

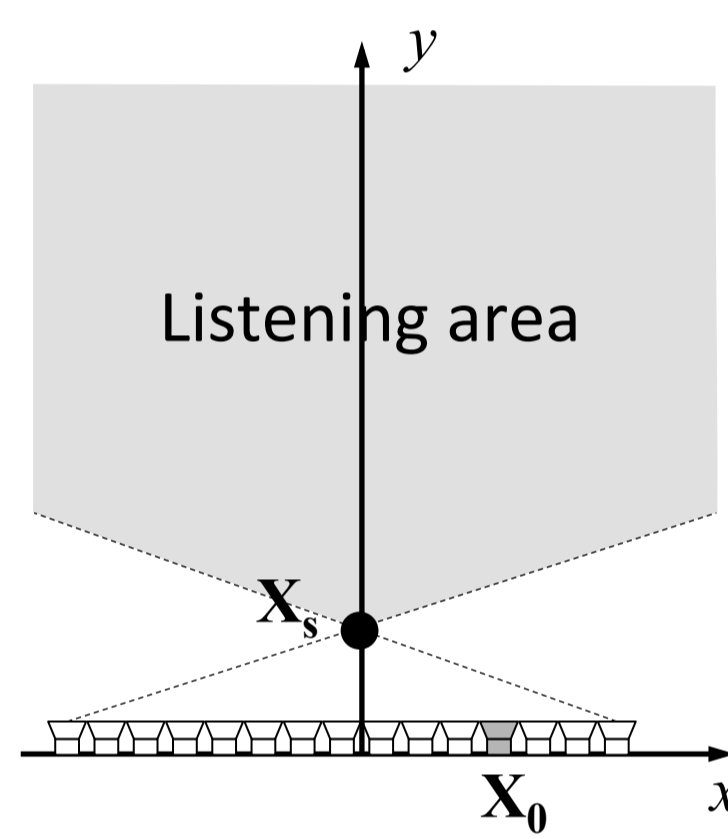


Goal: Create sound fields from directional sources in the listening areas of focused sources with linear loudspeaker array.
Question: How can we introduce directivity to virtual sound sources between a loudspeaker array and audience seats?

Proposed: 1. Create a cluster of focused sources to form multipoles in the listening area.
 2. Analytical conversion of circular harmonics into weights for each multipole.
Results: Reproduced directivity on the basis of multipoles comprising multiple focused sources.

Introduction

- Virtual sound sources closing in on the audience seats for live viewings and theaters
- Focused source method [1]: Creating sound fields from monopole sources between audience seats and a linear loudspeaker array



- Sound field and driving function

$$P(\mathbf{r}) = \int_{-\infty}^{\infty} D(\mathbf{x}_0, \mathbf{x}_s) G_{2D}(\mathbf{r} - \mathbf{x}_0) d\mathbf{x}_0$$

$$D(\mathbf{x}_0, \mathbf{x}_s) = -\frac{jk}{2} c_0 \frac{y_0 - y_s}{|\mathbf{x}_0 - \mathbf{x}_s|} H_1^{(1)}(k|\mathbf{x}_0 - \mathbf{x}_s|)$$

- Limited listening areas due to the nature of sound focusing

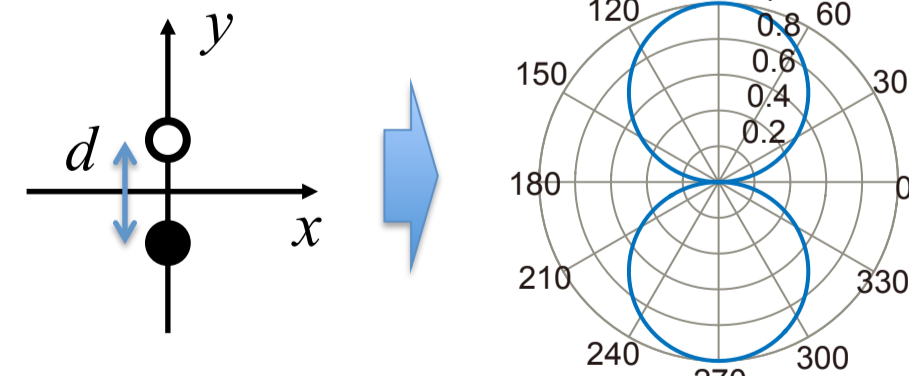
- Accurate reproduction of sound directivities help the audience feel more realistic sensations [2]

- Our goal: Creating sound fields from directional sources in the listening areas of focused sources**

Multipole

- Multipole: Directional source comprising a collection of point sources with opposite phases

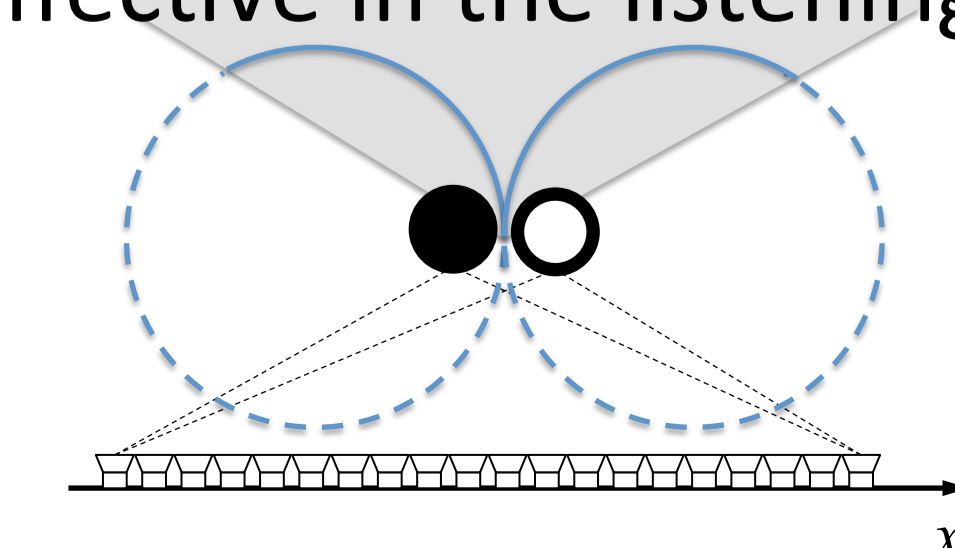
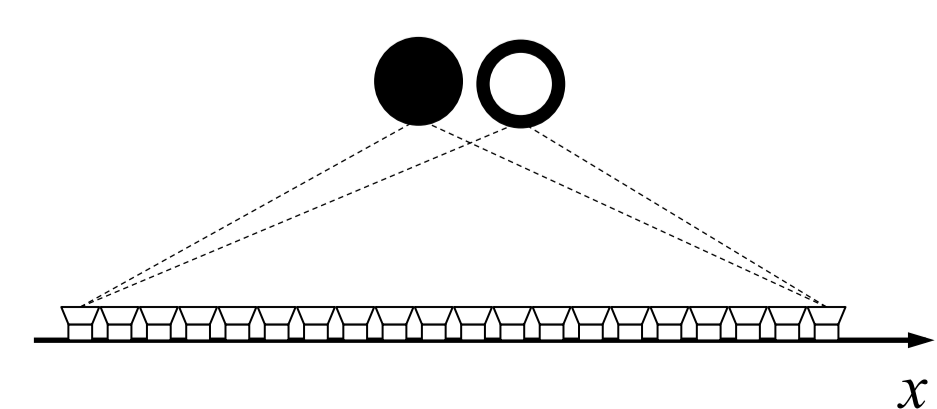
$$S(\mathbf{r}) = \sum_{m,n} d_{mn} \cdot \frac{\partial^{m+n}}{\partial x^m \partial y^n} G(\mathbf{r})$$



- Sound field from a multipole reproduced by a cluster of focused sources

Create multiple focused sources

Sound field of the multipole effective in the listening area



Proposed method

- Basic idea: The analytical conversion of circular harmonic coefficients into weights for each multipole
 - Circular harmonic decomposition of arbitrary sound fields in 2D

$$S(\mathbf{x}) = \sum_{v=-\infty}^{\infty} \tilde{S}_v^{(2)} H_v^{(2)}(kr) e^{jv\alpha} = \sum_{v=-\infty}^{\infty} \tilde{S}_v^{(2)} H_v^{(2)}(kr) (\cos\alpha + j\sin\alpha)^v$$

Euler's formula

By applying binomial expansion

$$S(\mathbf{x}) = \tilde{S}_0^{(2)} H_0^{(2)}(kr) + \sum_{N=1}^{\infty} \sum_{m=1}^N j^{N-m} \binom{N}{m} \tilde{S}_N^{(2)} H_N^{(2)}(kr) \cdot \cos^m \alpha \cdot \sin^{N-m} \alpha + \sum_{N=1}^{\infty} \sum_{m=1}^N (-j)^{N-m} \binom{N}{m} \tilde{S}_{-N}^{(2)} H_{-N}^{(2)}(kr) \cdot \cos^m \alpha \cdot \sin^{N-m} \alpha$$

- Taylor's expansion at the origin of the coordinate

$$S(\mathbf{x}) = \sum_{N=0}^{\infty} \sum_{m=0}^N \frac{\partial^N S(\mathbf{x})}{\partial x^m \partial y^{N-m}} \Big|_{\mathbf{x}=0} \frac{x^m \cdot y^{N-m}}{m! (N-m)!} \rightarrow \sum_{N=0}^{\infty} \sum_{m=0}^N \frac{\partial^N S(\mathbf{x})}{\partial x^m \partial y^{N-m}} \Big|_{\mathbf{x}=0} \frac{\cos^m \alpha \cdot \sin^{N-m} \alpha}{m! (N-m)!}$$

where $d = m + n$ On the unit circle

- By comparing coefficients of the term $\cos^m \alpha \cdot \sin^n \alpha$

$$\frac{\partial^{m+n} S(\mathbf{x})}{\partial x^m \partial y^n} \Big|_{\mathbf{x}=0} = j^n (m+n)! \{ \tilde{S}_{m+n}^{(2)} H_{m+n}^{(2)} + (-1)^n \tilde{S}_{-m-n}^{(2)} H_{-m-n}^{(2)} \}$$

- The driving functions for sound fields from a sound source based on superposition of multipoles**

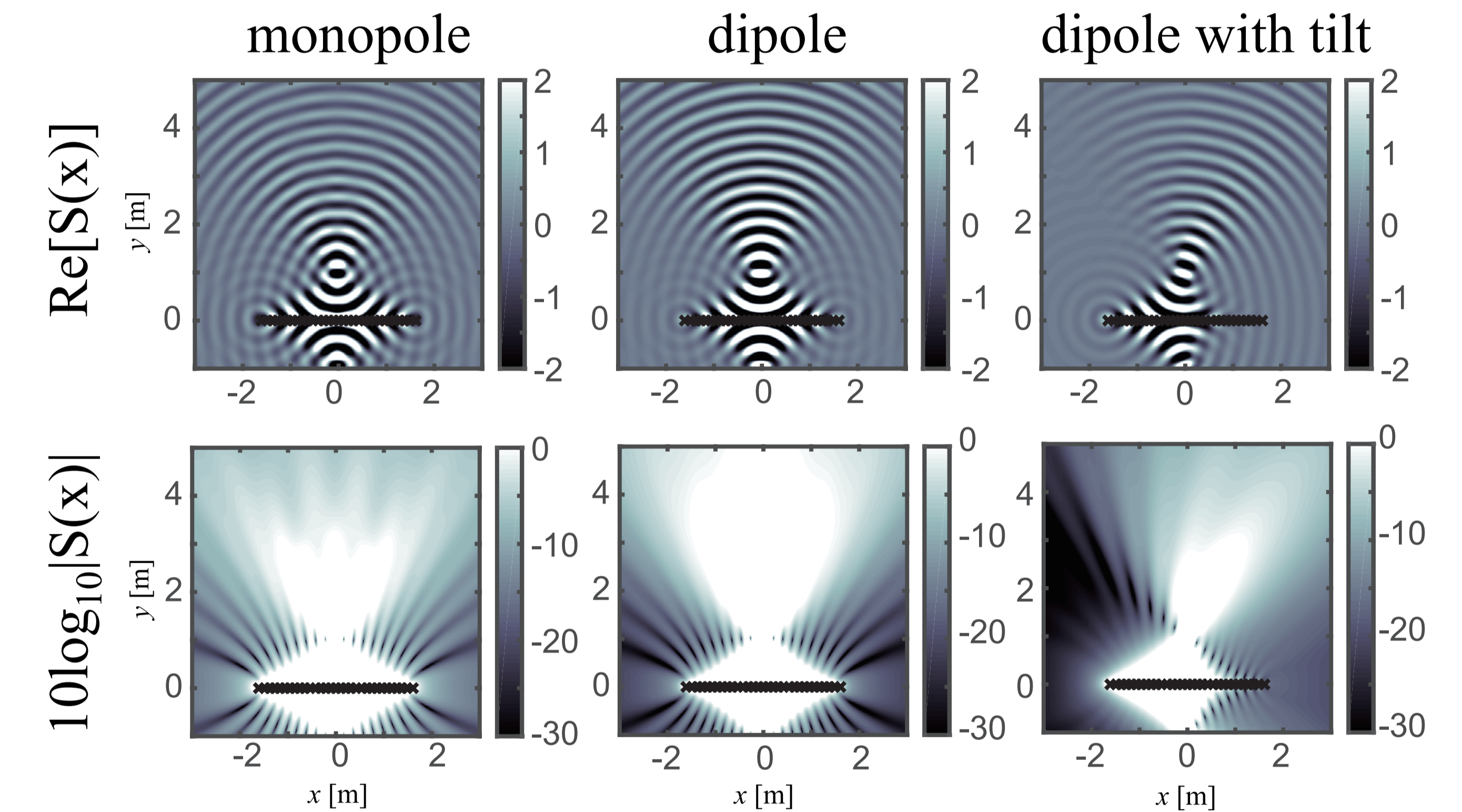
$$D(\mathbf{x}_0) = \sum_{m,n} \sum_{\substack{\mathbf{x}_s^{m,n} \in X^{m,n} \\ g_s^{m,n} \in G^{m,n}}} \frac{\partial^{m+n} S(\mathbf{x})}{\partial x^m \partial y^n} \Big|_{\mathbf{x}=0} \cdot \frac{g_s^{m,n}}{(j k)^{m+n}} \cdot \frac{D(\mathbf{x}_0, \mathbf{x}_s^{m,n})}{m! n!}$$

$$X^{m,n} = \{ (x^m, y^n) \mid x^m = x^{m-1} \pm \Delta, y^n = y^{n-1} \pm \Delta \}$$

$$G^{m,n} = \{ g^{m,n} = g^m \cdot g^n \mid g^m = (\pm 1) g^{m-1} \}$$

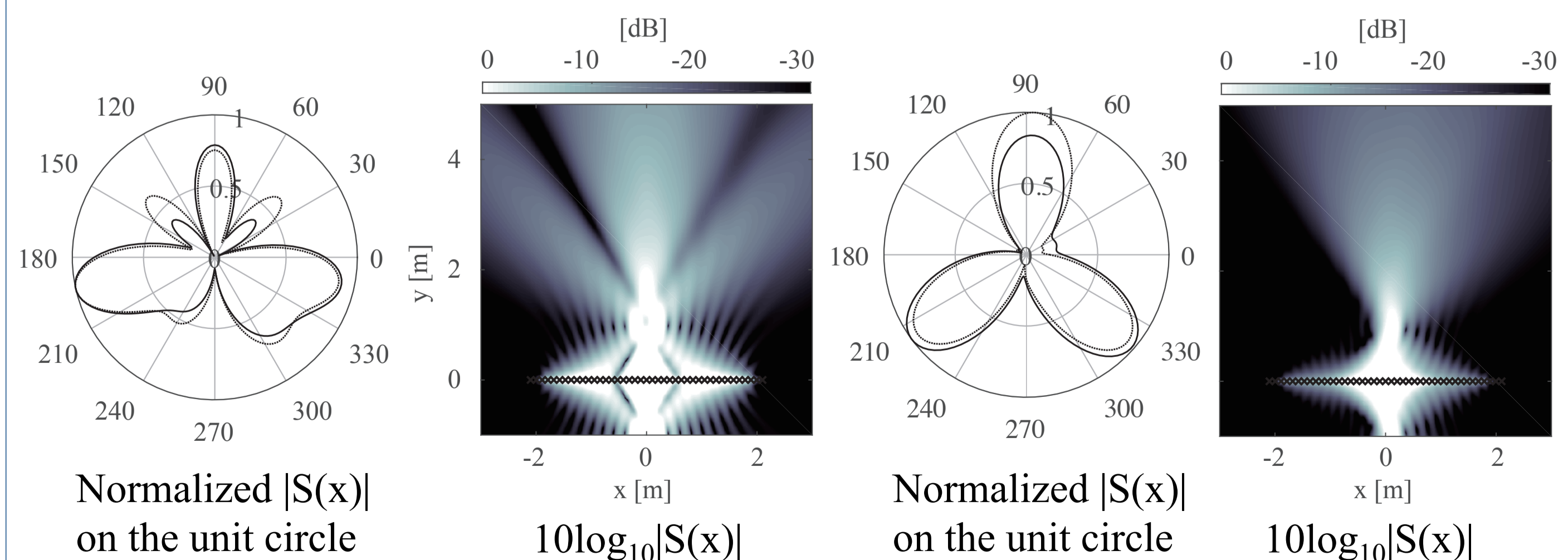
Computer simulations

- Sound field reproduction of multipoles and dipoles comprising focused sources (sine waves)
 - 41-ch linear array with 0.1 m intervals, $f = 1$ kHz
 - virtual sound sources at $(x,y) = (0, 1)$



- Directional source on the basis of superposition of multipoles comprising a cluster of focused sources

- Simulation setup: same as the above cases
- The order of circular harmonics and multipole: $N = 4$
- Original sound field: modeled by randomly generated $\tilde{S}_v^{(2)}$



Selected references

[1] S. Spors +, AES Conv. 127, 2009. [2] K. Maki +, AST. Vol. 67, 2011.