# Classifying Pump-probe Images of Melanocytic Lesions using the Weyl Transform


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## 1. Overview

- We introduce new variant of the Weyl transform, which we call the Patch Weyl transform.
- We show empirically that the Patch Weyl transform coefficients perform much better in classification of the lesion images compared to the standard descriptors.
- We claim that the Patch Weyl transform has several desirable properties that are useful for signal processing tasks.

## 2. Original Weyl transform (WT)

Define *D(a,b)*, indexed by binary *m*-tuple *a* and *b*, as the following:

$$D(a,b) := (X^{a_{m-1}} Z^{b_{m-1}}) \otimes \ldots \otimes (X^{a_0} Z^{b_0})$$

where, $$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

How to compute the Weyl coefficients?

1. Vectorize columnwise the image patch *P* and find the covariance matrix of the vectorized input.

Image (P) (n x n) → Cov ($n^2$ x $n^2$)

2. Compute the trace inner product of the covariance matrix with D(a,b) of same dimension.

$$W_{a,b} = Tr(\text{Cov} \bullet D(a,b))$$

* Given a vectorized input *y* of length $2^m$, we define its Weyl coefficients to be:

$$w_{a,b}(y) := \frac{1}{2^{m/2}} Tr[yy^T \cdot D(a,b)]$$

## 3. Patch Weyl transform (PWT)

- Comparison between **WT** and **PWT**
  - **PWT** decomposes the image patch *P* into a pair of symmetric and skew-symmetric matrix (*size of the input is maintained).

Image (P) → Symme-tric $(P + P^T)/2$ ; Skew-symme tric $(P - P^T)/2$

  - **PWT** coefficients are also computed by the trace inner product with *D(a,b)s* with same dimension as that of the original image patch *P*.
  - **PWT** generates two sets of coefficients per input.
    - ❑ $W_R$ (from the symmetric matrix) and $W_Q$ (from the skew-symmetric matrix).
- Important Properties of PWT
  - Not limited by the size of the input.
  - Detects periodicity.
  - Allows convolution operation, extending its application to powerful convolutional neural network.

## 4. Feature Selection Algorithm

Ex) Finding top K Features from the $W_R$ coefficients for classifying class *X*. (Same method applied for $W_Q$ coefficients and other classes)

1. Compute the $W_R$ coefficients of each training image and vectorize them into columns.

X image (Train), Non-X image (Train) → $W_R$, $W_R$

2. For every pair of X and non-X training image, compute the magnitudes of the difference in the coefficients and take the mean of the differences for each coefficient.

mean(abs( $W_R$ - $W_R$ ) ) = Y.

3. Rank the $W_R$ coefficients in the descending order of *Y*.

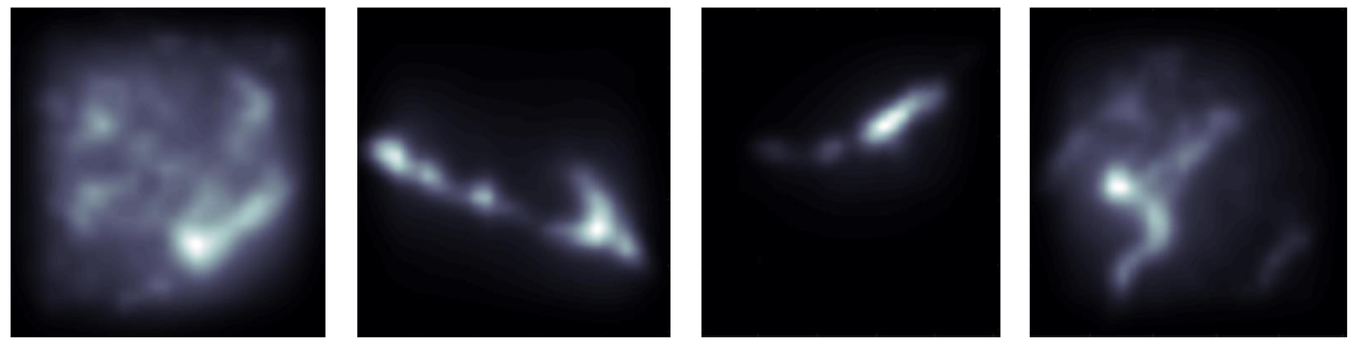
* Greater the difference, more effective the particular coefficient is at distinguishing class X from others.

## 5. Dataset

637 Preprocessed pump-probe images of vulvamelanoma lesions.

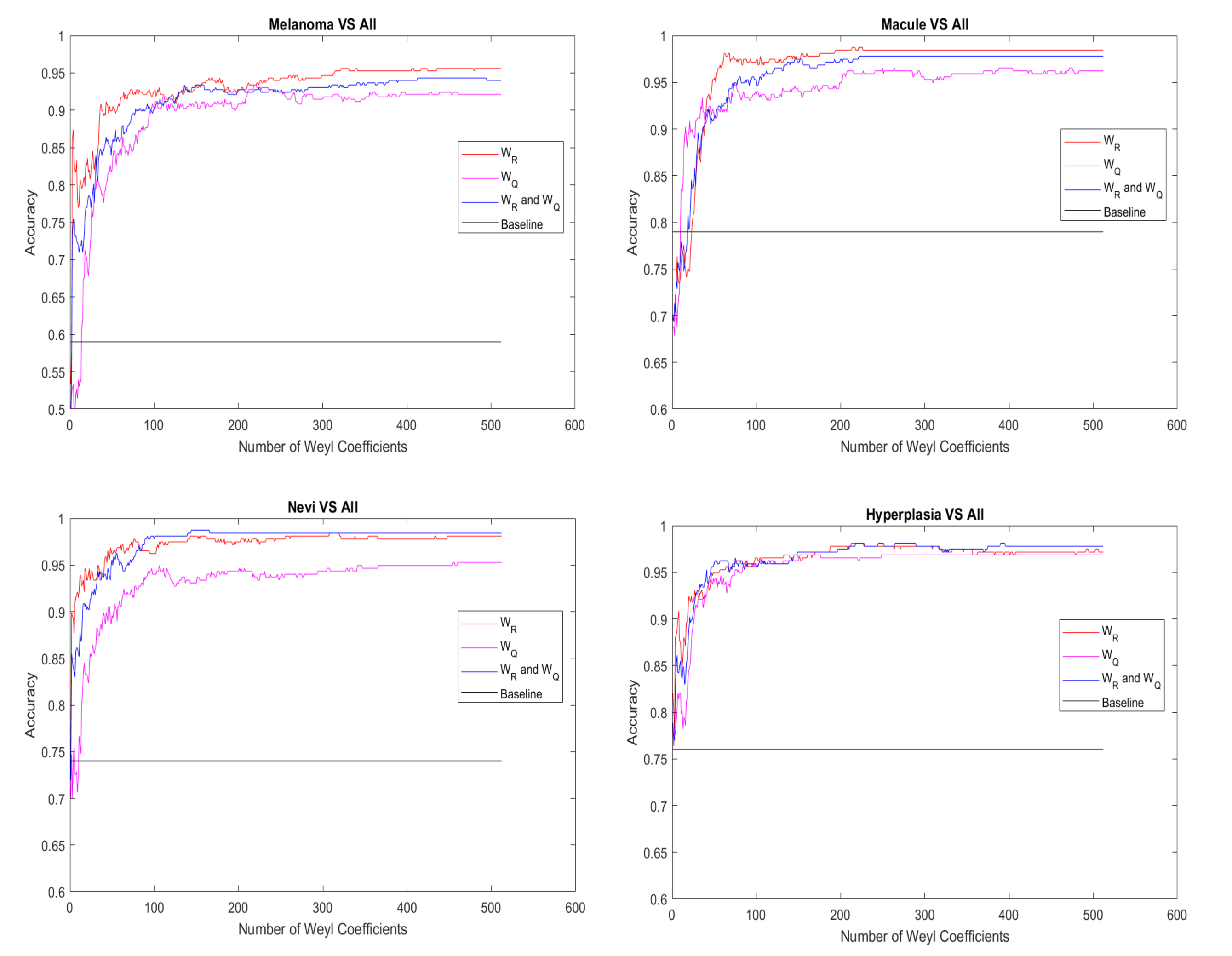
- 4 classes: *Melanoma*, *Macule*, *Nevi*, *Hyperplasia.*
- Sample images below

Melanoma · Macule · Nevi · Hyperplasia



## 6. Quantitative Results

| Classification | Base | HOG | LBP | PCA | Fourier | $W_R$ | $W_Q$ | $W_R$ and $W_Q$ |
|---|---|---|---|---|---|---|---|---|
| Melanoma VS Other | 59% | 65% | 68% | 70% | 78% | 96% | 93% | 94% |
| Macule VS Other | 79% | 80% | 77% | 86% | 87% | 99% | 97% | 98% |
| Nevi VS Other | 74% | 76% | 75% | 83% | 90% | 98% | 95% | 99% |
| Hyperplasia VS Other | 76% | 83% | 85% | 89% | 89% | 98% | 97% | 98% |




## 7. Conclusion

- **PWT** coefficients outperform the baseline measure and other commonly used descriptors in the classification of the lesion.
- **PWT** is far more practical and diverse in application than the **WT** because it is not limited by the size of the input.
- **PWT** can be incorporated in deep learning architecture, such as the convolutional neural networks (CNNs).

## 8. Reference


Q. Qiu, A. Thompson, R. Calderbank and G. Sapiro, "Data Representation using the Weyl Transform," IEEE Transactions on Information Theory, vol. 64, no. 7, pp. 1844–1853, 2016.