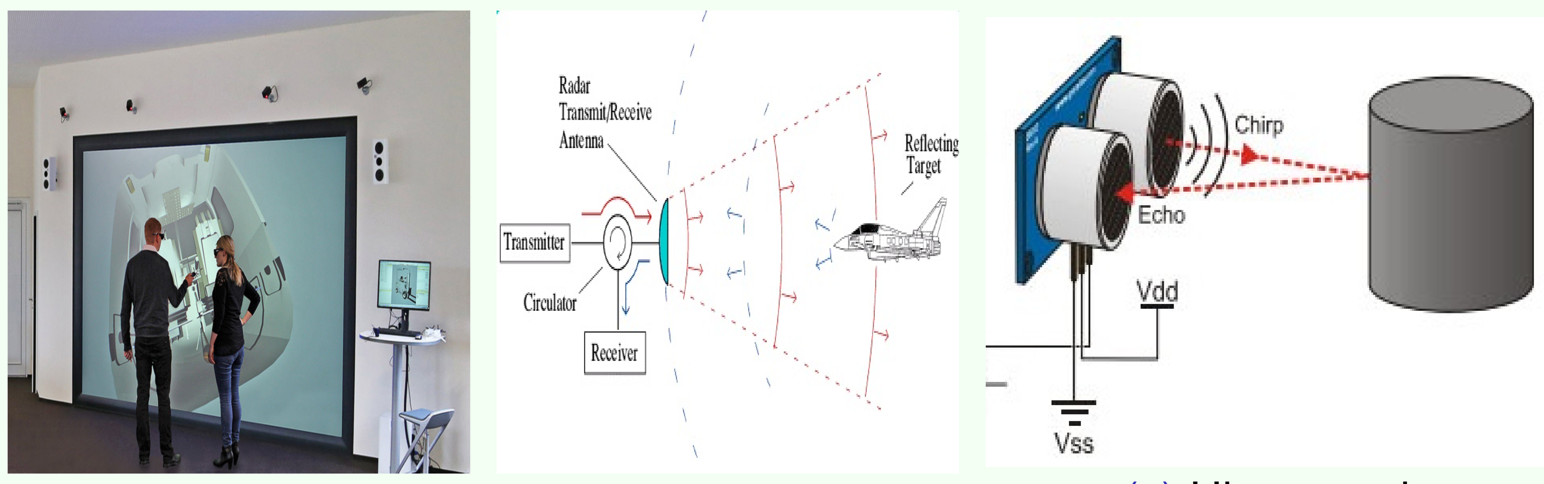
High Accuracy Acoustic Estimation of Multiple Targets

Mohammed H. AlSharif[1], Mohamed Saad[1], Mohamed Siala[2], Hatem Boujemaa[2], Tarig Ballal[1], Tareq Y. Al-Naffouri[1]

[1] King Abdullah University of Science and Technology (KAUST), Saudi Arabia
[2] Higher School of Communication of Tunis (SUP'COM), University of Carthage, Tunisia

Introduction

- Precise estimation of multiple targets is required in many of nowadays technologies.
- Many types of signals have been used for multiple targets estimation:

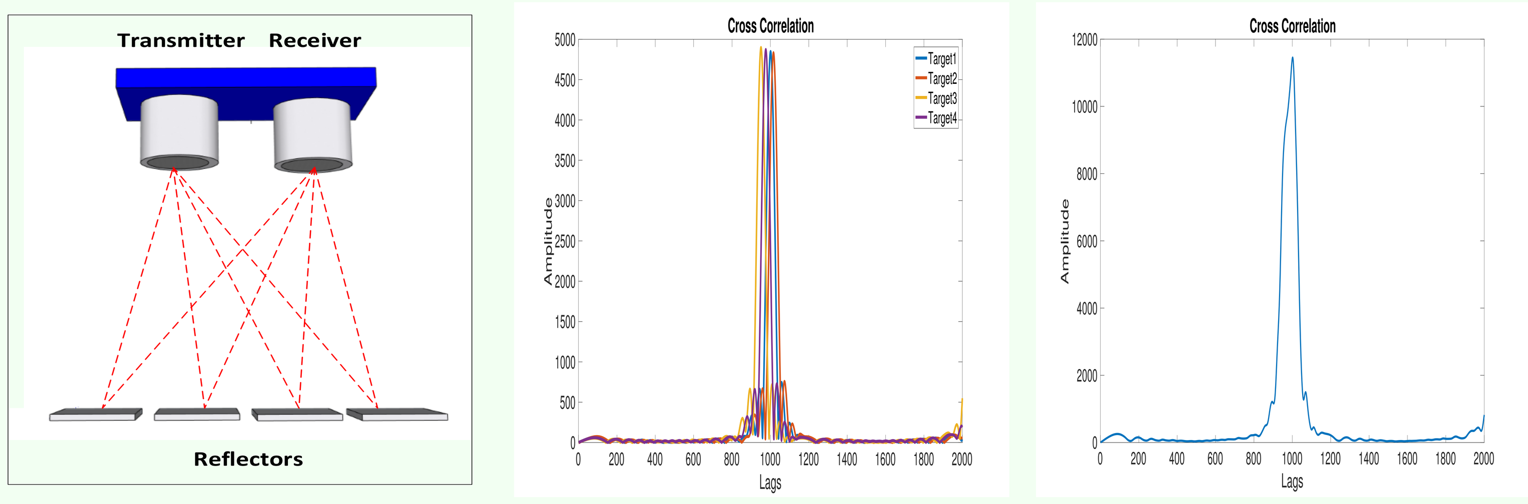


(a) Infrared (b) Radar TOA (c) Ultrasound

- Time of arrival (TOA) estimation using cross-correlation is simple but its resolution is dictated by the available bandwidth:

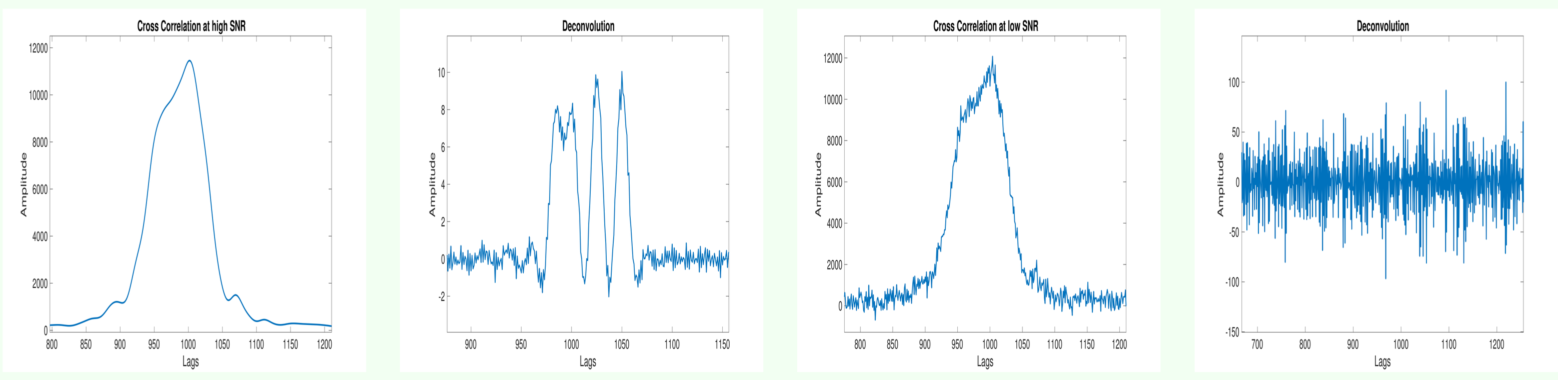
$$R = \frac{2v_s}{BW} \qquad (1)$$

where v_s is the wave propagation speed in the medium and BW is the bandwidth of the system.



(a) Multiple reflections (b) Targets XCorr (c) The result of XCorr

- The challenge is to resolve the TOAs of the reflected signals from the cross-correlation.
- The deconvolution method provides high resolution estimation TOAs of the reflected signals at high SNR, but the accuracy significantly degrades under low SNR.



(a) XCorr at high SNR (b) Deconv. at high SNR (c) XCorr at low SNR (d) Deconv. at low SNR

- Denoising the cross-correlation vector is required before applying the deconvolution method.

Multiple TOAs Estimation

- Transmitting an m-sequence and cross-correlating Rx with Tx gives the system impulse response (SIR): $r_{yx}(l) = h(l) * r_{xx}(l)$
- The Gaussian echo model (GEM) is a good model for SIR [1]:

$$s(\theta; t(nT_s)) = \alpha e^{-\beta(t(nT_s)-\tau)^2}\cos(2\pi f_c(t(nT_s)-\tau)+\phi),$$

where $\theta = [\beta, \tau, f_c, \phi, \alpha]$, β is a BW factor, τ is the TOA, f_c is the center frequency, ϕ is the phase and α is the amplitude of the target signal.

- The SIR in the presence of multiple targets can be modeled as:

$$q(t(nT_s)) = \sum_{m=1}^{M} h(t(nT_s))_m = \sum_{m=1}^{M} s(\theta_m; t(nT_s)) + n(t(nT_s)), \qquad (2)$$

where M is the number of targets, $h(t(nT_s))$ is the impulse response for a single target,and $n(t(nT_s))$ is the noise coming from the discrepancy between the system impulse response and the GEM.

- For a single target, we need to solve:

$$\min_{\theta} \| h(t(nT_s)) - s(\theta, t(nT_s)) \|^2 \qquad (3)$$

- Using Levenberg-Marquardt (LM) algorithm [2], the step vector:

$$p_k = \begin{cases} -(H_k^T H_k)^{-1} H_k^T r_k, & \text{if } \|p_k^{GN}\| \leq \Delta_k, \\ -(H_k^T H_k + \lambda \text{diag}(H_k^T H_k))^{-1} H_k^T r_k, & \text{if } \|p_k^{GN}\| > \Delta_k. \end{cases}$$

where H_k is the gradient vector of the model with respect to the parameter vector θ, r_k is the residual function, Δ_k is the trust region radius, λ is the damping factor, and $p_k^{GN} = -(H_k^T H_k)^{-1} H_k^T r_k$.

- The update of the parameter vector is: $\theta^{(k+1)} = \theta^{(k)} + p_k$.
- For multiple targets, the Expectation-Maximization (EM) algorithm is used [1] to solve the following optimization problem:

$$\min_{\theta_m} \| q - \sum_{m=1}^{M} s(\theta_m) \|^2 \qquad (4)$$

where q is the observed data vector which represents the cross-correlation.

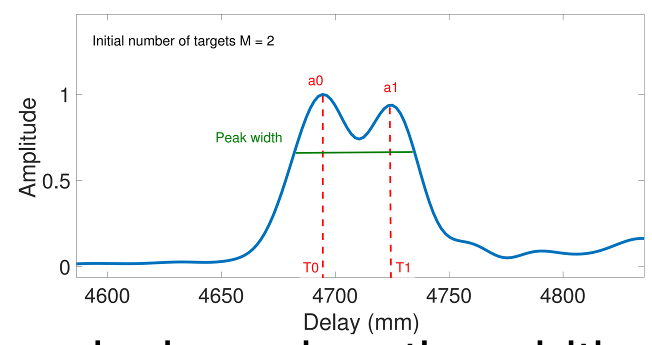
- The EM algorithm defines M unobserved data vectors as $h_m = s(\theta_m) + n_m$, where n_m is an AWGN sequence. The relation between the observed and the unobserved data is: $q = \sum_{m=1}^{M} h_m$.
- The expectation of the unobserved data h_m can be computed as [1]

$$\hat{h}_m^{(k)} = s(\theta_m^{(k)}) + \frac{1}{M}(q - \sum_{l=1}^{M} s(\theta_l^{(k)})) \qquad (5)$$

- The M-step iterates the parameter vector $\theta_m^{(k)}$ by minimizing:

$$\theta_m^{(k+1)} = \arg_{\theta_m} \min \| \hat{h}_m^{(k)} - s(\theta_m) \|^2 \qquad (6)$$

- The initial guess of $\beta_m^{(0)}$,and $f_{cm}^{(0)}$ depends on the available hardware bandwidth and center frequency and the initial phase $\phi_m^{(0)}$ is set to zero.

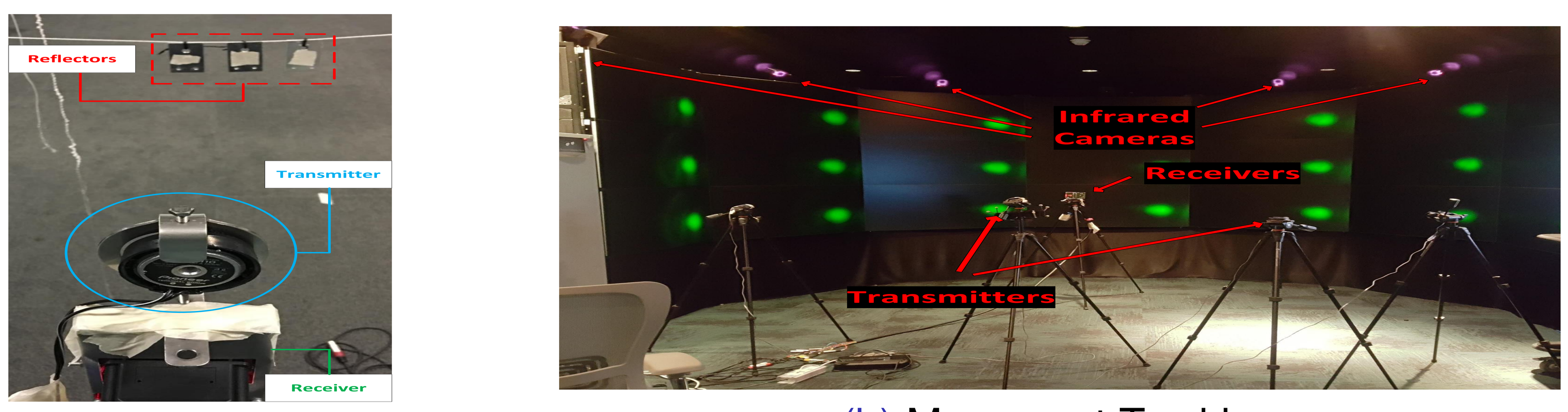


- Determine if there are overlapping peaks based on the width of each peak and the inverse-square law (amplitude $\propto \frac{1}{\text{distance}^2}$).
- The initial guess for the overlapping targets is the same as the initial guess of the m^{th} target except that the TOAs are perturbed.
- The deconvolution method provides high resolution:

$$Z(f) = \frac{Q(f)}{S(f)} \implies z(t) = \text{IFFT}(Z(f)) = \sum_{k=1}^{M} c_k \delta(t - \tau_k) \qquad (7)$$

where $Q(f)$ is the FFT of the estimated GEM of the cross-correlation, $z(t)$ is the high resolution output of the deconvolution method, c_k is the amplitude of the k^{th} target signal, and $\delta(t)$ is the Dirac delta function.

Experimental Setup



(a) Static Ranging (b) Movement Tracking

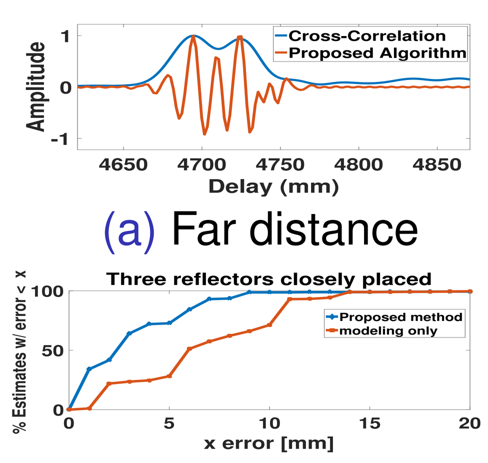
Results

Exps	Proposed Algorithm, RMSE (mm)	σ (mm)	M	GEM estimation RMSE (mm)	σ (mm)	M	CC M
Exp 1	2.11	2.10	3	2.86	2.26	3	3
Exp 2	5.89	5.81	3	10.78	5.91	3	2
Exp 3	3.52	3.48	3	7.32	4.54	3	2
Exp 4	5.11	4.13	3	8.67	3.15	3	1
Exp 5	3.39	2.43	4	3.46	3.34	4	2
Exp 6	3.99	3.08	3	8.52	5.61	3	2

Table: Estimated multiple targets RMSE, σ and M

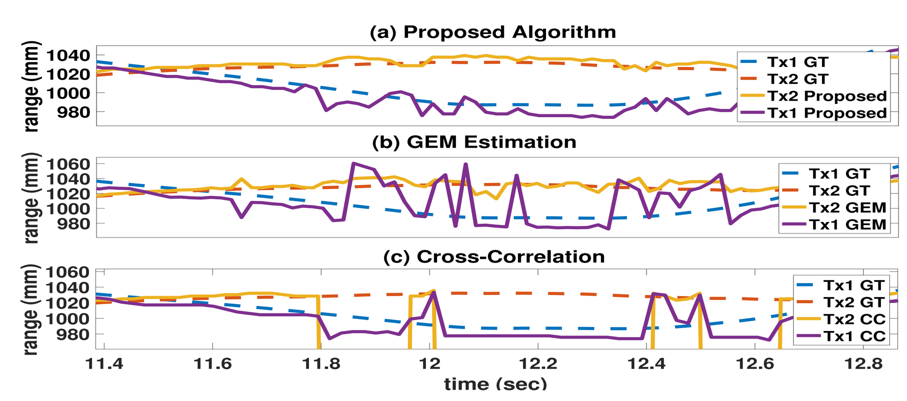
Rx-Tx	Proposed Algorithm RMSE (mm)	σ (mm)	% error < 5 mm	GEM estimation RMSE (mm)	σ (mm)	% error< 5 mm
1-1	2.30	1.48	99.40	1.07	0.97	100
2-1	3.11	3.10	88.80	3.60	3.01	84.43
3-1	2.86	2.38	96.80	2.26	2.20	95.41
4-1	2.35	2.35	97.40	3.28	2.97	90.82
1-2	3.98	2.53	76.65	3.02	2.51	93.41
2-2	8.30	6.35	27.54	9.03	6.15	27.74
3-2	7.21	5.70	50.9	6.21	5.75	42.32
4-2	6.02	4.56	66.67	11.13	10.78	57.68

Table: Tx1 and Tx2 tracking results



(a) Far distance

(b) Three close reflectors



(a) Tracking of two transmitters with the ground truth (GT)

References

1 Ramazan Demirli and Jafar Saniie, "Model-based estimation of ultrasonic echoes. part i: Analysis and algorithms," IEEE transactions on ultrasonics, ferroelectrics, and frequency control, vol. 48, no. 3, pp. 787-802, 2001.
2 Stephen J Wright and Jorge Nocedal, "Numerical optimization," Springer Science, vol. 35, no. 67-68, pp. 7, 1999.