L1 Patch-Based Image Partitioning Into Homogeneous Textured Regions

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1 Introduction

Segmentation is the partition of the image into regions which share common features.

We propose a variational segmentation method that considers adaptive patches to characterize, in an affine way, the local structure of each homogeneous region of the image.

To be able to give a sharp representative disc associated to each region we use the $L1$-norm.

Goals

- Characterizing the image regions that have homogeneous texture regardless of the point of view.
- Obtaining the region representative disc.

2 Preprocessing: Adaptive Patches

We associate to each pixel an affine invariant tensor and ellipse-shaped patches that automatically adapt their size and orientation. They are iteratively computed by:

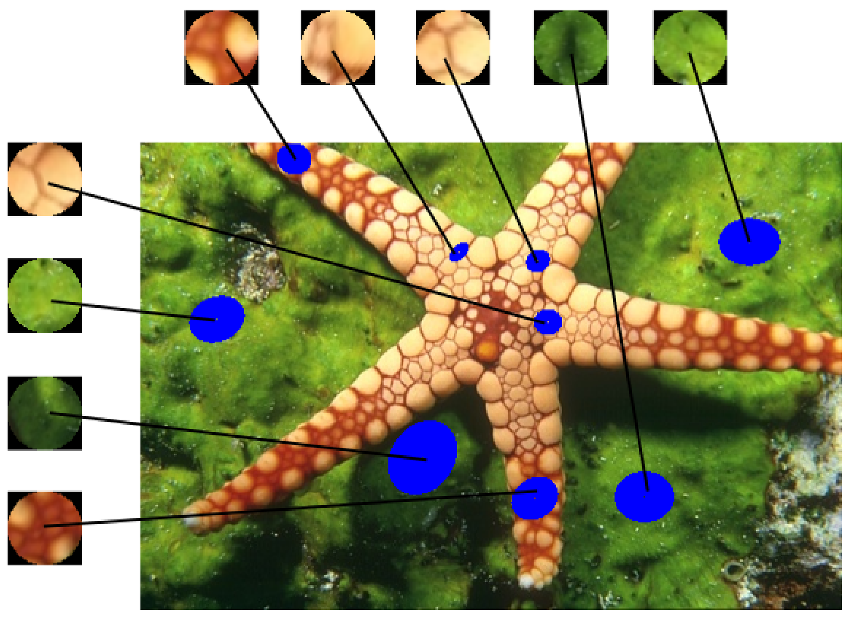
$$\mathcal{E}_{T^0(u)}(x,r) = \{y : |Du(x)(y-x) < r\}$$
$$\mathcal{E}_{T^k(u)}(x,r) = \{y : \langle T^{(k)}(u)(x)(y-x), (y-x)\rangle \leq r^2\}$$

where $T^{(k)}(u)(x) = \dfrac{\int_{\mathcal{E}_{T^{(k-1)}(u)}(x,r)} Du(y)\otimes Du(x)}{\text{Area}(\mathcal{E}_{T^{(k-1)}(u)}(x,r)}\underset{k}{\to} T_u(x)$.

$T_u(x)$ allows to normalize a patch

$$\mathbf{p}_u(x) := \mathbf{p}_u(x,h) = u(x + T_u(x)^{-\frac{1}{2}}R_u(x)^{-1}h),$$

to a fixed disc Δ_r (with $h \in \Delta_r$), where $R_u(x) = R(\theta)$ is a rotation matrix and θ the dominant orientation.



3 Proposed Model

Let $u : \Omega \to \mathbb{R}^M$ be an image with a patch $\mathbf{p}_u(x)$ associated to each pixel and $\mathcal{P}_u = \{\mathbf{p}_u(x), x \in \Omega\}$ the set of all patches. From now on, with an abuse of notation, $\mathbf{p}_u(x)$ will denote the normalized discs. We propose to simplify $\mathcal{P}_u$ by estimating a representative set of discs $\{\mathbf{p}_{\Omega_1}, \ldots, \mathbf{p}_{\Omega_N}\}$. It is modelled with the energy:

$$E(\mathbf{p},\chi) = \underbrace{\sum_{i=1}^{N}\int_\Omega |\nabla\chi_{\Omega_i}(x)|\,\mathrm{d}x}_{\text{Regularity term}} + \lambda\underbrace{\sum_{i=1}^{N}\int_\Omega \mathcal{D}_t^{\mathrm{a},1}(\mathbf{p}(x),\mathbf{p}_u(x))\,\chi_{\Omega_i}(x)\mathrm{d}x}_{\text{Data term}}, \qquad \lambda \geq 0$$

where $\mathbf{p} = \sum_i \mathbf{p}_{\Omega_i}\chi_{\Omega_i}$, Ω_i is a region.

Optimization.

1. Characteristic functions belong to a space which is not convex: change to **fuzzy membership** functions ω:

$$E(\mathbf{p},\omega) = \underbrace{\sum_{i=1}^{N}\int_\Omega |\nabla\omega_i(x)|\,\mathrm{d}x}_{E_s(\omega)} + \lambda\underbrace{\sum_{i=1}^{N}\int_\Omega \mathcal{D}_t^{\mathrm{a},1}(\mathbf{p}(x),\mathbf{p}_u(x))\,\omega_i(x)\mathrm{d}x}_{E_d(\mathbf{p},\omega)}$$

2. We introduce an auxiliary variable $\mathbf{v}$ to decouple the optimization problem:

$$E(\mathbf{p},\omega,\mathbf{v}) = E_s(\omega) + \lambda E_d(\mathbf{p},\mathbf{v}) + \frac{1}{2\theta}\sum_{i=1}^{N}\int_\Omega (\omega_i(x) - v_i(x))^2\,\mathrm{d}x$$

3. As E is convex w.r.t each variable, it is minimized by alternatively fixing two variables and minimize w.r.t the third one, and iterate until convergence:

3.1. ω-subproblem: Dual Formulation [3].

$$\omega_i(x) = v_i(x) + \theta\mathrm{div}(\xi(x))$$
$$\xi^{n+1}(x) = \frac{\xi^n(x) + \tau\nabla(\theta\mathrm{div}(\xi(x)) + v_i(x))}{1 + |\theta\mathrm{div}(\xi(x)) + v_i(x)|}$$

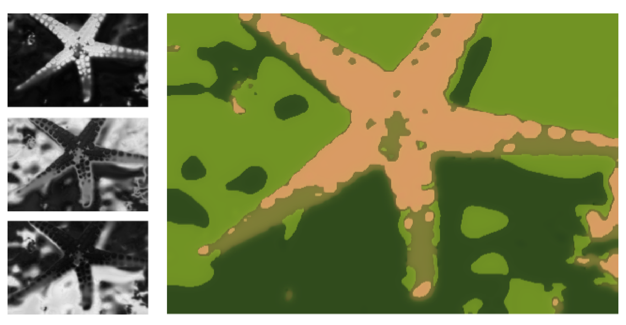
3.2. v-subproblem:

$$v_i(x) = \min\{\max\{\omega_i(x) - \lambda\theta\mathcal{D}_t^{\mathrm{a},1}(\mathbf{p}(x),\mathbf{p}_u(x)), 0\}, 1\}$$

3.3. p-subproblem: the solution is given by a weighted median vector [1]:

$$\begin{cases}\mathbf{p}_{\mathrm{wvm}}(x) \in \{\mathbf{p}_i(x), i = 1\ldots N\}\\ \sum_{i=1}^{N} v_i(x)\|\mathbf{p}_{\mathrm{wvm}}(x) - \mathbf{p}_i(x)\|_{L_1} \leq \sum_{i=1}^{N} v_i(x)\|\mathbf{p}_j(x) - \mathbf{p}_i(x)\|_{L_1} \qquad \forall j\end{cases}$$

Postprocessing and Output



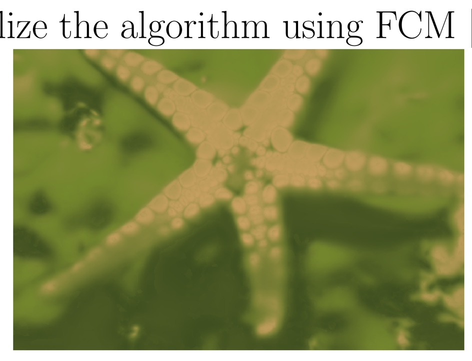
Fuzzy functions output.
$I(x) = \sum_i \mathbf{p}_{\Omega_i}(x)\omega_{\Omega_i(x)}$

$$\chi_{\Omega_i}(x) = \begin{cases}1, & \text{if } i = \arg\max_j \omega_j(x)\\ 0, & \text{else}\end{cases}$$

Characteristic functions output and associated discs.
$I(x) = \sum_i \mathbf{p}_{\Omega_i}(x)\chi_{\Omega_i(x)}$

Initialization. As the functional is not jointly-convex the final result has a high dependence on the initialization. We initialize the algorithm using FCM [2]

$$\omega_{\Omega_i}(x_k) = \sum_{j=1}^{N}\frac{\mathcal{D}_t^{\mathrm{a},2}(\mathbf{p}_{\Omega_i}(x_k),\mathbf{p}_u(x_k))}{\mathcal{D}_t^{\mathrm{a},2}\left(\mathbf{p}_{\Omega_j}(x_k),\mathbf{p}_u(x_k)\right)} \qquad \forall i,k$$
$$\mathbf{p}_{\Omega_i}(x) = \frac{\sum_k(\omega_{\Omega_i}(x_k))^2\mathbf{p}_u(x_k)}{\sum_{k=1}^{N}(\omega_{\Omega_i}(x_k))^2} \qquad \forall i$$



4 Results

Input & patch	Li. et. al [5]	Ours	Discs

All these experiments are done with $r = 90$ and disc size 51×51px and $\lambda = 0.04$.

5 Conclusions

Our **model** considers:

- Similarity among discs in an $L1$ data term.
- TV of fuzzy membership functions as relaxed length of the region boundaries.

We provide as **output** :

- Partition of the image into local homogeneous regions.
- Texture disc associated to each region.

6 Future work

- Use patch texture features to compare the patches.
- Fill-in the region with the texture from the representative patch.

7 References

[1] M. Barni (IEEE Trans. Image Processing, 1998)
[2] J. C. Bazdek (Med. Phys, 1993)
[3] A. Chambolle (J. Math. Imag. Vis, 2004)
[4] V. Fedorov et al. (SIAM J. Imaging Sci, 2015)
[5] F. Li et al (J. Sci. Comput, 2016)