

# 3D IMAGE RECONSTRUCTION FROM MULTI-FOCUS MICROSCOPE: AXIAL SUPER-RESOLUTION AND MULTIPLE FRAME PROCESSING

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## Introduction to MFM

**Multi-focus microscopy (MFM)** is a microscopy that captures multiple focal planes with a single shot [1]. A diffractive grating splits the light from different focal depth to form an array of  $K \times K$  image tiles on a camera. It is able to capture dynamic scenes in biological samples, such as movement of a cell or a molecule.

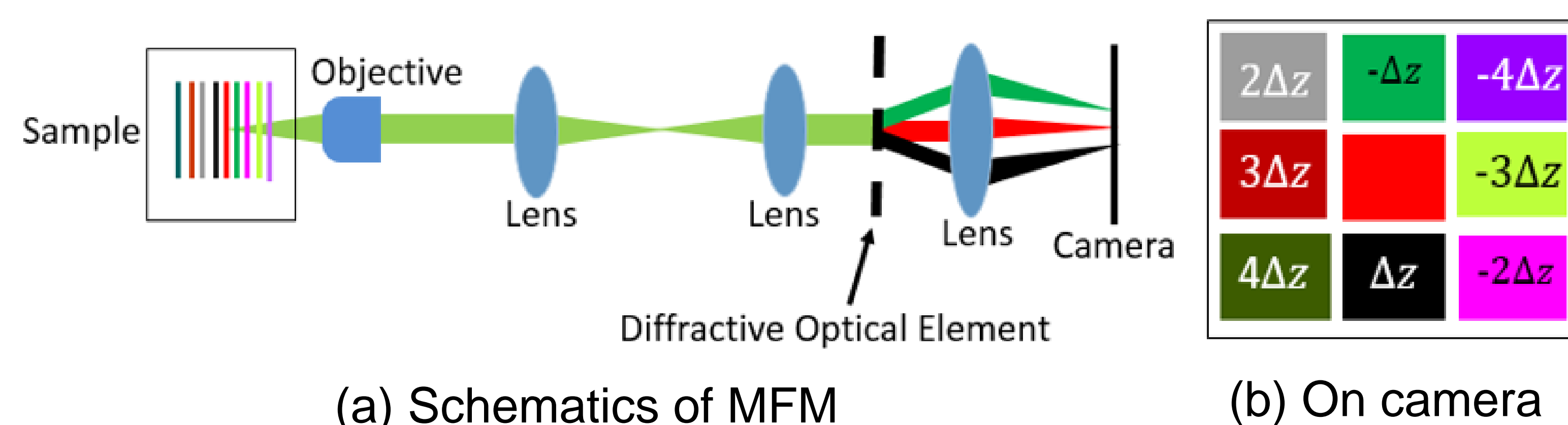


Fig 1. Multi-focus microscopy

A naïve method to reconstruct a 3D image from a MFM image is stacking the tiles with alignment, but this method has several limitations: number of planes is limited by the number of tiles, the spacing in the z-axis are limited by the  $\Delta z$ , and out-of-focus blur remains.

## Single-Frame (SF) MFM Reconstruction

Xiang *et al.* presented a 3D image reconstruction algorithm from MFM images [2]. The forward model of MFM can be represented as a linear model assuming Gaussian noise as

$$\mathbf{g} = \mathbf{H}\mathbf{f} + \boldsymbol{\epsilon}$$

$$\begin{aligned} \mathbf{f} &\in \mathbb{R}^{N_x N_y N_z \times 1} \\ \mathbf{g} &\in \mathbb{R}^{M_x M_y \times 1} \\ \mathbf{H} &\in \mathbb{R}^{M_x M_y \times N_x N_y N_z} \\ \boldsymbol{\epsilon} &\in \mathbb{R}^{M_x M_y \times 1} \end{aligned}$$

and we formulate a TV-regularized least squares to reconstruct a 3D image as follows:

$$\begin{aligned} \hat{\mathbf{f}} &= \arg \min_{\mathbf{f}} \|\mathbf{g} - \mathbf{H}\mathbf{f}\|_2^2 + \lambda\Phi(\mathbf{f}) \\ &\text{subject to } \mathbf{f} \geq 0 \\ \Phi(\mathbf{f}) &= \sum_i \sqrt{(\Delta_i^x \mathbf{f})^2 + (\Delta_i^y \mathbf{f})^2 + (\Delta_i^z \mathbf{f})^2} \end{aligned}$$

## Multiple-Frame (MF) MFM Reconstruction

Given a sequence of MFM images, we can utilize multiple MFM images to achieve a higher quality 3D image reconstruction. The forward model of MFM in a dynamic scene is represented as

$$\mathbf{g}_k = \mathbf{H}\mathbf{f}_k + \boldsymbol{\epsilon}_k$$

and it can be extended as

$$\mathbf{g}_k = \mathbf{H}\mathbf{M}_{l,k}(\boldsymbol{\alpha}_{l,k})\mathbf{f}_l + \boldsymbol{\epsilon}_{l,k}$$

where  $\boldsymbol{\alpha}_{l,k}$  is a set of motion parameters and  $\mathbf{M}_{l,k}$  is the corresponding warping matrix.

We propose two MF MFM reconstruction algorithms:

### (1) Batch approach

$$\begin{aligned} \{\hat{\mathbf{f}}_l, \hat{\boldsymbol{\alpha}}_{l,k}\} &= \arg \min_{\mathbf{f}_l \geq 0, \boldsymbol{\alpha}_{l,k}} \sum_{k=l-m}^{l+m} \|\mathbf{g}_k - \mathbf{H}\mathbf{M}_{l,k}(\boldsymbol{\alpha}_{l,k})\mathbf{f}_l\|_2^2 \\ &+ \lambda\Phi(\mathbf{f}_l) + \omega \sum_{k=l-m}^{l+m} \|\boldsymbol{\alpha}_{l,k}\|_2^2 \end{aligned}$$

### (2) Recursive approach

$$\begin{aligned} \{\hat{\mathbf{f}}_k, \hat{\boldsymbol{\alpha}}_{k-1,k}\} &= \arg \min_{\mathbf{f}_k \geq 0, \boldsymbol{\alpha}_{k-1,k}} \|\mathbf{g}_k - \mathbf{H}\mathbf{f}_k\|_2^2 + \lambda\Phi(\mathbf{f}_k) \\ &+ \eta \|\mathbf{f}_k - \mathbf{M}_{k-1,k}(\boldsymbol{\alpha}_{k-1,k})\mathbf{f}_{k-1}\|_2^2, \end{aligned}$$

## Experimental Results

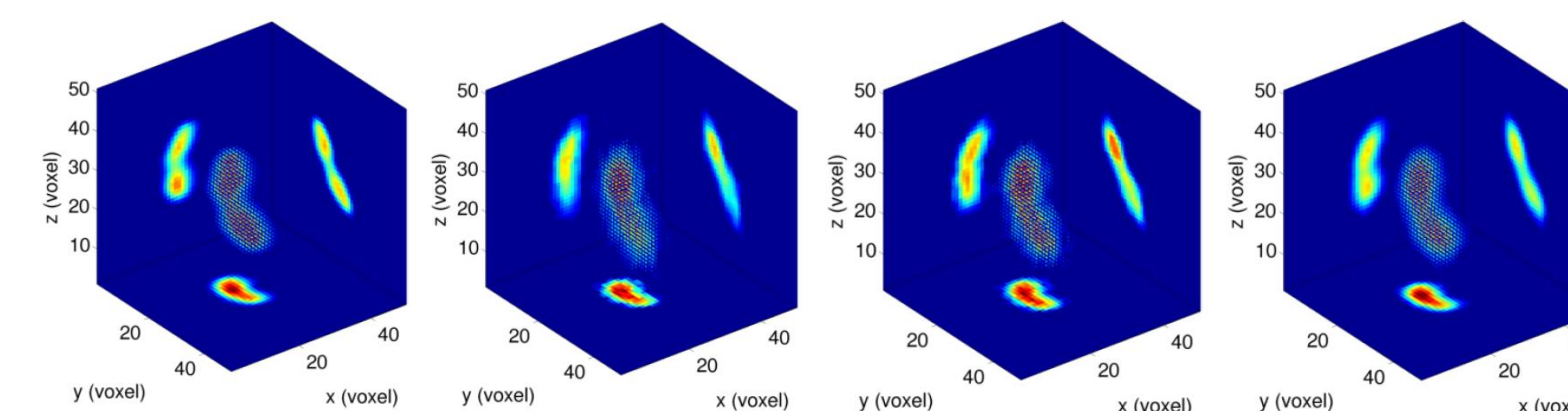


Fig 2. MFM reconstruction results. (a) Ground truth, (b) SF, (c) MF batch, and (d) MF recursive

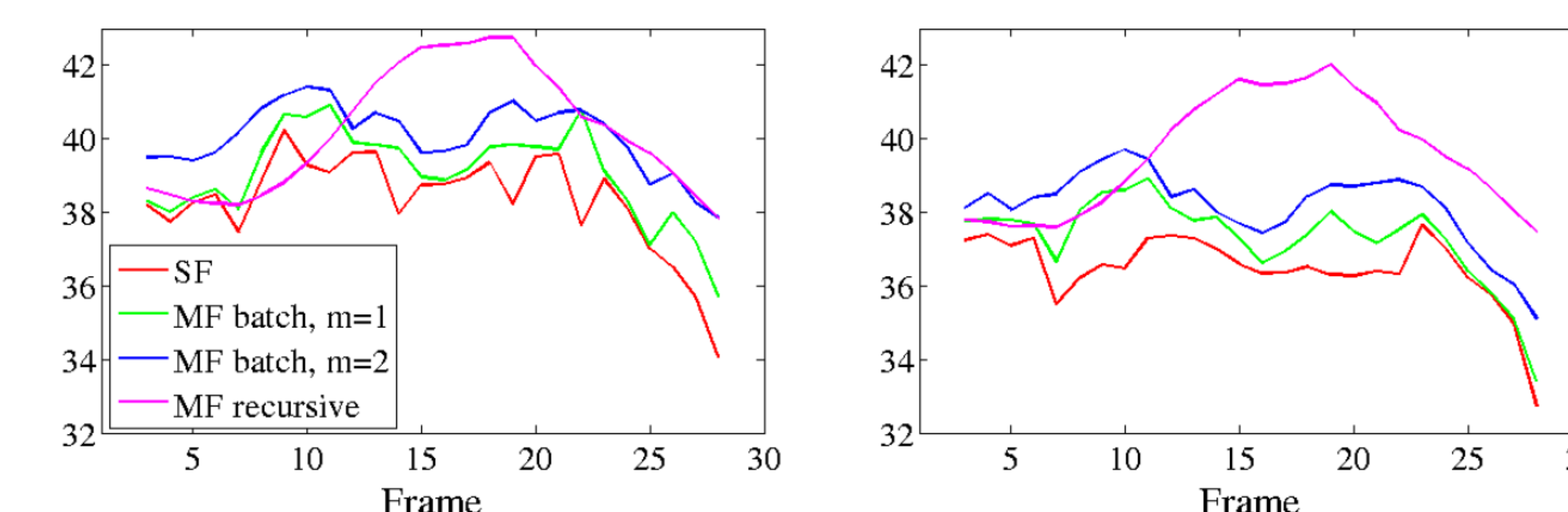


Fig 3. Performance of simulation with different noise std 0.01, 0.02

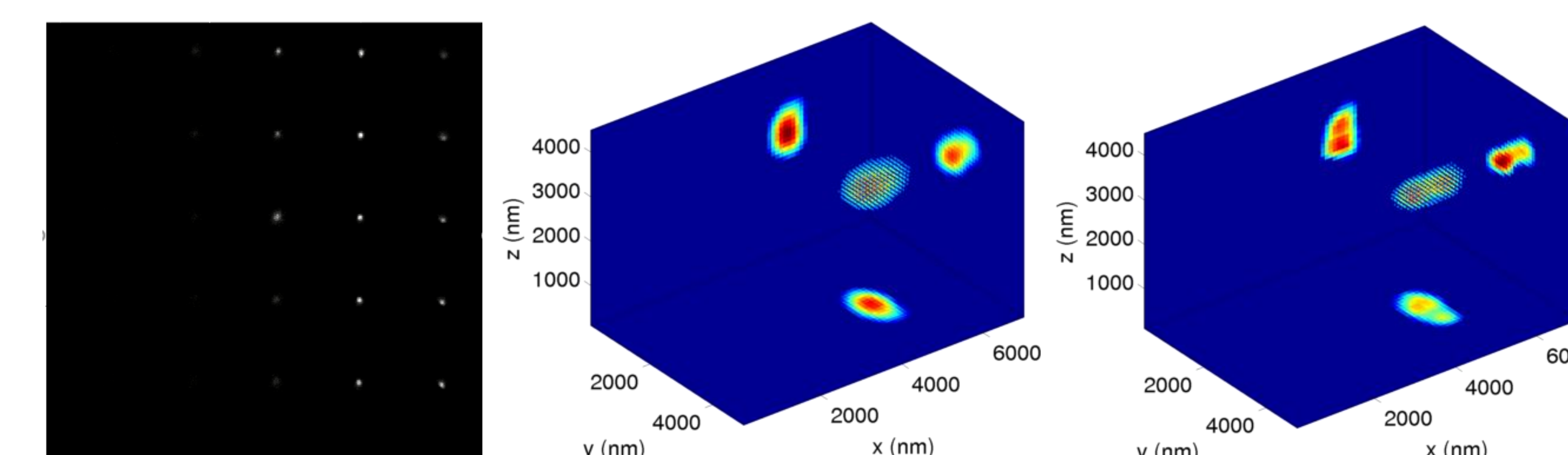


Fig 4. Real data experiment. (a) MFM image, (b) SF, (c) MF

## References

- [1] S. Abrahamsson *et al.*, "Fast multicolor 3D imaging using aberration-corrected multifocus microscopy," *Nature Methods*, vol. 10, no. 1, pp. 60–63, 2012.
- [2] X. Huang, *et al.*, "3D snapshot microscopy of extended objects," arXiv:1802.01565, 2018.