UNBIASED DISTANCE BASED NON-LOCAL FUZZY MEANS

Xiaoyao Li [1,2], Yingcong Zhou [2], Jing Zhang [1], Lianhong Wang [1]

[1] College of Electrical and Information Engineering, Hunan University, Changsha 410082, China
[2] Department of Computer and Information Science, University of Macau, Macau 999078, China
Email: lxyimayday@gmail.com, yicongzhou@umac.mo

INTRODUCTION

Image denoising: It aims at removing the noise effectively and also performing well in detail preservation.

Non-local Means (NLM):

Given a noisy image as **Y = X + N**, where **X** is a clean image and **N is** a noise model, NLM calculates the weight ω_{ij} for each pixel in a search window S_i, and then obtains the target patch $\hat{\mathbf{X}}_i$ as

$$\omega_{ij} = \exp(-\frac{||\mathbf{Y}_i - \mathbf{Y}_j||^2}{h^2}), \qquad \hat{\mathbf{X}}_i = \frac{\sum_{j\in\mathbf{S}_i}\omega_{ij}\mathbf{Y}_j}{\sum_{j\in\mathbf{S}_i}\omega_{ij}}.$$

Limitation:

NLM and its improvements consider weight w_{ij} as a constant, which means they only calculate ω ij once and keep it unchanged during later iterative denoising processes. This is improper because the denoised image and patch similarity will change after each iteration.

Our contributions:

- We propose three unbiased distances, namely pixel-pixel unbiased distance, patch-patch unbiased distance and combined unbiased distance, which are robust to measure the similarity between image pixels or between image patches.
- We propose Unbiased Distance based NLFM (UDNLFM), which considers weight ω_{ij} as a variable and updates its value in each denoising iteration via computing the combined unbiased distances between patches.

PROPOSED METHOD

Optimization Models

Method	Optimization Model
Mean Filter	$\hat{\mathbf{X}}(i) = \arg\min_{\mathbf{X}(i)} \sum_{j\in\mathbf{S}_i} (\mathbf{X}(i) - \mathbf{Y}(j))^2$
Gaussian Filter	$\hat{\mathbf{X}}(i) = \arg\min_{\mathbf{X}(i)} \sum_{j\in\mathbf{S}_i} \omega_{ij}(\mathbf{X}(i) - \mathbf{Y}(j))^2$
Median Filter	$\hat{\mathbf{X}}(i) = \arg\min_{\mathbf{X}(i)} \sum_{j\in\mathbf{S}_i} \lvert\mathbf{X}(i) - \mathbf{Y}(j)\rvert$
NLM	$\hat{\mathbf{X}}_i = \arg\min_{\mathbf{X}_i} \sum_{j\in\mathbf{S}_i} \omega_{ij}\lVert\mathbf{X}_i - \mathbf{Y}_j\rVert^2$
NLEM	$\hat{\mathbf{X}}_i = \arg\min_{\mathbf{X}_i} \sum_{j\in\mathbf{S}_i} \omega_{ij}\lVert\mathbf{X}_i - \mathbf{Y}_j\rVert$
INLEM	$\hat{\mathbf{X}}_i = \arg\min_{\mathbf{X}_i} \sum_{j\in\mathbf{S}_i} \sqrt{\omega_{ij}}\lVert\mathbf{X}_i - \mathbf{Y}_j\rVert$
PNLM	$\hat{\mathbf{X}}_i = \arg\min_{\mathbf{X}_i} \sum_{j\in\mathbf{S}_i} f_{ij}\lVert\mathbf{X}_i - \mathbf{Y}_j\rVert^2$
NLFM	$\{\hat{\mathbf{X}}_i, \hat{\omega}_{ij}\} = \arg\min_{\mathbf{X}_i,\omega_{ij}} \sum_{j\in\mathbf{S}_i} \omega_{ij}^m\lVert\mathbf{X}_i - \mathbf{Y}_j\rVert^2$

Unbiased Distances

- squared pixel-pixel unbiased distance

$$\mathbb{D}_{\mathrm{U}}^2\big(\mathbf{Y}(i),\mathbf{Y}(j)\big) = \big(\mathbf{Y}(i) - \mathbf{Y}(j)\big)^2 - 2\sigma^2$$

- squared patch-patch unbiased distance

$$\mathbb{D}_{\mathrm{U}}^2\big(\mathbf{Y}_i,\mathbf{Y}_j\big) = ||\mathbf{Y}_i - \mathbf{Y}_j||^2 - 2||\mathbf{P}||\sigma^2$$

$$\mathbb{D}_{\mathrm{U}}^2\big(\hat{\mathbf{X}}_i,\mathbf{Y}_j\big) = ||\hat{\mathbf{X}}_i - \mathbf{Y}_j||^2 - ||\mathbf{P}||\Big(\sum_{l\in\mathbf{S}_i}\omega_{il}^2 - 2\omega_{ij} + 1\Big)\sigma^2$$

where $\sum_{l\in\mathbf{S}_i}\omega_{il} = 1$ and $\hat{\mathbf{X}}_i = \sum_{l\in\mathbf{S}_i}\omega_{il}\mathbf{Y}_l$.

- squared combined unbiased distance

$$\mathbb{D}_{\mathrm{C}}^2\big(\hat{\mathbf{X}}_i,\mathbf{Y}_j\big) = \alpha\cdot\max\big[0, \mathbb{D}_{\mathrm{U}}^2(\hat{\mathbf{X}}_i,\mathbf{Y}_j)\big] + \beta\cdot\big(\bar{\hat{x}}_i - \bar{\hat{x}}_j\big)^2$$

$\bar{\hat{x}}_i$ and $\bar{\hat{x}}_j$ are average pixel values of the related denoised patch. α and β are trade-off parameters.

UDNLFM

Using this combined unbiased distance, we introduce the optimization model of UDNLFM as

$$\{\hat{\mathbf{X}}_i, \hat{\omega}_{ij}\} = \arg\min_{\mathbf{X}_i,\omega_{ij}} \sum_{j\in\mathbf{S}_i} \omega_{ij}\mathbb{D}_{\mathrm{C}}^2\big(\mathbf{X}_i,\mathbf{Y}_j\big).$$

In order to solve this optimization problem, we initialize $\hat{\mathbf{X}}_i^{(0)} = \mathbf{Y}_i$ and then update ω_{ij} and $\hat{\mathbf{X}}_i$ alternatively using the following two equations.

$$\omega_{ij}^{(t+1)} = \exp\left(-\frac{\mathbb{D}_{\mathrm{C}}^2\big(\hat{\mathbf{X}}_i^{(t)},\mathbf{Y}_j\big)}{h^2}\right)\cdot H_{ij},$$

$$\hat{\mathbf{X}}_i^{(t+1)} = \frac{\sum_{j\in\mathbf{S}_i}\omega_{ij}^{(t)}\mathbf{Y}_j}{\sum_{j\in\mathbf{S}_i}\omega_{ij}^{(t)}},$$

where $H_{ij} = \exp\left(-\frac{(i-j)^2}{h_s}\right)$

Algorithm 1 UDNLFM

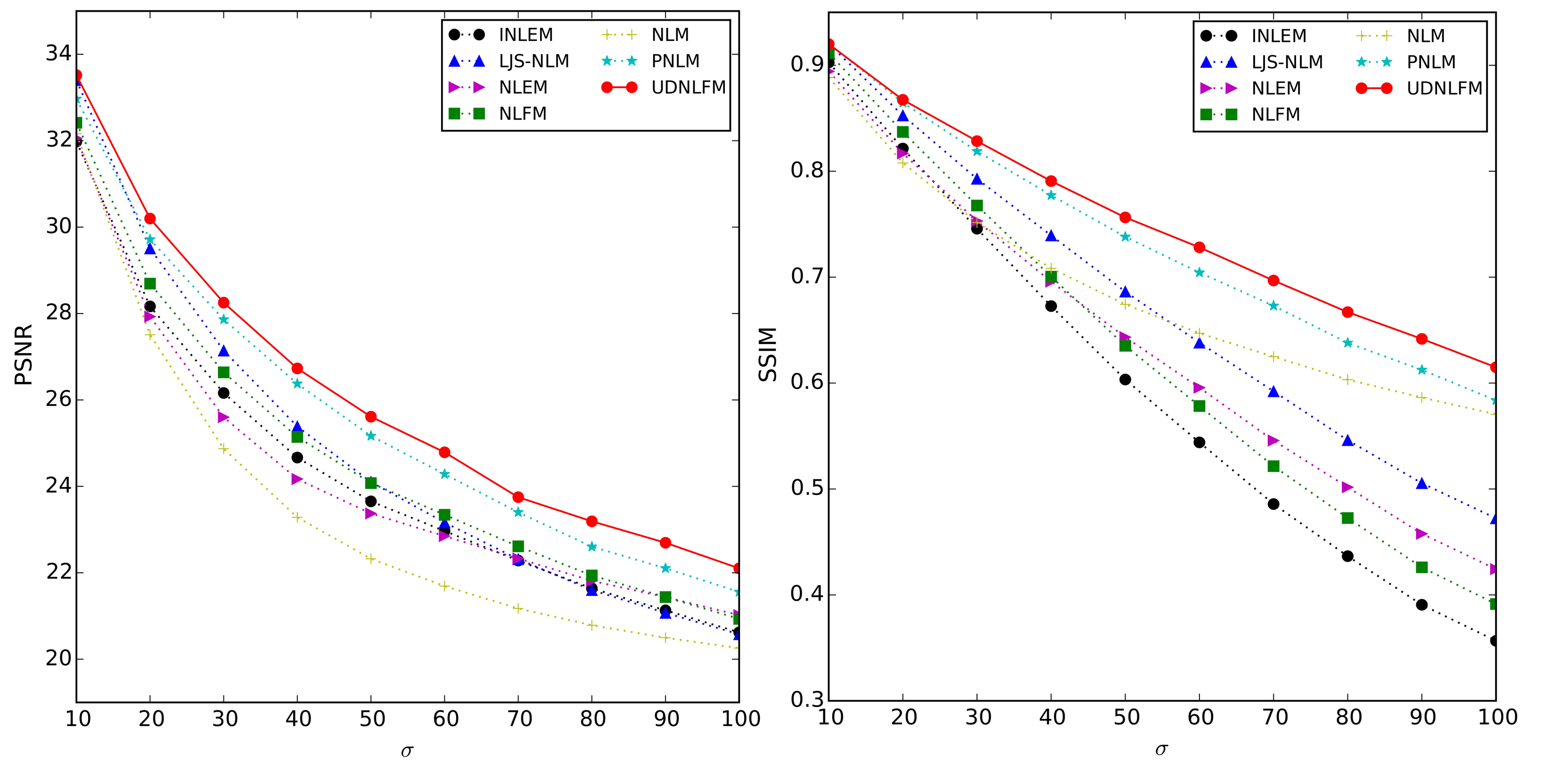
Input: The noisy image $\mathbf{Y}$, the radius of patch k, the radius of search window s and other parameters h, h_s, α, β.

- Step 1: Extract a patch $\mathbf{Y}_i$ with radius k centered at each pixel i in $\mathbf{Y}$.
- Step 2: For each pixel i, do
  - (a) Use $\hat{\mathbf{X}}_i^{(0)} = \mathbf{Y}_i$ as initial values, and iteratively find $\{\hat{\mathbf{X}}_i, \hat{\omega}_{ij}\} = \arg\min_{\mathbf{X}_i,\omega_{ij}} \sum_{j\in\mathbf{S}_i}\omega_{ij}\mathbb{D}_{\mathrm{C}}^2\big(\mathbf{X}_i,\mathbf{Y}_j\big)$ by Eq. (9) and (10).
  - (b) Assign $\hat{\mathbf{X}}(i)$ as the center pixel value in $\hat{\mathbf{X}}_i$.

Output: Denoised image $\hat{\mathbf{X}}$.

EXPERIMENTS

Average values of (a) PSNR and (b) SSIM on all the test images.

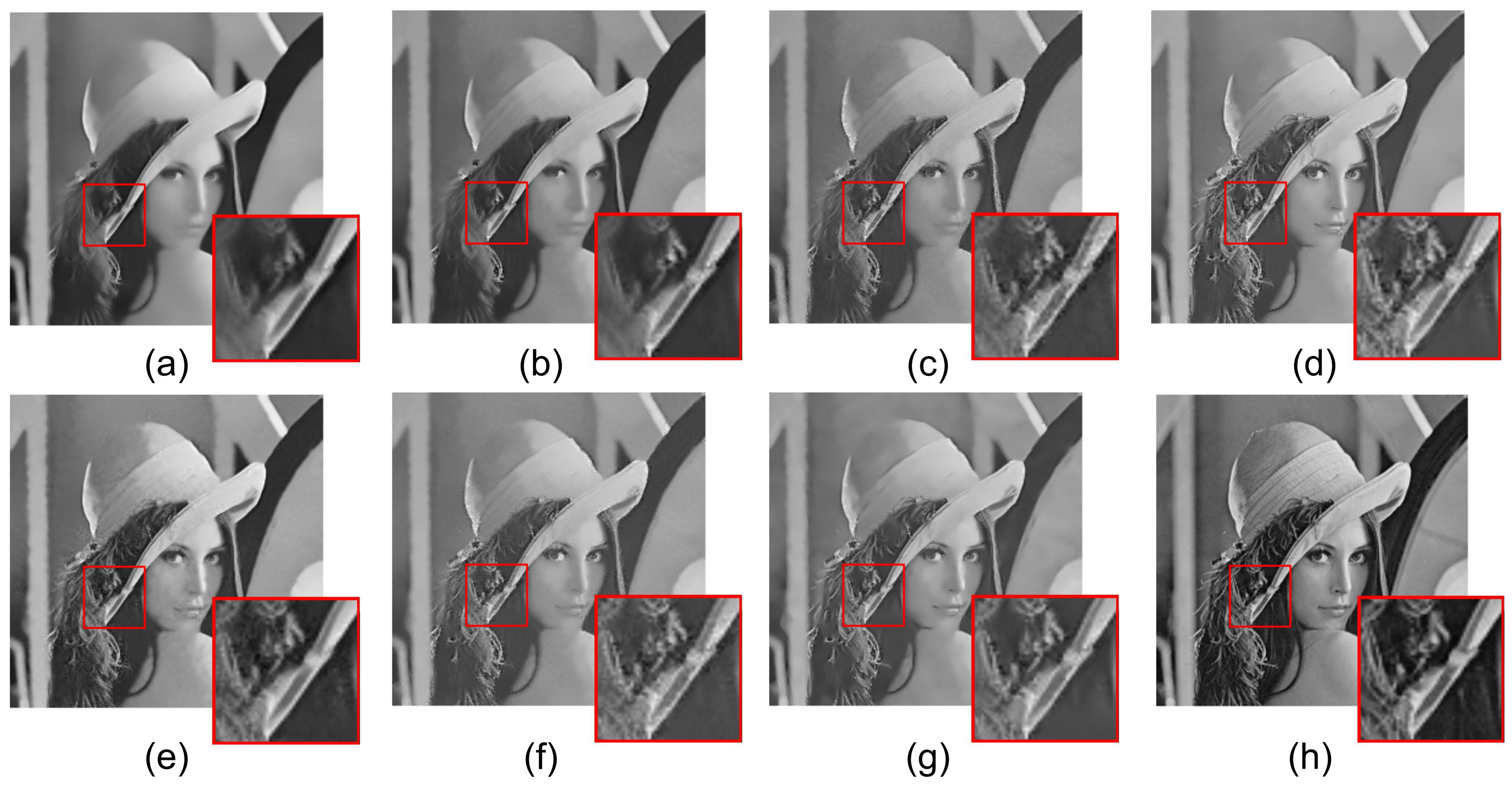


(a) (b)

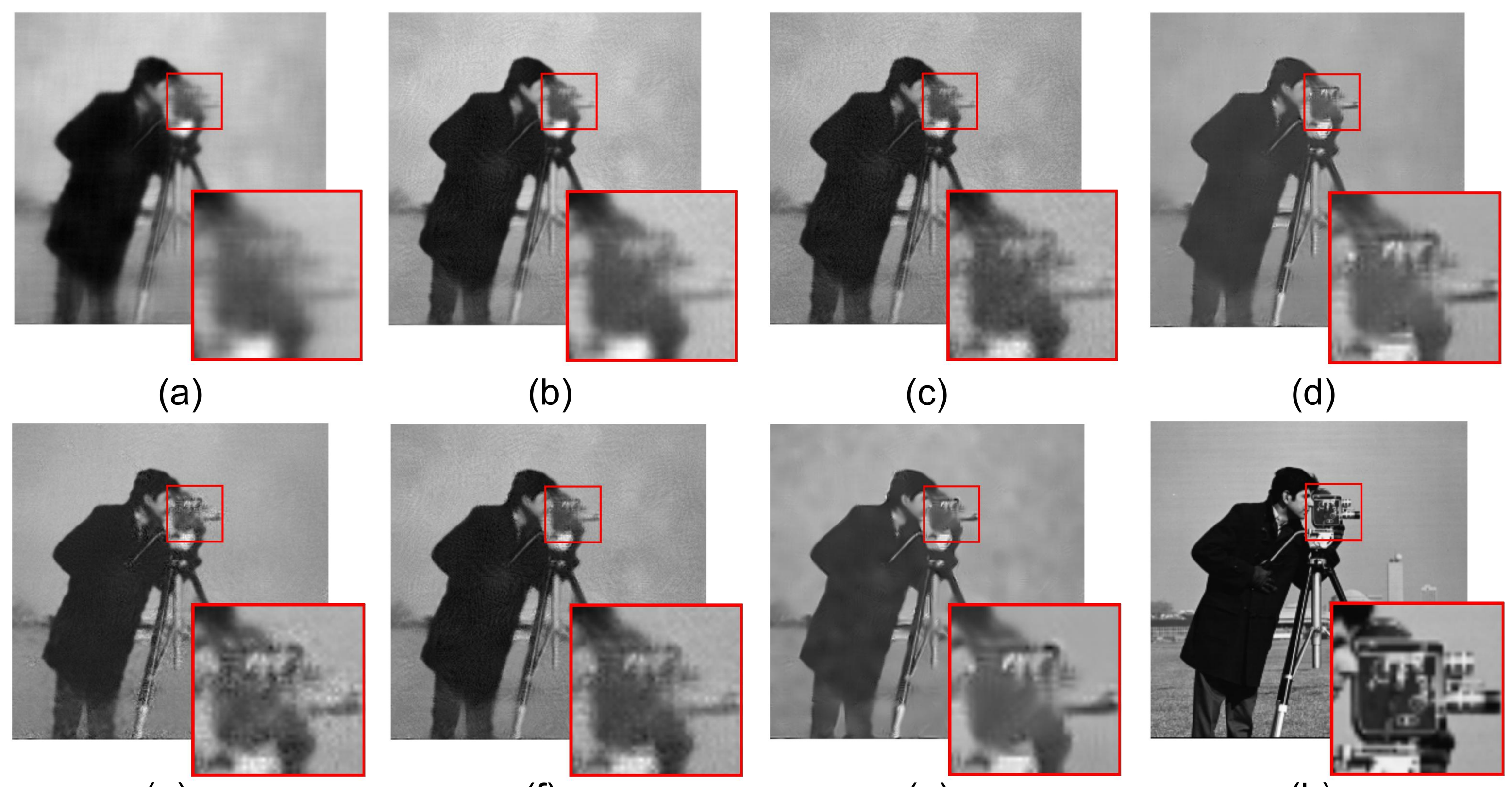
PSNR and SSIM results of UDNLFM and other methods at noise levels σ=10,20,...,100

Image	Method\σ	PSNR(dB) 10	20	30	40	50	60	70	80	90	100	SSIM(%) 10	20	30	40	50	60	70	80	90	100
c.man	NLM	31.49	27.98	24.46	22.61	21.50	20.96	20.43	19.87	19.64	19.43	87.86	81.97	76.29	71.78	67.99	65.33	62.98	60.16	58.25	56.77
	NLEM	31.37	28.11	25.16	23.26	22.27	21.96	21.53	20.98	20.76	20.40	88.52	81.98	74.72	68.13	61.65	56.93	52.17	47.50	43.11	40.38
	INLEM	31.80	27.81	25.81	24.08	22.76	22.25	21.64	20.96	20.60	20.14	90.38	81.50	73.55	65.96	57.69	51.85	46.29	41.17	36.76	33.86
	PNLM	32.36	28.89	27.23	26.06	24.84	24.05	**22.97**	22.05	21.69	21.01	**92.49**	**85.85**	81.54	78.60	75.02	71.99	68.44	64.61	62.58	58.99
	LJS-NLM	**33.06**	29.36	26.94	25.25	23.77	22.97	22.09	21.20	20.78	20.20	92.30	85.83	79.49	74.92	69.20	64.70	60.09	54.72	50.63	46.97
	NLFM	32.15	28.14	26.16	24.75	23.38	22.72	21.99	21.23	20.88	20.40	91.60	82.72	75.58	69.24	61.51	55.74	50.13	44.87	40.28	37.27
	UDNLFM	32.98	**29.37**	**27.64**	**26.44**	**25.13**	**24.37**	22.41	**22.45**	**22.01**	**21.36**	92.21	85.70	**82.13**	**79.55**	**75.94**	**73.83**	**69.13**	**66.49**	**63.88**	**60.64**
lena	NLM	31.58	26.49	24.23	22.85	22.00	21.38	20.84	20.48	20.27	20.07	87.79	77.42	71.07	66.27	62.45	59.39	56.91	54.93	53.45	52.26
	NLEM	31.44	27.06	25.05	24.01	23.34	22.90	22.40	21.77	21.42	21.03	88.50	79.29	73.16	68.59	63.93	59.67	54.95	50.51	46.60	43.25
	INLEM	31.38	27.73	25.75	24.43	23.55	22.95	22.30	21.57	21.08	20.60	89.49	81.27	73.70	67.19	60.91	55.44	49.74	44.78	40.48	37.09
	PNLM	32.16	28.76	27.10	25.90	24.61	23.74	22.89	21.97	21.58	21.15	91.03	84.83	79.71	75.26	70.49	66.42	62.90	58.85	56.02	53.96
	LJS-NLM	32.59	28.66	26.52	24.89	23.62	22.67	21.74	21.00	20.54	20.13	91.10	83.51	77.46	71.90	66.01	60.90	55.96	51.41	47.57	44.97
	NLFM	31.76	28.13	26.21	24.82	23.83	23.18	22.52	21.77	21.33	20.87	90.32	82.82	75.80	69.52	63.42	58.15	52.68	47.83	43.56	40.17
	UDNLFM	**32.72**	**29.31**	**27.44**	**26.08**	**24.91**	**24.16**	**23.55**	**22.65**	**22.29**	**21.90**	**91.20**	**85.35**	**80.73**	**76.39**	**72.39**	**68.99**	**66.25**	**62.42**	**60.22**	**58.34**
peppers	NLM	32.29	27.66	24.84	22.91	21.75	20.87	20.41	20.04	19.72	19.39	90.17	82.81	76.84	71.84	67.71	64.19	61.82	59.39	57.73	55.36
	NLEM	32.01	28.18	25.70	24.03	23.13	22.29	21.72	21.27	20.80	20.42	90.41	83.89	78.02	72.09	67.19	62.07	57.46	53.28	48.62	44.54
	INLEM	31.56	28.25	26.14	24.49	23.49	22.53	21.81	21.23	20.65	20.15	90.43	83.90	77.21	69.89	63.45	57.34	51.86	47.21	42.32	38.16
	PNLM	32.74	29.79	27.94	26.32	25.09	24.07	23.06	22.22	21.62	20.94	91.88	87.35	83.28	79.00	74.98	71.66	68.33	64.82	62.19	58.39
	LJS-NLM	33.25	29.51	27.01	25.07	23.68	22.46	21.59	20.89	20.26	19.64	**92.01**	86.19	80.69	75.03	69.51	64.41	59.78	55.44	51.18	47.14
	NLFM	32.04	28.60	26.50	24.89	23.93	23.00	22.22	21.59	20.97	20.47	91.15	85.09	79.02	72.37	66.41	60.72	55.43	50.84	45.89	41.65
	UDNLFM	**33.58**	**30.58**	**28.54**	**26.77**	**25.83**	**24.88**	**23.99**	**23.20**	**22.41**	**21.83**	91.97	**88.34**	**85.12**	**81.21**	**78.14**	**75.21**	**72.42**	**69.59**	**66.74**	**63.27**
house	NLM	33.28	27.91	25.96	24.76	24.05	23.56	23.03	22.76	22.36	22.14	89.60	80.92	76.43	73.45	71.57	69.92	68.26	66.81	65.10	**63.81**
	NLEM	33.41	28.36	26.50	25.38	24.75	24.26	23.68	23.24	22.77	22.24	90.27	81.57	75.31	69.51	64.57	59.59	53.73	49.38	44.76	41.57
	INLEM	33.18	28.88	26.95	25.66	24.80	24.13	23.39	22.80	22.20	21.60	90.77	81.89	73.83	66.02	59.30	52.97	46.42	41.46	36.72	33.55
	PNLM	34.61	31.42	29.18	27.23	26.12	25.28	24.67	24.18	23.53	23.12	**92.81**	**87.84**	83.00	78.05	74.75	71.67	69.51	66.91	64.19	62.13
	LJS-NLM	34.70	30.49	28.10	26.34	25.33	24.57	23.84	23.31	22.70	22.33	92.69	85.52	79.50	73.90	69.81	65.24	61.06	56.86	52.81	49.84
	NLFM	33.71	29.90	27.67	26.12	25.17	24.47	23.73	23.15	22.57	21.99	91.54	84.19	76.64	69.07	62.74	56.71	50.38	45.49	40.68	37.45
	UDNLFM	**34.78**	**31.53**	**29.39**	**27.63**	**26.58**	**25.74**	**25.05**	**24.47**	**24.07**	**23.33**	92.63	87.57	**83.38**	**79.12**	**76.09**	**73.22**	**70.93**	**68.29**	**65.84**	63.71

Denoised results of the *lena* image with noise σ = 20: (a) NLM; (b) NLEM; (c) INLEM; (d) PNLM; (e) LJS-NLM; (f) NLFM; (g) **UDNLFM**; (h) Clean image.



Denoised results of the *cameraman* image with noise σ = 60: (a) NLM; (b) NLEM; (c) INLEM; (d) PNLM; (e) LJS-NLM; (f) NLFM; (g) **UDNLFM**; (h) Clean image.



ACKNOWLEDGEMENT

This work was supported in part by the Macau Science and Technology Development Fund under Grant FDCT/016/2015/A1 and by the Research Committee at University of Macau under Grant MYRG2016-00123-FST.