A Refined Analysis of the Gap between Expected Rate for Partial CSIT and the Massive MIMO Rate Limit

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Objectives

- Weighted Sum Rate (WSR) optimal Beamformer design under partial Channel State Information at Transmitter (CSIT).
- Quantify the impact (Gap) of using an approximate (more tractable) utility function.

MIMO IBC Signal Model

The $N_k \times 1$ received signal at user k in cell b_k with M_k antennas is,

$$\mathbf{y}_k = \underbrace{\mathbf{H}_{k,b_k}\mathbf{G}_k\mathbf{x}_k}_{\text{signal}} + \underbrace{\sum_{\substack{i\neq k\\ b_i=b_k}} \mathbf{H}_{k,b_k}\mathbf{G}_i\mathbf{x}_i}_{\text{intracell interf.}} + \underbrace{\sum_{j\neq b_k}\sum_{i:b_i=j} \mathbf{H}_{k,j}\mathbf{G}_i\mathbf{x}_i}_{\text{intercell interf.}} + \mathbf{v}_k$$

- $\mathbf{H}_{k,b_k}$ is the $N_k \times M_{b_k}$ channel from BS b_k to user k.
- $\mathbf{G}_k$ is the $M_{b_k} \times d_k$ Tx beamformer(BF) for d_k streams. $\mathbf{Q}_k = \mathbf{G}_k\mathbf{G}_k^H$.

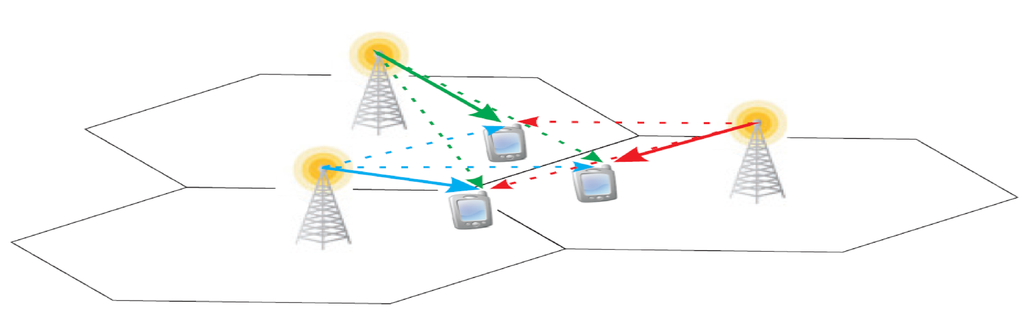

Figure 1: Illustration of a MIMO IBC scenario.

The Gaussian CSIT model for the partial CSIT is,

$$\mathbf{H}_{k,b_k} = \overline{\mathbf{H}}_{k,b_k} + \widetilde{\mathbf{H}}_{k,b_k}\mathbf{C}_{t,k,b_k}^{1/2}$$

where $\overline{\mathbf{H}}_{k,b_k} = \mathbf{E}\mathbf{H}_{k,b_k}$, $\mathbf{C}_t$ is the Tx side covariance matrix and $\widetilde{\mathbf{H}}_{k,b_k} \sim \mathcal{CN}(\mathbf{0},\mathbf{I})$.

Expected WSR (EWSR)

$$\text{EWSR}(\mathbf{G}) = \mathbf{E}\sum_{k=1}^{K} u_k \left(\ln|\mathbf{I}+\mathbf{H}_k\mathbf{Q}\mathbf{H}_k^H| - \ln|\mathbf{I}+\mathbf{H}_k\mathbf{Q}_{\bar{k}}\mathbf{H}_k^H|\right)$$

$$\mathbf{H}_k = [\mathbf{H}_{k,b_1}\cdots\mathbf{H}_{k,b_{k-1}}\ \mathbf{H}_{k,b_k}\ \mathbf{H}_{k,b_{k+1}}\cdots\mathbf{H}_{k,b_K}] = \overline{\mathbf{H}}_k + \widetilde{\mathbf{H}}_k\mathbf{C}_{t,k}^{\frac{1}{2}}$$

$\mathbf{Q}$ is a block diagonal matrix with i^{th} diagonal block being $\sum_{l:b_l=b_i}\mathbf{Q}_l$. $\mathbf{Q}_{\bar{k}}$ is similar to $\mathbf{Q}$ but with the k^{th} block diagonal set to $\sum_{l:b_l=b_k}^{l\neq k}\mathbf{Q}_l$.

MaMIMO limit and ESEI-WSR

$$\mathbf{H}\mathbf{Q}\mathbf{H}^H \overset{M\to\infty}{\longrightarrow} \mathbf{E}\,\mathbf{H}\mathbf{Q}\mathbf{H}^H = \overline{\mathbf{H}}\mathbf{Q}\overline{\mathbf{H}}^H + \text{tr}\{\mathbf{Q}\mathbf{C}_t\}\,\mathbf{I}$$

$$\text{ESEI-WSR}(\mathbf{G}) = \sum_{k=1}^{K} u_k \left(\ln|\mathbf{I}+\mathbf{E}\,\mathbf{H}_k\mathbf{Q}\mathbf{H}_k^H| - \ln|\mathbf{I}+\mathbf{E}\,\mathbf{H}_k\mathbf{Q}_{\bar{k}}\mathbf{H}_k^H|\right)$$

- ESEI-WSR can be solved using well known techniques for full CSIT.

Can the beamformer matrix G be derived from ESEI-WSR instead of the EWSR ?

Monotonicity of gap with SNR

- For an SNR ρ, define $\Gamma_k(\rho) = \ln|\mathbf{I}+\rho\mathbf{E}\mathbf{H}_k'\mathbf{H}_k'^H| - \mathbf{E}\ln|\mathbf{I}+\rho\mathbf{H}_k'\mathbf{H}_k'^H|$, where $\mathbf{H}_k' \sim \mathcal{CN}(\frac{1}{\sqrt{\rho}}\overline{\mathbf{H}}_k\mathbf{Q}^{\frac{1}{2}}, \frac{1}{\rho}\mathbf{C}_{t,k}^{\frac{1}{2}}\mathbf{Q}\mathbf{C}_{t,k}^{\frac{1}{2}})$.

$\Gamma_k(\rho)$ monotonically increases with ρ; $\Gamma_k(0) = 0$

- As a result, we can write,

$$\text{ESEI-WSR}(\mathbf{G}) - \sum_{k=1}^{K} u_k\Gamma_k(\infty) \le \text{EWSR}(\mathbf{G}) \le \text{ESEI-WSR}(\mathbf{G}) + \sum_{k=1}^{K} u_k\Gamma_{\bar{k}}(\infty)$$

MISO zero mean corr. channel

$$0 \le \ln(1+\rho\sum_{i=1}^{p}\lambda_i) - \mathbf{E}\ln(1+\rho||\mathbf{h}||^2) \le \gamma - \left(\sum_{i=1}^{p}\frac{\ln\lambda_i}{\pi_{l\neq i}(1-\lambda_l/\lambda_i)} - \ln(\sum_{i=1}^{p}\lambda_i)\right),$$

where λ_is correspond to the p eigen values of $\mathrm{E}\mathbf{h}^H\mathbf{h}$, ρ is the SNR, γ is Euler constant.
In the case of p identical Eigenvalues,

$$0 \le \Gamma(\infty) \le \gamma - \left(\sum_{k=1}^{p}\frac{1}{k} - \ln(p)\right) + \frac{1}{p}$$

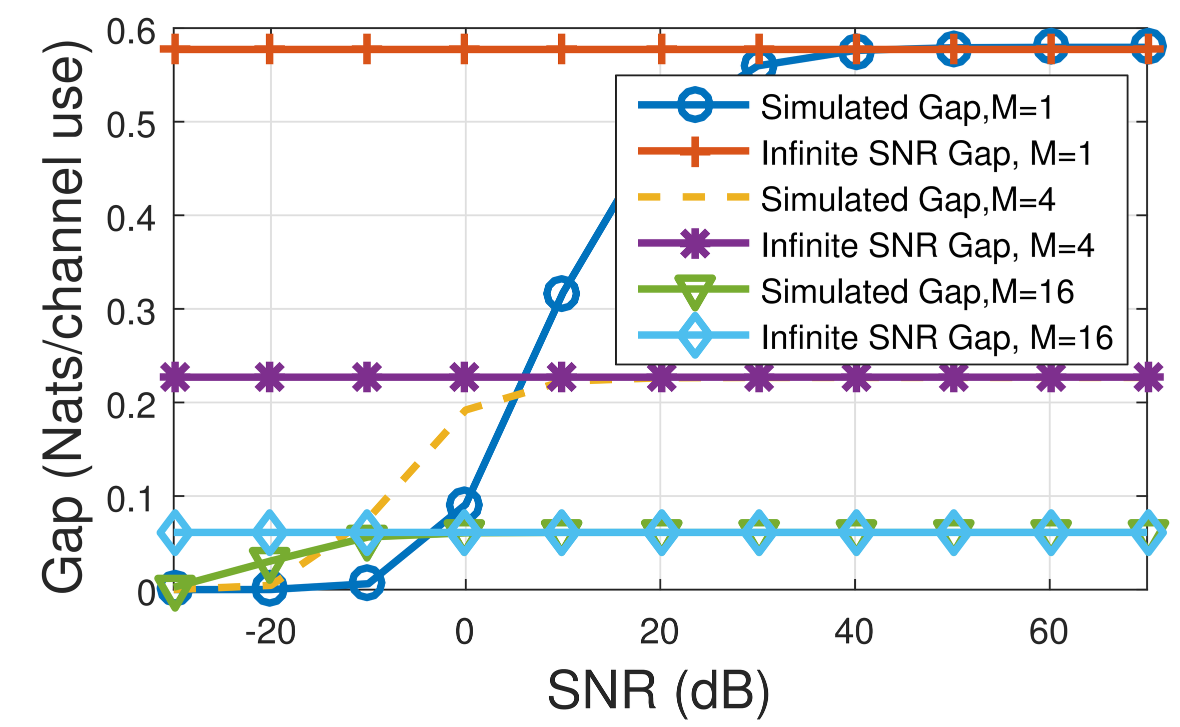

Figure 2: Gap between ESEI-WSR and EWSR for the MISO correlated scenario for different values of transmit antennas.

MIMO zero mean i.i.d channel

Bartlett decomposition

$$\mathbf{H}\mathbf{H}^H = \mathbf{L}\mathbf{D}\mathbf{L}^H = (\mathbf{L}\mathbf{D}^{\frac{1}{2}})(\mathbf{L}\mathbf{D}^{\frac{1}{2}})^H$$

where lower triangular matrix $\mathbf{L}$ has unit diagonal and $\mathbf{D}$ is a diagonal matrix with diagonal entries $(\mathbf{D}_i)$ greater than zero.

$$\mathbf{D}_i \sim \frac{1}{2}\chi^2_{2(M-i+1)},\ i \in 1\cdots N_k;\ \mathbf{L}_{i,j}\mathbf{D}_i^{\frac{1}{2}} \sim \mathcal{CN}(0,1),\ i > j$$

Hence, $\ln|\mathbf{H}\mathbf{H}^H| = \sum_{i=1}^{N_k}\ln|\mathbf{D}_i|$ and this can be treated as a sum of MISO i.i.d scenarios.

Second-Order Taylor Series approximation for Gap

- Taylor expansion of $\mathbf{E}\ln|\mathbf{I}+\rho\mathbf{H}\mathbf{H}^H|$ around $\mathbf{X} = \mathbf{I}+\rho\mathbf{E}\,\mathbf{H}\mathbf{H}^H$.
- Retaining only the second order terms,

$$\mathbf{E}\ln|\mathbf{I}+\rho\mathbf{H}\mathbf{H}^H| - \ln|\mathbf{I}+\rho\mathbf{E}\mathbf{H}\mathbf{H}^H| \approx \Gamma_2(\rho) = \frac{\rho^2}{2}\text{tr}\left\{\text{tr}\{\mathbf{X}^{-1}\}^2\mathbf{C}^2 + 2\text{tr}\{\mathbf{X}^{-1}\}\overline{\mathbf{H}}^H\mathbf{X}^{-1}\overline{\mathbf{H}}\mathbf{C} - (\overline{\mathbf{H}}^H\mathbf{X}^{-1}\overline{\mathbf{H}})^2\right\}.$$

Actual EWSR Gap

- **Can we quantify the gap $|\text{EWSR}(\mathbf{G}^*) - \text{EWSR}(\mathbf{G}^{**})|$**, where $\mathbf{G}^*$ are the optimal BFs that maximize the EWSR and $\mathbf{G}^{**}$ are the BFs that maximize ESEI-WSR.

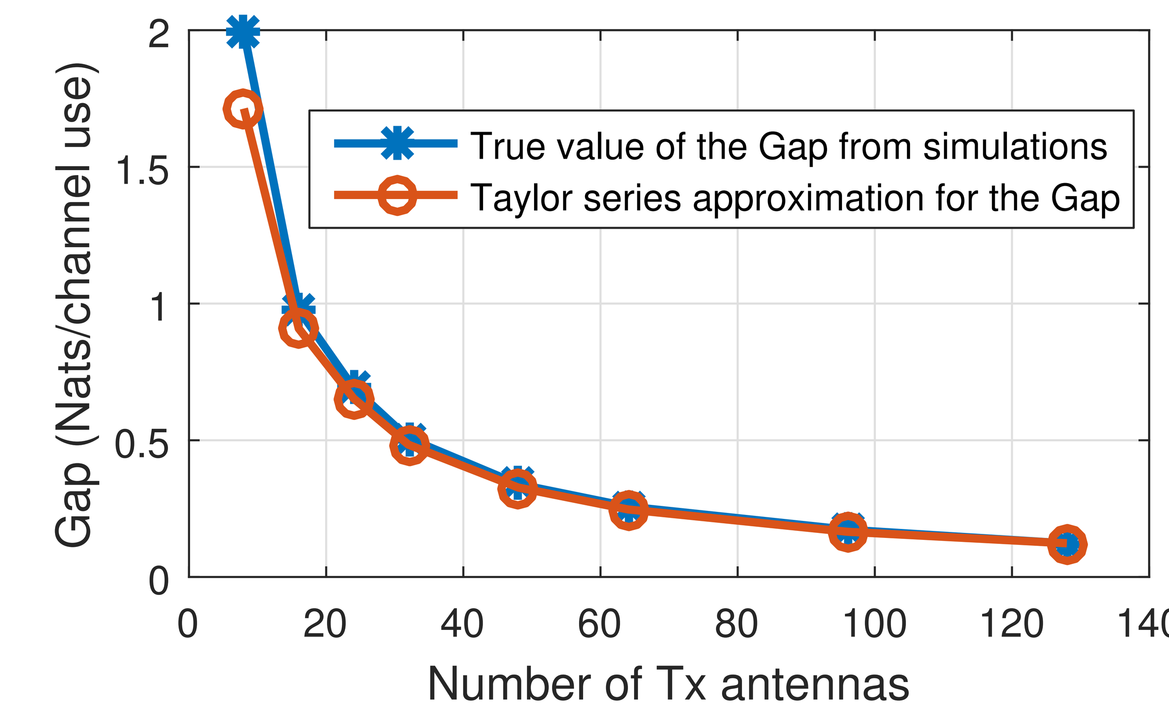

Figure 3: Gap obtained from the second order Taylor series approximation vs. the true value of the gap for a MIMO zero mean correlated scenario. Number of Rx antennas $N_k = 4$.

Actual EWSR Gap (Cont'd)

- Assume interference limited to a subspace and sufficient number of Tx antennas at each BS.
- Then, at high SNR, optimal beamformers perform Zero Forcing (ZF).

For a zero mean correlated MISO IBC channel allowing covariance CSIT based ZF, at infinite SNR, $|\text{EWSR}(\mathbf{G}^*) - \text{EWSR}(\mathbf{G}^{**})| = 0$.

- In the MIMO case, consider a per stream approach. At the output of a linear Rx $\mathbf{f}_k$, the signal estimate for stream k would be,

$$\widehat{x}_k = \mathbf{f}_k^H\mathbf{H}_{k,b_k}\mathbf{g}_k x_k + \sum_{i\neq k}\mathbf{f}_k^H\mathbf{H}_{k,b_i}\mathbf{g}_i x_i + \mathbf{f}_k^H\mathbf{v}_k.$$

- The scenario is now identical to that of the MISO scenario.

Conclusions and Future work

- Analyzed the Gap for certain MISO and MIMO scenarios.
- Initiated a discussion on actual EWSR gap.
- General case of MIMO correlated scenario to be addressed.
- Need for further analysis on actual EWSR gap.