Probability Reweighting in Social Learning: Optimality and Suboptimality

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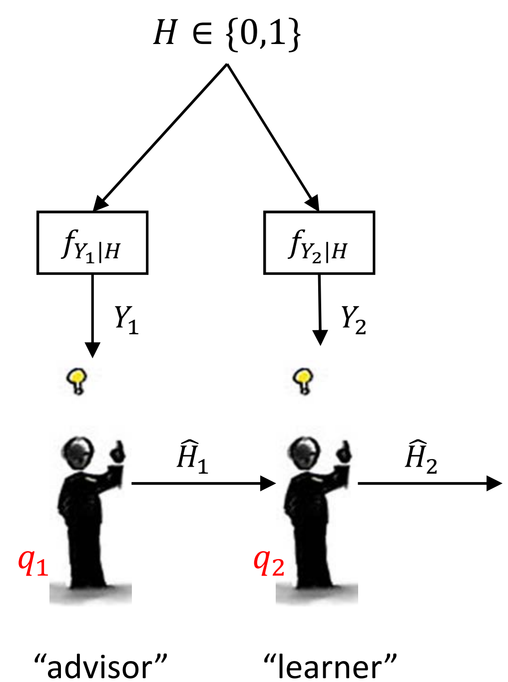
Social learning

- Our decision is influenced not only by private observation, but also by prior decisions by others
  - Buying a cell-phone

- Rhim and Goyal (2013) coined "social teaching"
  - Sequential social learning
  - Combination of agents with beliefs differing from prior could outperform that of agents with exact prior

*Rhim and Goyal, "Social teaching: Being informative vs. Being right in sequential decision making," ISIT 2013

Sequential social learning*



- $H \in \{0, 1\}$, $\mathbb{P}(H = 0) = p_0$
- Each agent has private observation $Y_i = H + Z_i$
  - $Z_i \sim N(0, \sigma_i^2)$
- Each agent is unaware of p_0, and has own belief q_i
- Agent 2 also observes prior decisions $\hat{H}_1$
- Each agent makes a selfish decision to minimize own Bayes risk as if q_i is the true prior
- "Advisor" and "Learner"
  - Eg. AI-assisted human decision

*Rhim and Goyal, "Social teaching: Being informative vs. Being right in sequential decision making," ISIT 2013

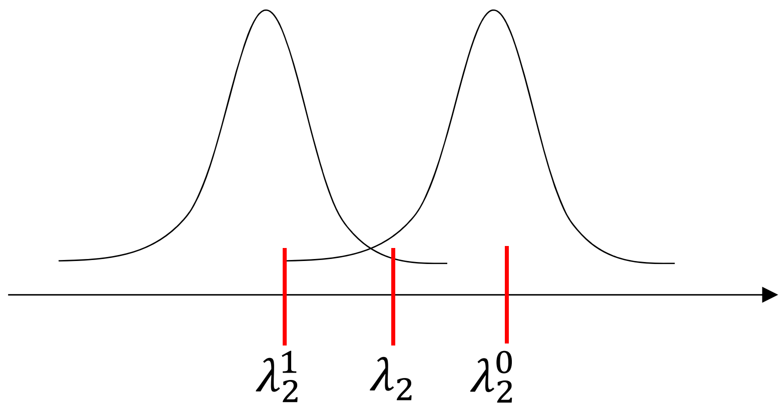
Sequential social learning

- Sequential binary hypothesis testing
  - Classical binary hypothesis test: $\frac{f(y_i|1)}{f(y_i|0)} \lesseqgtr \frac{c_{10} p_0}{c_{01}(1-p_0)}$
  - Agent 1: $\frac{f(y_1|1)}{f(y_1|0)} \lesseqgtr \frac{c_{10} q_1}{c_{01}(1-q_1)}$
  - Agent 2: $\frac{f(y_2,\widehat{H}_1|1)}{f(y_2,\widehat{H}_1|0)} \lesseqgtr \frac{c_{10} q_2}{c_{01}(1-q_2)}$
  - Due to independence, $\frac{f(y_2,\widehat{H}_1|1)}{f(y_2,\widehat{H}_1|0)} = \frac{f(y_2|1)}{f(y_2|0)} \cdot \frac{p_{[2]}(\widehat{H}_1|1)}{p_{[2]}(\widehat{H}_1|0)}$
  - Agent 2: $\frac{f(y_2|1)}{f(y_2|0)} \lesseqgtr \frac{c_{10} q_2}{c_{01}(1-q_2)} \cdot \frac{p_{[2]}(\widehat{H}_1|0)}{p_{[2]}(\widehat{H}_1|1)} =: \gamma$

Sequential social learning

- Decision threshold when Gaussian noise $N(0, \sigma^2)$

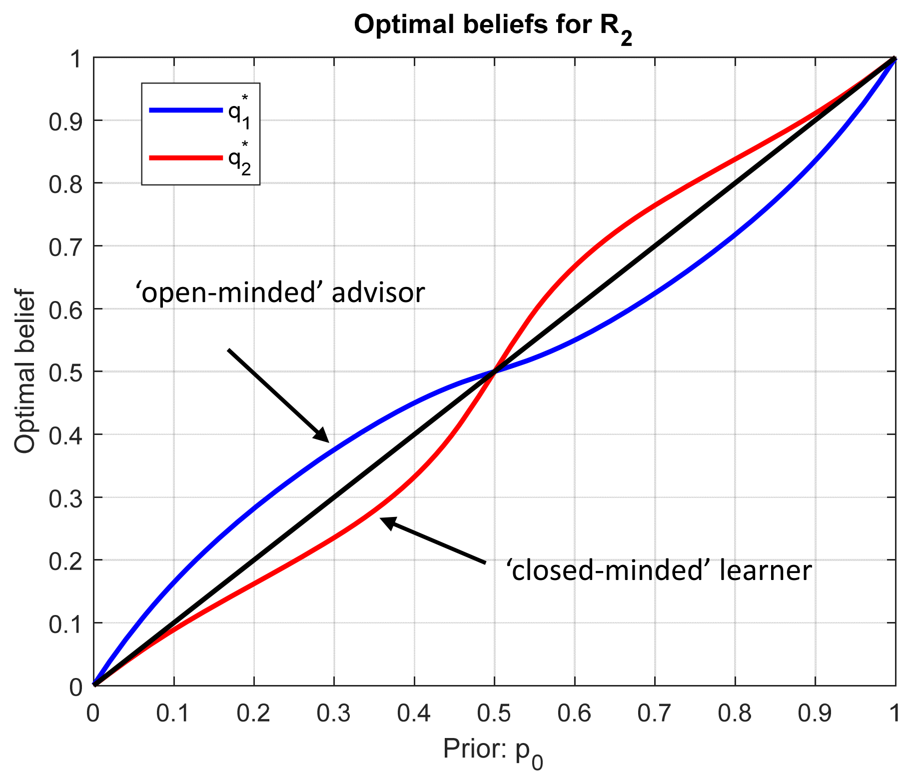
$$\lambda(\gamma, \sigma^2) = \frac{1}{2} + \sigma^2 \log \gamma$$



- (by Oracle) $R_i = c_{10} p_0 p_{\hat{H}_i|H}(1|0) + c_{01}(1 - p_0) p_{\hat{H}_i|H}(0|1)$

- Q: $q_1 = q_2 = p_0$ works the best for R_2?

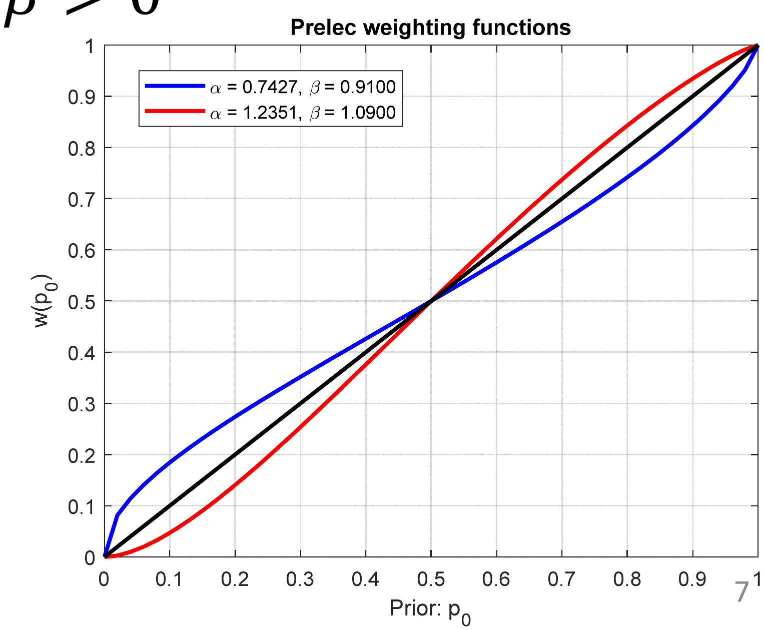
Sequential social learning



- True prior is NOT optimal for agent 2, even though each makes selfish decision

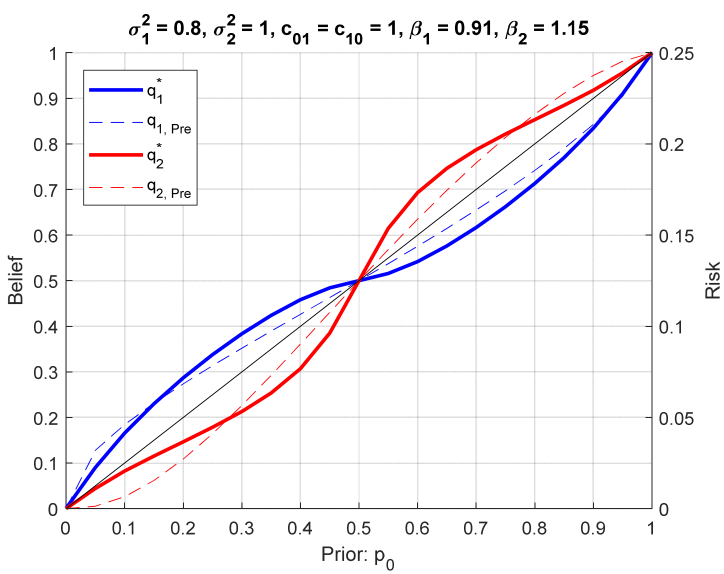
Prelec reweighting function

- Comes from cumulative prospect theory in behavioral economics
  - Kahneman (2002), Thaler (2017) won Nobel prize in economics
- Explains irrational human behaviors:
  - Winning probability of lottery
- $w(p;\alpha,\beta) = e^{-\beta(-\ln p)^{\alpha}}$ for $\alpha, \beta > 0$
  - $\alpha < 1$: open-minded
  - $\alpha > 1$: closed-minded

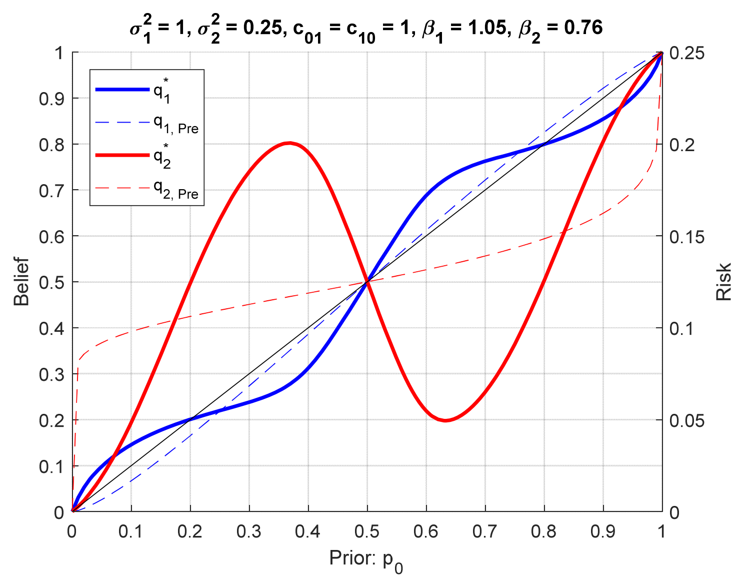


Diverse expertise levels

- Model expertise of each person through observation noise variance: σ_i^2
  - $\sigma_1^2 < \sigma_2^2$: advisor has more expertise
  - $\sigma_1^2 > \sigma_2^2$: learner has more expertise



<Expert advisor>



<Expert learner>

Diverse expertise levels

- Model expertise of each person as observation noise variance: σ_i^2
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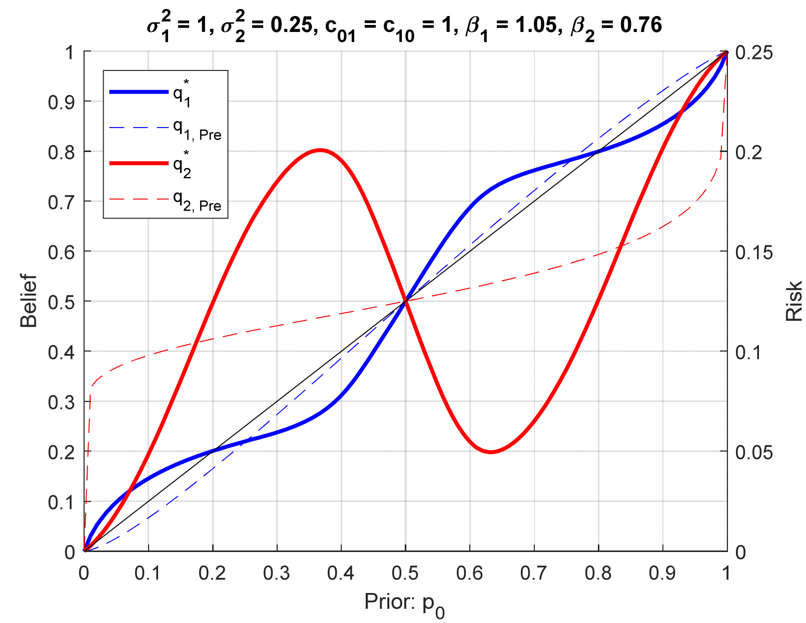
Thm. For any (σ_1^2, σ_2^2), (q_1^*, q_2^*) satisfies

- q_1^* is below p_0 if and only if $q_2^* \geq \frac{c_{01}}{c_{01}+c_{10}}$, with equality at $q_2^* = \frac{c_{01}}{c_{01}+c_{10}}$
- $p_0 = q_1^* = q_2^*$ if and only if

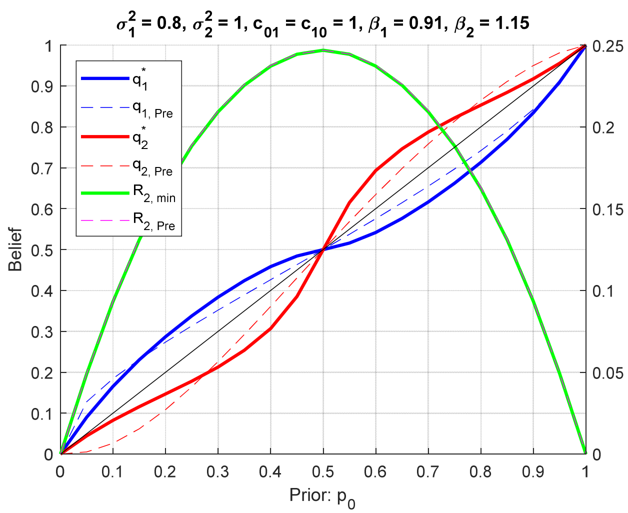
$$p_0 \in \{0, \frac{c_{01}}{c_{01}+c_{10}}, 1\}$$



Prelec function in social learning

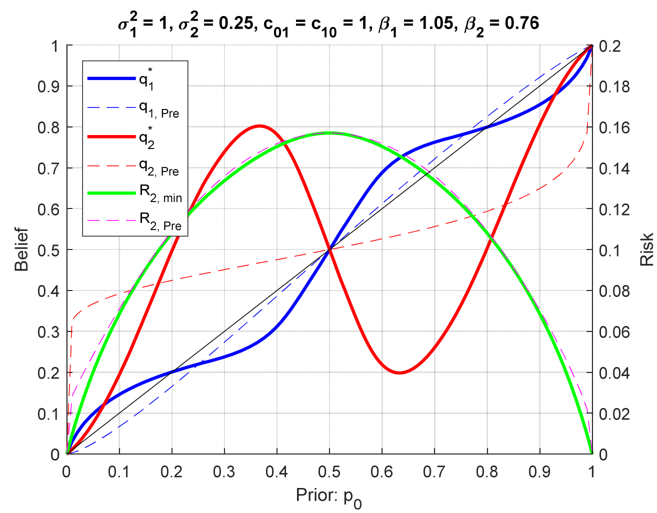
- Prelec curve fitting for (q_1^*, q_2^*)
  - Among Prelec functions that cross the same fixed point $p_0^* = \frac{c_{10}}{c_{10}+c_{01}}$,
  - Pick the minimax Prelec function such that

$$\underset{\alpha,\beta}{\operatorname{argmin}} \|q_n(\cdot) - w(\cdot;\alpha,\beta)\|_\infty$$



Max Prelec loss ≈ 0.0009

<Expert advisor>



Max Prelec loss ≈ 0.0187

<Expert learner>

Team construction criterion

- When a social planner, aware of p_0, couples a team as follows

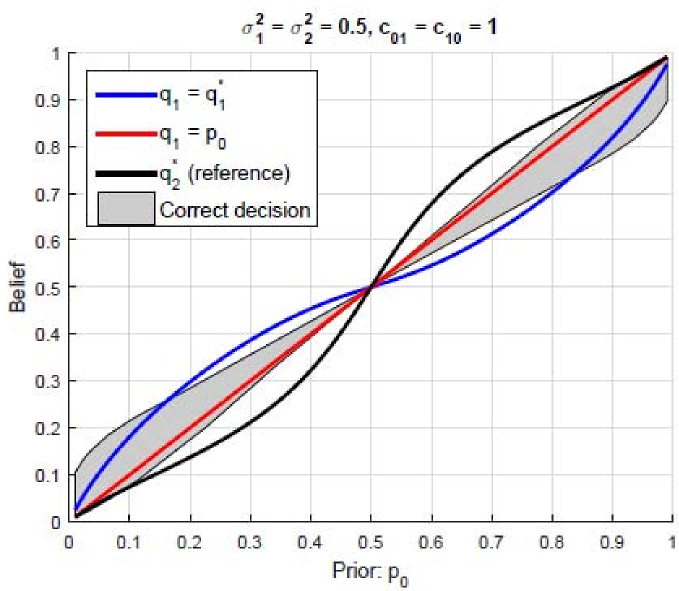
> Thm. Consider two advisors $q_1 < q_1{}'$. Then, advisor with q_1 is a better choice if and only if
>
> $$\frac{\mathbb{P}_1[\hat{H}_1 = \hat{H}_2 = 1, \hat{H}_{1'} = \hat{H}_{2'} = 0]}{\mathbb{P}_0[\hat{H}_1 = \hat{H}_2 = 1, \hat{H}_{1'} = \hat{H}_{2'} = 0]} \geq \frac{c_{10} p_0}{c_{01}(1 - p_0)}$$

- So when learner, unaware of p_0, picks a better advisor based on q_2 if and only if

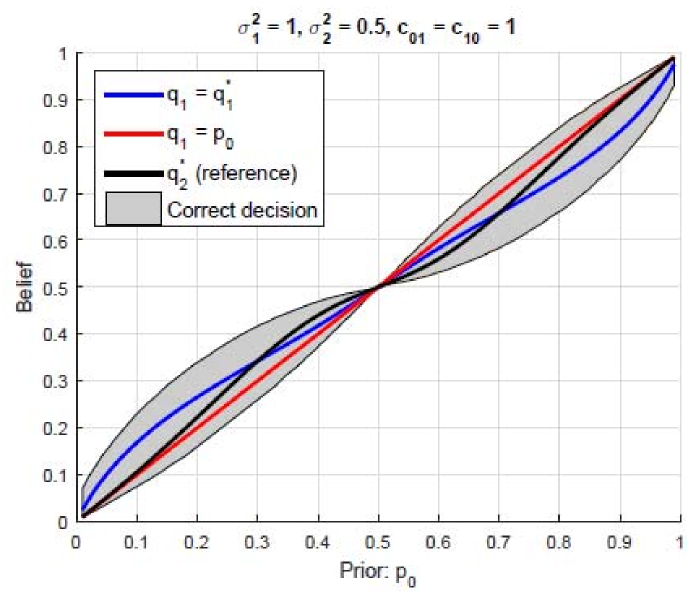
$$\frac{\mathbb{P}_1[\hat{H}_1 = \hat{H}_2 = 1, \hat{H}_{1'} = \hat{H}_{2'} = 0]}{\mathbb{P}_0[\hat{H}_1 = \hat{H}_2 = 1, \hat{H}_{1'} = \hat{H}_{2'} = 0]} \geq \frac{c_{10} p_0}{c_{01}(1 - p_0)}$$

Team construction criterion

- Consider two advisors with $q_1 = q_1^*, q_1' = p_0$
- With what belief the learner can pick better advisor?



Equal expertise: optimal q_2^* never satisfies the iff condition



Expert learner: optimal q_2^* will always pick advisor with q_1^*

Conclusion

- Diverse expertise in social learning
  - When advisor has more expertise, overall behavior of optimal agents remains similar to Rhim & Goyal's result
  - While when learner has more expertise, it shows different nature
- Prelec function approximation
  - Prelec function is nearly optimal when advisor has more expertise, otherwise suboptimal
- Self-organizing team criterion
- Implication on AI supported human inference
  - Support of suboptimal AI system could help human decision more, and vice versa