COMBINING RANGE AND DIRECTION FOR IMPROVED LOCALIZATION

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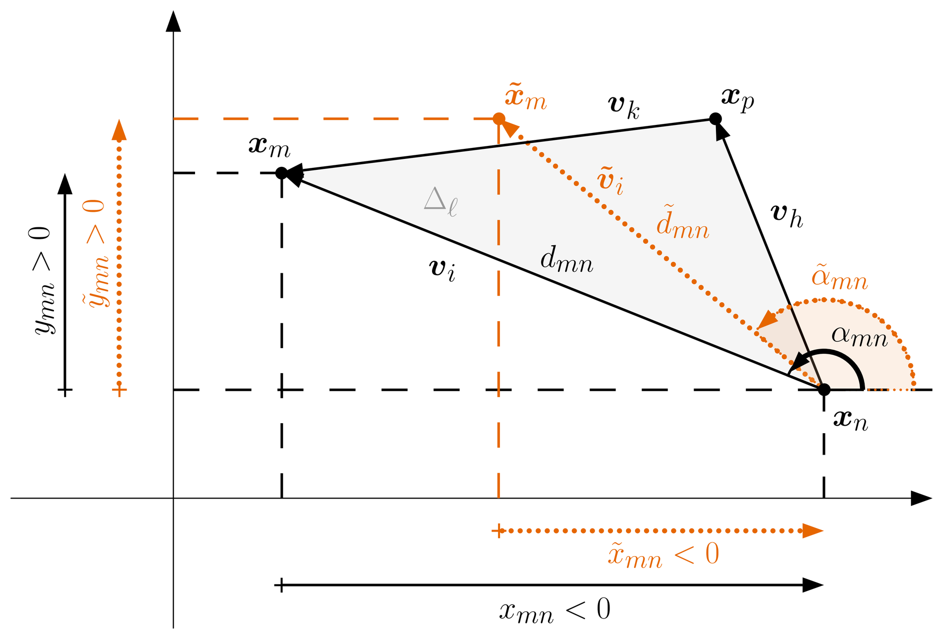
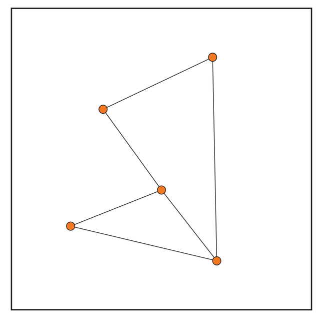
Problem Statement

- **Goal:** multimodal self-localization of nodes in a sensor network
- **Approach:** recover the locations of the nodes given **distance** and **angle** measurements
- **Contributions:** two algorithms that outperform the state-of-the art

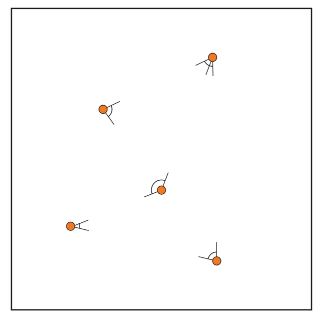


Background



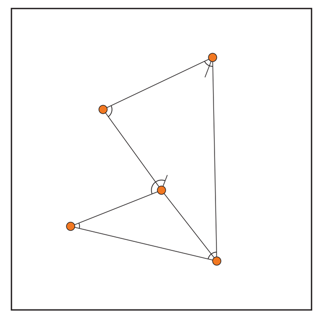
Distance-based approaches

- **How to measure distances:** time-of-arrival, received signal strength
- **Methods:** Euclidean distance matrices, MDS [1]



Angle-based approaches

- **How to measure angles:** antenna arrays, leveraging phase differences
- **Methods:** angle-of-arrival [2]



Hybrid approaches

- Combination of distances and angles
- **Method:** edge-multidimensional scaling [3]

Edge-Kernel, prior work

Edge-multidimensional scaling (E-MDS) [3]

- **Edge kernel matrix**

$$(\boldsymbol{K}_E)_{ij} = \langle \boldsymbol{v}_i, \boldsymbol{v}_j \rangle = \langle \boldsymbol{x}_m - \boldsymbol{x}_n, \boldsymbol{x}_q - \boldsymbol{x}_p \rangle$$

$$\boldsymbol{K}_E = \boldsymbol{V}\boldsymbol{V}^\top$$

- **Solution:** recover $\boldsymbol{V}$ via eigenvalue decomposition of $\boldsymbol{K}_E$

Constrained Edge-Kernel, proposed approach

Constrained E-MDS

- **Our contribution:** enforce additional constraints on $\boldsymbol{K}_E$
- **Constraints:** (1) $\boldsymbol{K}_E$ is PSD and of rank 2
  (2) the entries of $\boldsymbol{K}_E$ satisfy the triangle equality
- **Solution:** lift-and-project, alternative projections onto the sets satisfying constraints (1) and (2)

Coordinate Difference Matrices (CDMs)

Definitions

CDM	$\boldsymbol{S} = \boldsymbol{s}\mathbb{1}^\top - \mathbb{1}\boldsymbol{s}^\top$
Noise matrix	$\boldsymbol{Z}$
Mask matrix	$\boldsymbol{W}$
Degree matrix	$\Lambda = \operatorname{diag}\left(\sum_{n=1}^{N} W_{mn}\right)^{-1}$

Input Incomplete noisy measurements $\tilde{\boldsymbol{S}} = (\boldsymbol{S} + \boldsymbol{Z}) \circ \boldsymbol{W}$

Cost function $\min_{\boldsymbol{s}} \left\| \boldsymbol{W} \circ (\boldsymbol{s}\mathbb{1}^\top - \mathbb{1}\boldsymbol{s}^\top - \tilde{\boldsymbol{S}}) \right\|_F^2$

Algorithm Decompose the edge vectors $\boldsymbol{V}$ into 1D coordinate differences s_x and s_y.

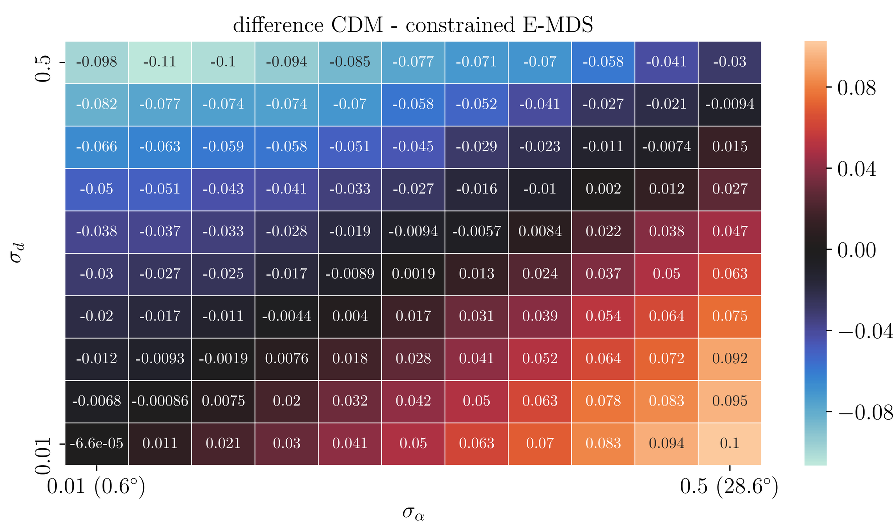
For each dimension

Compute $\boldsymbol{W}'$, $\tilde{\boldsymbol{S}}'$ and $\boldsymbol{\Lambda}'$ by removing the first row and column of $\boldsymbol{W}$, $\tilde{\boldsymbol{S}}$ and $\boldsymbol{\Lambda}$.
$\boldsymbol{d}' = \boldsymbol{\Lambda}'(\tilde{\boldsymbol{S}}' \circ \boldsymbol{W}')\mathbb{1}$
$\boldsymbol{A}' = \boldsymbol{I} - \boldsymbol{\Lambda}'\boldsymbol{W}'$
$\hat{\boldsymbol{s}} = (\boldsymbol{A}')^{-1}\boldsymbol{d}'$ where $(\boldsymbol{A}')^{-1}$ has a closed form.
Return $[0 \quad \hat{\boldsymbol{s}}]^\top$

Output The 2D points $[\hat{\boldsymbol{s}}_x \quad \hat{\boldsymbol{s}}_y]$

Comparison of the two methods

- **Experiment:** measure the RMSE of the reconstructed locations for different noise on distances (σ_d) or angles (σ_α)
- **Outcome:** CDM better for high distance noise, constrained E-MDS better with high angular noise



Comparison with state-of-the-art

State-of-the-art

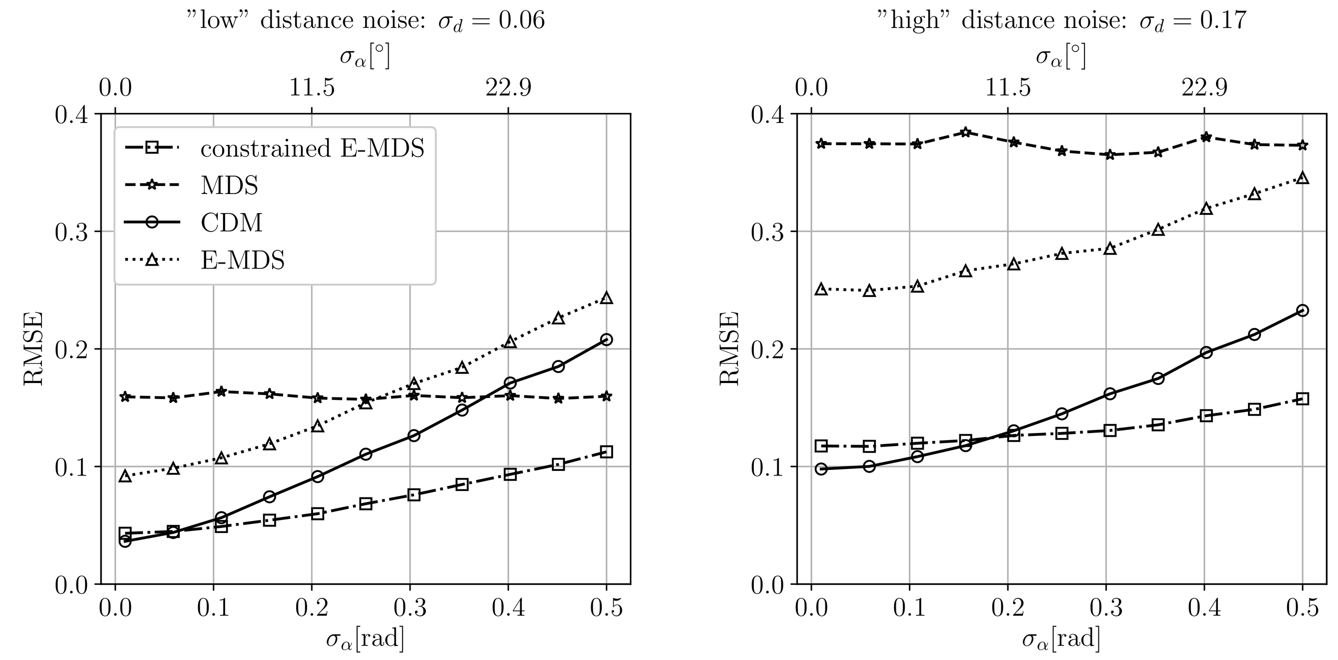
- **Distance-only:** MDS [1]
- **Distance + angles:** E-MDS [3]

Experiment

- 6 points chosen uniformly in the unit square
- Compute RMSE on reconstructed points

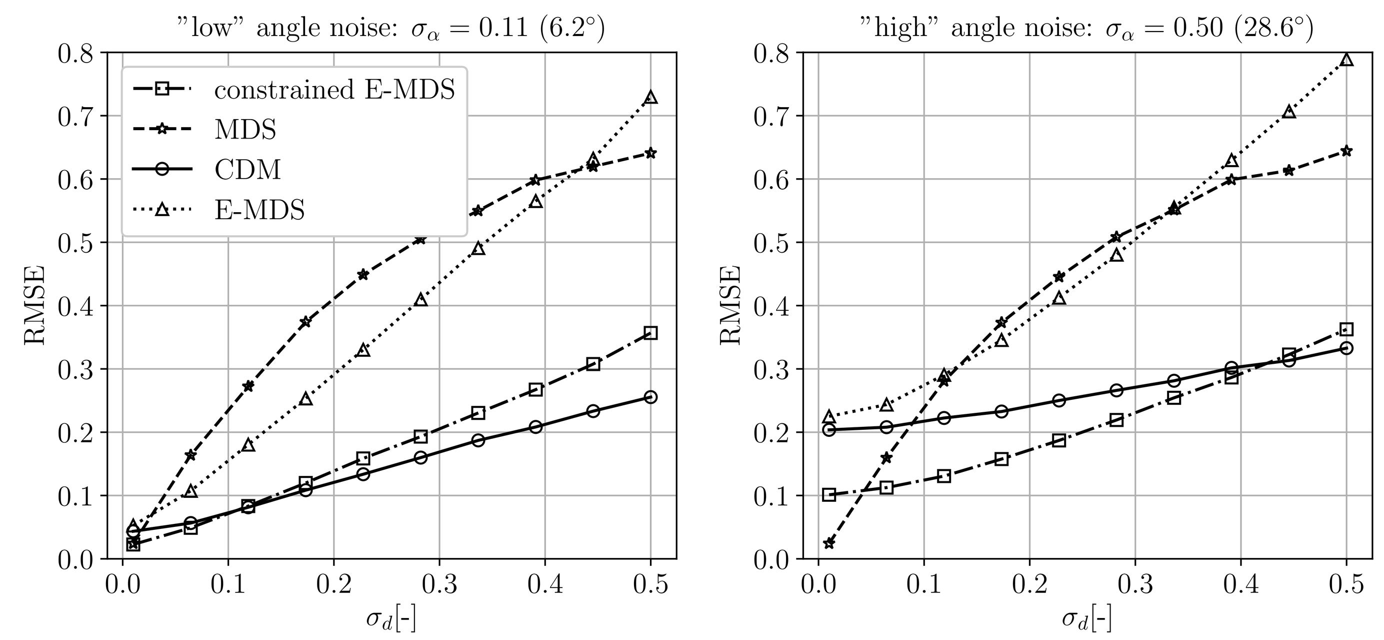
RMSE vs. angular noise

- RMSE vs. angular noise for two representative distance noise levels



RMSE vs. distance noise

- RMSE vs. distance noise for two representative angle noise levels



Conclusion

- We proposed two algorithms for multimodal sensor localization:
  - **Constrained Edge-Kernel** improves on the existing Edge-Kernel method by adding geometric constraints on triplets of points.
  - **Coordinate Difference Matrices** allow us to estimate the sensors' coordinates independently for each dimension.
- Numerical simulations demonstrate that both proposed methods significantly outperform existing distance-based and multimodal localization algorithms.

*The authors have equal contribution to this work and the order is alphabetical

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[1] I. Dokmanić, R. Parhizkar, J. Ranieri, and M. Vetterli, "Euclidean Distance Matrices: Essential theory, algorithms, and applications," IEEE Signal Processing Magazine, vol. 32, no. 6, pp. 12-30, 2015.
[2] R. Peng and M. L. Sichitiu, "Angle of arrival localization for wireless sensor networks," 3rd Annual IEEE Communications Society on Sensor and Ad Hoc Communications and Networks, vol. 1, pp. 374–382, 2006.
[3] D. Macagnano, G. Thadeu, F. D. Abreu, and S. Member, "Algebraic Approach for Robust Localization with Heterogeneous Information," IEEE Transactions on Wireless Communications, vol. 12, no. 10, pp. 5334–5345, 2013.