SLOW-TIME CODING FOR MUTUAL INTERFERENCE MITIGATION

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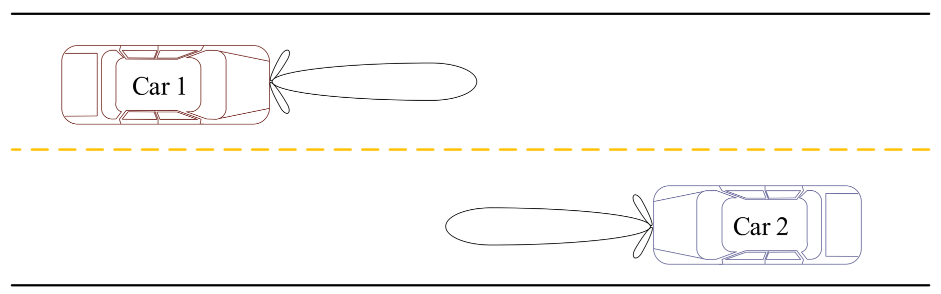
ABSTRACT

- The **mutual interference** between similar radar systems can result in **reduced radar sensitivity** and **increased false alarm rates**.
- To address the interference mitigation problems in similar radar systems, we propose herein two slow-time coding schemes to modulate the pulses within a coherent processing interval (CPI).
- The incorporation of the coding schemes only requires slight modification of the existing systems.

INTRODUCTION



Problem	• Massively produced civilian radar systems tend to be quite similar, which will result in severe mutual interferences.
Idea	• Slow-time coding • A heuristic coding scheme • An optimized coding scheme utilizing unimodular quadratic programming (UQP)

PROBLEM FORMULATION

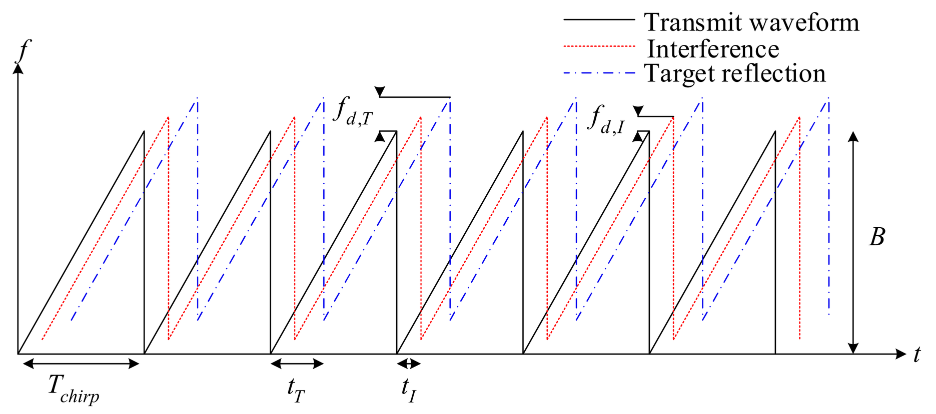


Fig.1. Time-frequency illustration of the transmit waveform, the target signal, and the interference

Consider two identical linear frequency-modulated continuous waveform (LFMCW) radar systems operating within the same frequency band. Their transmission is shown in Fig.1. The transmitted waveform can be written as

$$s(t) = \sum_{n=-\infty}^{\infty} u(t - T_{\text{chirp}}),$$

where $u(t) = \exp(j(2\pi f_c t + \pi K t^2)),\ 0 \le t \le T_{\text{chirp}}$, T_{chirp} is the chirp duration, f_c is the carrier frequency, and $K = B / T_{chirp}$ is the chirp rate.

When the two radar systems are operating simultaneously, the received signal can be written as:

$$r(t) = y_{\text{T}}(t) + y_{\text{I}}(t) + w(t),$$

$y_{\text{T}}(t)$: target returns

$y_{\text{I}}(t)$: interference signal

$w(t)$: noise

Range-Doppler image of received signal

After dechirping, the digital samples associated with the n^{th} period of a coherent processing interval (CPI) (N periods) is

$$r(m,n) = \alpha_{\text{T}} \exp(j2\pi(\tilde{f}_{B,\text{T}} m + \tilde{f}_{d,\text{T}} n))$$
$$+ \alpha_{\text{I}} \exp(j2\pi(\tilde{f}_{B,\text{I}} m + \tilde{f}_{d,\text{I}} n)) + w(m,n),$$

$\tilde{f}_{B,\text{T}} = f_{B,\text{T}} T_s,\ \tilde{f}_{B,\text{I}} = f_{B,\text{I}} T_s$		Normalized beat frequencies	
$\tilde{f}_{d,\text{T}} = f_{d,\text{T}} T_{\text{chirp}},\ \tilde{f}_{d,\text{I}} = f_{d,\text{I}} T_{\text{chirp}}$		Normalized Doppler frequencies	
$f_{B,\text{T}} = K\tau_{\text{T}} + f_{d,\text{T}}$		Target beat frequency	
$f_{B,\text{I}} = K\tau_{\text{I}} + f_{d,\text{I}}$		Interference beat frequency	
T_{chirp}	sampling period	$w(m,n)$	noise
$\alpha_{\text{T}},\ \tau_{\text{T}},\ f_{d,\text{T}}$	The amplitude, time delay and Doppler frequency associated with the target, respectively		
$\alpha_{\text{I}},\ \tau_{\text{I}},\ f_{d,\text{I}}$	The amplitude, time delay and Doppler frequency associated with the interference, respectively		

Assume M samples are collected for each period. Then we can obtain the range-Doppler image:

$$R(k,p) = \alpha_{\text{T}} D_M(\tilde{f}_{B,\text{T}} - k/M) D_N(\tilde{f}_{d,\text{T}} - p/N)$$
$$+ \alpha_{\text{I}} D_M(\tilde{f}_{B,\text{I}} - k/M) D_N(\tilde{f}_{d,\text{I}} - p/N) + W(k,p),$$

$D_n(x) = \sin(n\pi x)/\sin(\pi x)$: Dirichlet function

$W(k,p)$: 2-D FFT of noise

- **The interference signal will form a sharp peak in the range-Doppler image.**
- **The interference level can be high due to the one-way propagation characteristic.**

SLOW-TIME CODING SCHEME

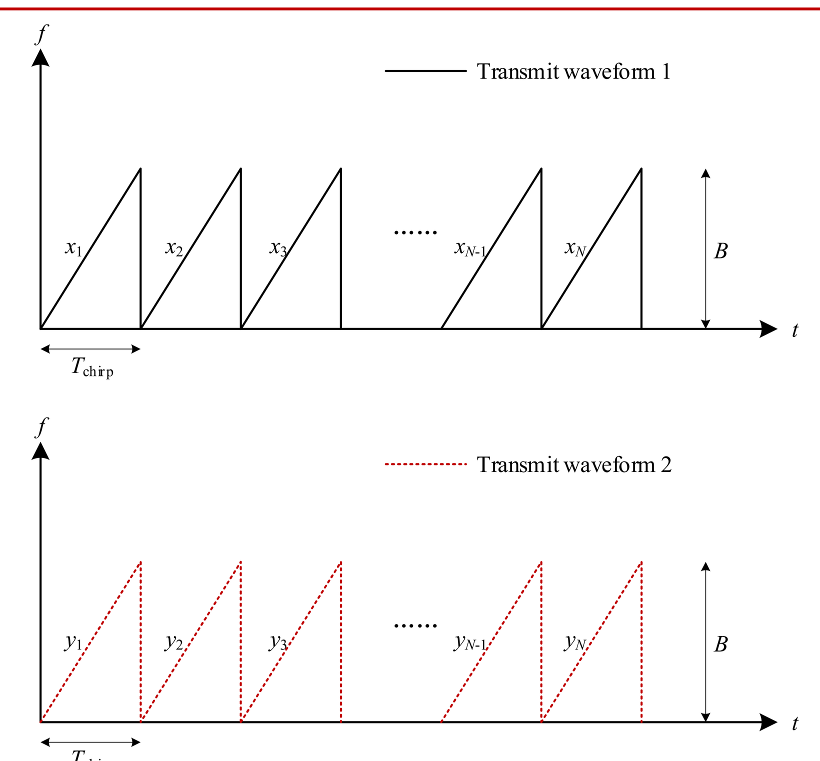


Fig.2. The proposed coding scheme

Denote the associated coding sequences by

$$\boldsymbol{x} = [x_1, x_2, ..., x_N]^T \text{ and } \boldsymbol{y} = [y_1, y_2, ..., y_N]^T.$$

In the n^{th} period of a CPI, the first radar system transmits $x_n u(t)$ and the second transmits $y_n u(t)$.

To keep constant transmit power,

$$|x_n| = |y_n| = 1,\ n = 1, 2, ..., N.$$

With the coding scheme, the range-Doppler is given by

$$R^c(k,p) = \alpha_{\text{T}} D_M(\tilde{f}_{B,\text{T}} - k/M) D_N(\tilde{f}_{d,\text{T}} - p/N)$$
$$+ \alpha_{\text{I}} D_M(\tilde{f}_{B,\text{I}} - k/M) r_{xy}^l(\tilde{f}_{d,\text{I}} - p/N) + W(k,p),$$

Where $r_{xy}^l(f) = \sum_{n=1}^{N} x_n^* y_{(n+l)\bmod N} \exp(j2\pi n f)$, which is the periodic cross-ambiguity function of $\boldsymbol{x}$ and $\boldsymbol{y}$, l is determined by the unsynchronized time $(-N+1 \le l \le N-1)$.

To suppress the interference power in the range-Doppler image, we aim at designing $\boldsymbol{x}$ and $\boldsymbol{y}$ to minimize $r_{xy}^l(f)$ in a range of interest for f.

Doppler-shifting scheme

A heuristic coding scheme : the coding vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ are given by

$$\boldsymbol{x} = [1, 1, ..., 1]^T,$$
$$\boldsymbol{y} = \begin{cases} [1, -1, ..., -1, 1]^T, & \text{if } N \text{ is odd}, \\ [1, -1, ..., 1, -1]^T, & \text{if } N \text{ is even}. \end{cases}$$

Thus we have $r_{xy}^l(f) = D_N(f + 1/2)$. The Doppler frequency of the interference signal is shifted to a higher frequency area.

Optimized Coding Scheme

Discretized version of $r_{xy}^l(f)$:

$$r_{lp} = \sum_{n=1}^{N} x_n^* y_{(n+l)\bmod N} \exp(-j2\pi np/N_f)$$
$$= \boldsymbol{x}^H \text{Diag}(\boldsymbol{f}_p) \mathbf{C}_l \boldsymbol{y}$$

N_f : number of discrete frequencies

$\mathbf{C}_l$: circular shift matrix, $\mathbf{C}_l = \begin{bmatrix} \mathbf{0} & \mathbf{I}_{N-l} \\ \mathbf{I}_l & \mathbf{0} \end{bmatrix}$

$\boldsymbol{f}_p$: a $1 \times N$ vector whose n^{th} element is $e^{-j2\pi np/N_f}$

To optimize $\boldsymbol{x}$ and $\boldsymbol{y}$ such that r_{lp} have small values in a desired area for any l, we can formulate the optimization problem as: $(-P \le p \le P$ is the desired area$)$

$$\min_{\boldsymbol{x},\boldsymbol{y}} \sum_{l=-N+1}^{N-1} \sum_{p=-P}^{P} |\boldsymbol{x}^H \text{Diag}(\boldsymbol{f}_p) \mathbf{C}_l \boldsymbol{y}|^2$$
$$\text{s.t. } |x_n| = 1,\ |y_n| = 1,\ n = 1, 2, ..., N,$$

The optimization problem is non-convex and difficult to solve. We propose to tackle it in a cyclic manner:

- **Optimization of $\boldsymbol{x}$ for fixed $\boldsymbol{y}$**

$$\min_{\boldsymbol{x}} \boldsymbol{x}^H \mathbf{B}_y \boldsymbol{x},\ \text{ s.t. } |x_n| = 1,\ n = 1, 2, ..., N,$$

where

$$\mathbf{B}_y = \sum_{l=-N+1}^{N-1} \sum_{p=-P}^{P} \text{Diag}(\boldsymbol{f}_p) \mathbf{C}_l \boldsymbol{y}\boldsymbol{y}^H \mathbf{C}_l^H \text{Diag}^H(\boldsymbol{f}_p)$$

It's a *UQP* problem, which can be solved using *power-method-like iterations*. In the k^{th} iteration, we update $\boldsymbol{x}$:

$$\boldsymbol{x}^{(k)} = \exp(j \arg(\mathbf{D}_y \boldsymbol{x}^{(k-1)}))$$

where $\mathbf{D}_y = \mu_y \mathbf{I}_N - \mathbf{B}_y$, μ_y is a constant to ensure $\mathbf{D}_y \succ \mathbf{0}$.

- **Optimization of $\boldsymbol{y}$ for fixed $\boldsymbol{x}$**

$$\min_{\boldsymbol{y}} \boldsymbol{y}^H \mathbf{B}_x \boldsymbol{y},\ \text{ s.t. } |y_n| = 1,\ n = 1, 2, ..., N,$$

where

$$\mathbf{B}_x = \sum_{l=-N+1}^{N-1} \sum_{p=-P}^{P} \mathbf{C}_l^H \text{Diag}^H(\boldsymbol{f}_p) \boldsymbol{x}\boldsymbol{x}^H \text{Diag}(\boldsymbol{f}_p) \mathbf{C}_l$$

Then we have

$$\boldsymbol{y}^{(k)} = \exp(j \arg(\mathbf{D}_x \boldsymbol{y}^{(k-1)}))$$

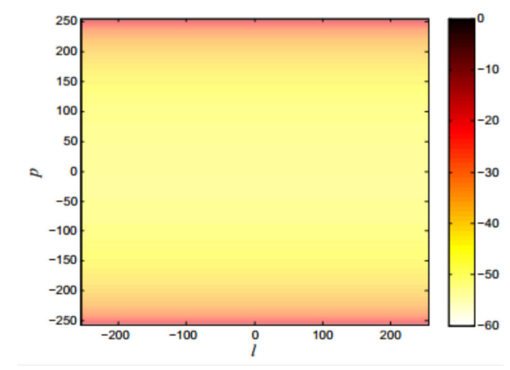
where $\mathbf{D}_x = \mu_x \mathbf{I}_N - \mathbf{B}_x$, μ_x ensures $\mathbf{D}_x \succ \mathbf{0}$.

- **Interestingly, if we fix $\boldsymbol{y} = \mathbf{1}_N$ and only optimize $\boldsymbol{x}$, we obtain similar results, which corresponds to a more practical coding methods, since no co-ordination between the two radar systems is needed.**

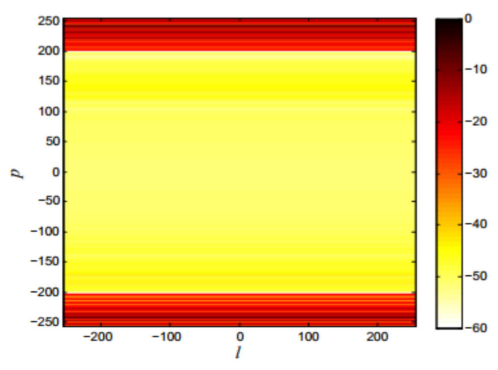
NUMERICAL EXAMPLES

Discrete periodic cross-ambiguity functions

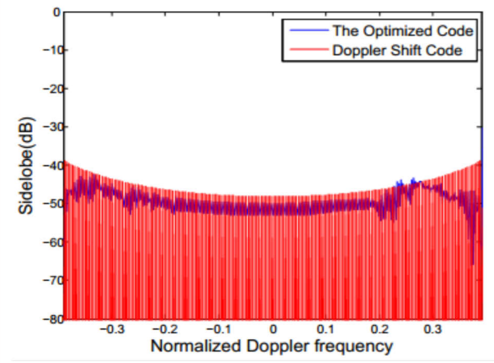
f_c(GHz)	B(MHZ)	T_{chirp}(μs)	N	P	N_f	$v_{r\max}$(km/ h)
24	150	20	256	200	512	87.9



Doppler-shifting scheme



The optimized coding scheme

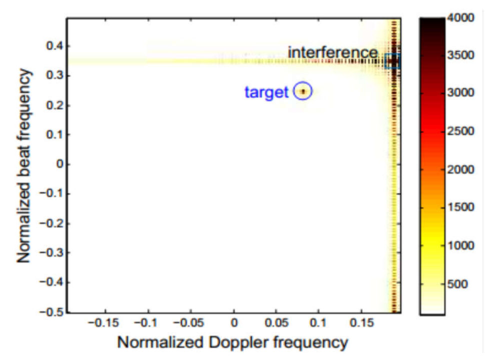


The zero-delay cut

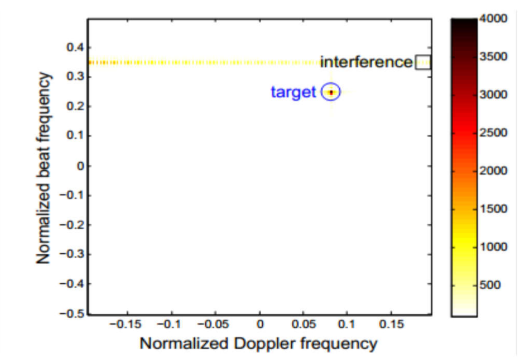
· Both coding schemes achieve very low side-lobes in the desired area
· The peak sidelobe of the optimized codes is 3.55dB lower than that of the Doppler-shifting

The range-Doppler image with slow-time coding

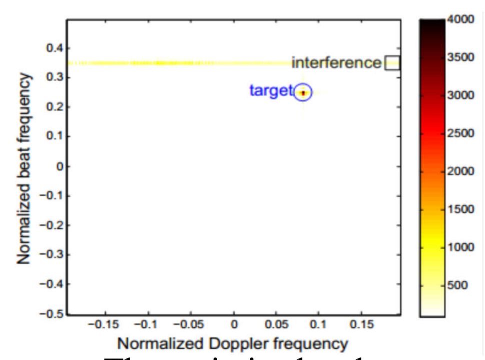
	Range(m)	v_r(km/ h)	SNR(dB)	f_s(MHz)	M
Target	50	36.432	30	4	200
Interference	140	84.42	60		



Without coding



Doppler-shifting codes



The optimized codes

· The interference level is significantly reduced applying our slow-time coding scheme

CONCLUSION

- ❑ We have proposed two slow-time coding schemes to mitigate the mutual interference in two identical or similar FMCW radar systems.
- ❑ We have presented efficient methods to construct the codes.
- ❑ We have shown that both coding schemes can be used to reduce the interference power level significantly.
- ❑ We have demonstrated that the optimized coding scheme can achieve a lower PSL than the first more intuitive coding scheme.

ACKNOWLEDGMENT

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