PRIMA: PROBABILISTIC RANKING WITH INTER-ITEM COMPETITION AND MULTI-ATTRIBUTE UTILITY FUNCTION

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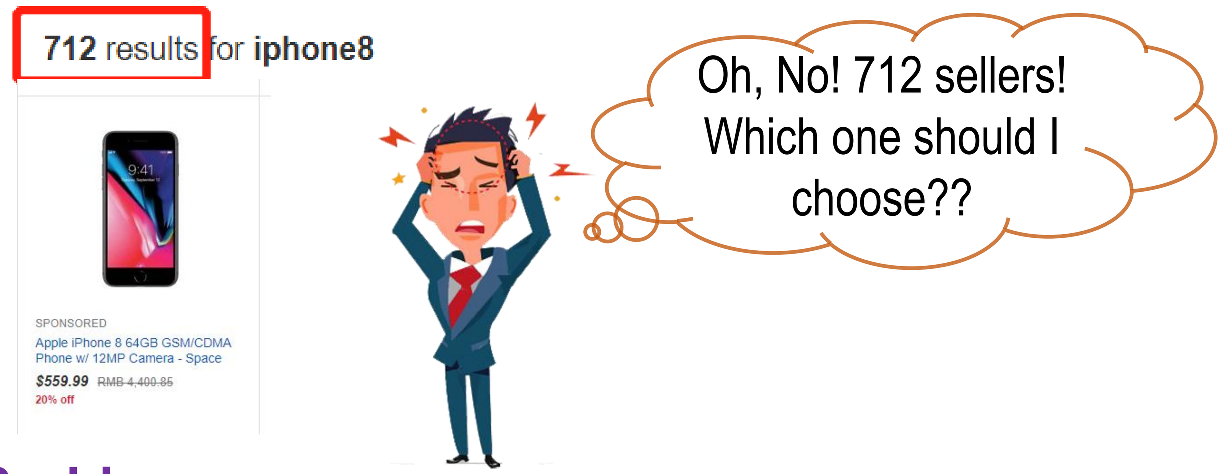
Introduction

Personalized service in E-commerce



Two Problems

- recommending new products [1,2]
- personalized ranking of sellers offering similar products ★

Challenges

- **the inter-attribute tradeoff**
- **the inter-item competition**

Prior Works

Multi-Criteria Decision Making Theory (MCDM) [3]

- explicit utility function
- ignore the inter-item competition

Multi-Attribute Probabilistic Selection (MAPS) [5]

- address the inter-item competition
- reduce information

Indifference Curve Based Method (IC) [6]

- higher accuracy than MAPS
- high complexity

Our solution: use an utility function to simplify the analysis of users' preferences; combine MCDM [3] and IC [6]

The Price-Reputation Plane

- Normalization: $p, r \in [0,1]$

  A larger value indicates a higher preference

- Important Concepts

1. Utility
2. Skyline Items
3. Indifference Curve
4. Marginal Rate of Substitution (MRS)

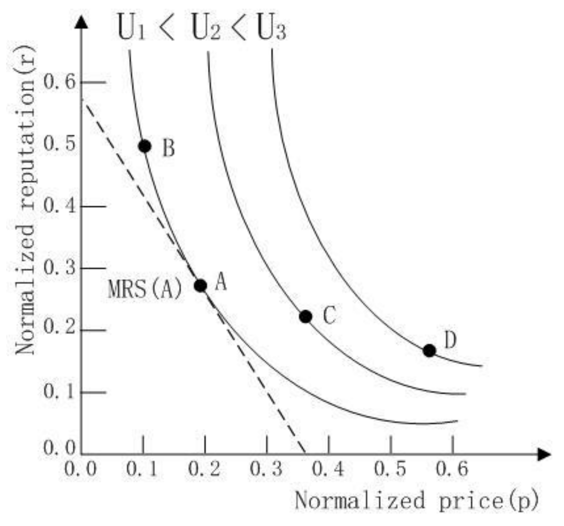


Fig.1 Indifference Curves

Our Proposed Method PRIMA

Ⓐ Utility Function $U(p,r)$

1. monotonicity

$$\frac{\partial U}{\partial p} > 0, \frac{\partial U}{\partial r} > 0$$

2. diminishing value

$$\frac{\partial^2 U}{\partial p^2} > 0, \frac{\partial^2 U}{\partial r^2} > 0$$

3. separable

$$U(p,r) = u_1(p) + u_2(r)$$

- one example is $U = \alpha \ln p + (1-\alpha)\ln r$
- $\alpha \in [0,1]$ describing the user's personalized preference on **the price-reputation tradeoff**

Ⓑ Parameter Estimation

- MRS Estimation based on IC [6]

  For each item $s_i = (p_i, r_i)$, PRIMA obtains the MRS range $[\underline{k}_i, \overline{k}_i]$

- α Estimation

1. for an item $s_i = (p_i, r_i)$, let k_i be the true MRS at s_i

$$k_i = -\frac{\alpha}{1-\alpha} \cdot \frac{r_i}{p_i} \quad \Rightarrow \quad \alpha = \frac{k_i p_i}{k_i p_i - r_i}$$

2. given one item (p_i, r_i) and $[\underline{k}_i, \overline{k}_i]$, PRIMA obtains $[\underline{\alpha}_i, \overline{\alpha}_i]$ by

$$\overline{\alpha}_i = \frac{\underline{k}_i p_i}{\underline{k}_i p_i - r_i}, \qquad \underline{\alpha}_i = \frac{\overline{k}_i p_i}{\overline{k}_i p_i - r_i}$$

3. given multiple items, PRIMA refines the range $[\underline{\alpha}, \overline{\alpha}]$ by

$$\overline{\alpha} = \min_{i=1}^{N} \overline{\alpha}_i, \qquad \underline{\alpha} = \max_{i=1}^{N} \underline{\alpha}_i$$

Ⓒ Probabilistic Ranking (The Inter-Item Competition)

Define P_i be the probability for item s_i to be selected

1. Two-Item Competition

$$s_i = (p_i, r_i),\ s_j = (p_j, r_j),\ \text{if} \quad U(p_i, r_i) > U(p_j, r_j)$$

$$\Rightarrow \begin{cases} \alpha > A_i(j), & \text{if } \{p_i > p_j, r_i < r_j\} \\ \alpha < A_i(j), & \text{if } \{p_i < p_j, r_i > r_j\} \end{cases}, \quad \text{where } A_i(j) = \frac{-\ln(r_i/r_j)}{\ln(p_i/p_j) - \ln(r_i/r_j)} > 0$$

2. Multi-Item Competition

s_i is the best choice $\Leftrightarrow$ $U(p_i, r_i) > U(p_j, r_j), \ \forall j \neq i$

$$\Rightarrow \alpha \in \left[\underline{\alpha}_{p_i}, \overline{\alpha}_{p_i}\right], \quad \text{where} \quad \underline{\alpha}_{p_i} = \max_{p_j < p_i} A_i(j), \quad \overline{\alpha}_{p_i} = \min_{p_j > p_i} A_i(j)$$

3. Assume α is uniformly distributed in $[\underline{\alpha}, \overline{\alpha}]$

$$\Rightarrow \quad \mathrm{P}_i = \frac{\min\left\{\overline{\alpha}, \overline{\alpha}_{p_i}\right\} - \max\left\{\underline{\alpha}, \underline{\alpha}_{p_i}\right\}}{\overline{\alpha} - \underline{\alpha}}$$

Real User Test

Ⓐ Data Collection and Processing

- Types of products:

  Cuisine coffee maker DCC-1200 (~$100)

  iTouch 5th generation (~$200)

  Canon EOS 5D Mark II camera (~$2000)

- Price and reputation information from eBay
- For each product, 15 item sets were generated, each with 4~6 skyline items
- 21 subjects were interviewed

Ⓑ Performance Metrics

- Ranking Quality (rq): $rq = (N - v_b)/(N-1)$, N is the number of items, and v_b is the ranking position of the user's true choice
- Success Rate: the frequency that PRIMA ranks the user's true choice in the first place

Ⓒ Results

Real user test results of ranking quality

	Coffee Maker	iTouch	Canon	Average
PRIMA	74.01%	**76.43%**	**77.80%**	76.08%
IC	**78.57%**	73.00%	77.75%	**76.44%**
MAPS	71.12%	76.12%	74.18%	73.80%

Real user test results of success rate

	Coffee Maker	iTouch	Canon	Average
PRIMA	**59.86%**	**58.84%**	**62.24%**	**60.32%**
IC	58.50%	56.80%	57.49%	57.60%
MAPS	38.10%	57.49%	46.60%	47.39%

- Both IC [6] and PRIMA give higher ranking quality and success rate than MAPS [5].
- PRIMA achieves comparable or even better performance than IC [6]. Note that PRIMA is also much simpler than IC [6] and mathematically tractable.

Conclusion

1. Personalized ranking of sellers offering similar products is an important problem in E-commerce

2. PRIMA: a novel personalized multi-attribute probabilistic ranking model

- addressing the inter-attribute tradeoff and the inter-item competition
- mathematical tractability, comparable accuracy to the state-of-the-art work
- estimating each item's probability of being the user's best choice; critical to personalized ranking, market analysis and pricing strategies

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