|\mathbf{v}_M\|}\right)$$

$$\mathcal{D}(\mathcal{P}_M) \triangleq \sum_{i=1}^{M}\frac{\mathbf{v}_i^T\mathbf{W}\mathbf{v}_i}{\mathbf{v}_i^T\mathbf{v}_i} = \mathrm{Tr}(\mathbf{X}^T\mathbf{W}\mathbf{X})$$

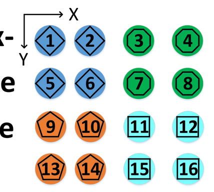
- After lifting and rank constraint relaxation, the SDR version is formulated as

$$\begin{aligned}&\max_{\mathbf{Y}}\ \mathrm{Tr}(\mathbf{Y}\mathbf{W})\\ &\text{s.t.}\quad \mathbf{Y}\mathbf{e}\le\mathbf{e},\\ &\qquad \mathrm{Tr}(\mathbf{Y})=M,\\ &\qquad \mathbf{Y}\ge 0,\\ &\qquad \mathbf{Y}\succeq 0.\end{aligned}$$

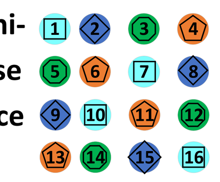
$$\mathbf{Y} = \sum_{i=1}^{M}\frac{\mathbf{v}_i\mathbf{v}_i^T}{\mathbf{v}_i^T\mathbf{v}_i}$$

- The cost function is chosen depending on the aim of the clustering. The actual physical distance of the elements can be used to ensure shorter and less complicated routing
- A useful weight matrix is composed of the correlation coefficients corresponding to different correlation steering vectors v_{js}
- Antenna elements that are most correlated with each other are spread across different clusters. This guarantees that the loss of a front-end imposes the smallest loss when the selection is implemented

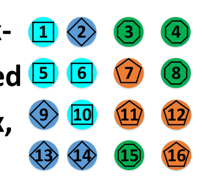
Clustering by maximizing the inverse Euclidean distance (Conf. 1)



Clustering by minimizing the inverse Euclidean distance (Conf. 2)

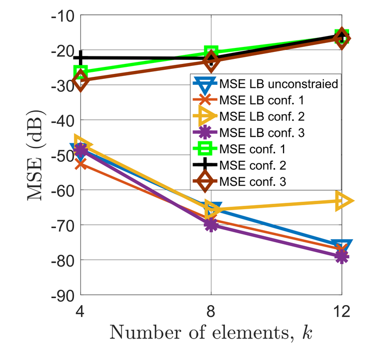


Clustering by maximizing the sampled correlation matrix, *Wr;T* (Conf. 3)

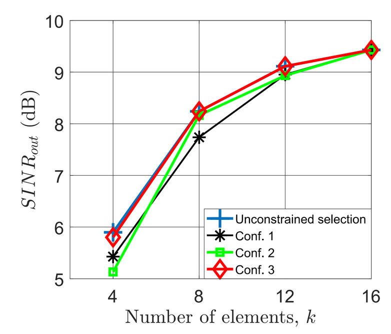


Results

- We employed a uniformly spaced rectangular array comprising 4 x 4 antennas. The elevation and azimuth of the signal of interest are fixed. On the other hand, the azimuth of the interference varies with a fixed elevation
- We considered three cases involving the selection of *k* = 4; 8, and 12 antennas from total 16 elements, arranged in *M* = 4 clusters.
- Two different mean-square-errors (MSE) are calculated. MSE between the optimum value obtained by exhaustive search and the SDR lower bound, and the MSE between the minimum achievable values of the SCC squared for the clustered and unconstrained arrays
- In the second example, the DOAs are fixed and the performance comparison is done in terms of output SINR for different cluster cardinalities
- Although all configurations are capable of providing acceptable SINRs compared to the full array, the clustering based on Conf. 3 outperforms the other two for all considered cardinality values



Top: MSE between clustered selection and unconstrained selection. Bottom: MSE between the optimum value obtained by exhaustive search and SDR



The maximum achievable output SINR corresponding to different cluster cardinalities for unconstrained spatial thinning and spatial thinning under different connectivity constraints

Conclusion

- We studied spatial array thinning for interference cancellation under connectivity constraints
- We formulated the problem as a generalized clique problem in graph optimization and use semidefinite relaxation to solve it
- We proposed appropriate connectivity constraints for different criteria and evaluated their performance using simulations. We demonstrated that the relaxed solutions attain their respective lower bounds
- We Showed that the unconstrained thinning performance was achieved by optimum clustering scheme resulting in smaller number of required connections and lower computational complexity