Semi-Blind Channel Estimation in Massive MIMO Systems with Different Priors on Data Symbols

UC San Diego
Electrical and Computer Engineering
JACOBS SCHOOL OF ENGINEERING

Elina Nayebi and Bhaskar Rao
Department of Electrical and Computer Engineering
University of California, San Diego

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Objectives

Semi-Blind Estimation: Using information in unknown data symbols to estimate channel coefficients instead of using pilots only

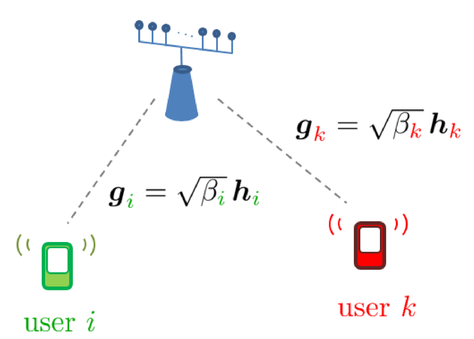
Advantages:

- More accurate channel estimates in TDD/FDD
- Utilizing smaller number of pilot sequences
- More accurate downlink precoders in TDD sys.
- More accurate channel estimates as number of antennas increases in Massive MIMO systems

System Model

- Uplink TDD transmission in a single cell
- M antennas at BS and K single antenna users
- Rayleigh fading channel:

$$G = H \times \begin{bmatrix} \sqrt{\beta_1} & & 0 \\ & \ddots & \\ 0 & & \sqrt{\beta_K} \end{bmatrix}$$



  - β_k: Large scale fading
  - $h_k \sim \mathcal{CN}(0, I_k)$: Small Scale Fading

- Uplink signal received by BS at time n

$$\boldsymbol{y}[n] = G\boldsymbol{s}[n] + \boldsymbol{v}[n], \quad n = 0, \cdots, N-1$$

- $S = \Big[\underbrace{\boldsymbol{s}[0], \cdots, \boldsymbol{s}[L-1]}_{S_{\text{pilot}}:\ \text{pilot sequences}}, \underbrace{\boldsymbol{s}[L], \cdots, \boldsymbol{s}[N-1]}_{S_{\text{data}}:\ \text{data symbols}}\Big]$
- Complete received signal $Y = [Y_{\text{pilot}},\ Y_{\text{data}}]$

ML Estimators

- **Training Pilots:** Estimation based on pilots

$$\hat{G}^{\text{tr}}_{\text{ML}} = \underset{G}{\text{argmax}} \log p\left(Y_{\text{pilot}} | G, S_{\text{pilot}}\right) = \left(Y_{\text{pilot}} S_{\text{pilot}}\right)\left(S_{\text{pilot}} S^H_{\text{pilot}}\right)^{-1}$$

- **Full Data**: All data symbols are known

$$\hat{G}^{\text{Full}}_{\text{ML}} = \underset{G}{\text{argmax}} \log p\left(Y | G, S\right) = (YS)\left(SS^H\right)^{-1}$$

- **Semi-Blind Estimation**:

$$\hat{G}_{\text{EM}} = \underset{G}{\text{argmax}}\ \log p\left(Y | G, S_{\text{pilot}}\right)$$

No closed form solution → EM/MM Algorithms

Semi-Blind Estimation

- Expectation-maximization (EM) algorithm with latent variable S_{data} is expressed as

$$\hat{G}_{\ell+1} = \underset{G}{\text{argmax}}\ \mathbb{E}_{p(S_{\text{data}}|Y, \hat{G}_\ell)} \left(\log p\left(Y, S_{\text{data}} | G\right)\right)$$

- At $(\ell+1)$th iteration channel is given by (M-step)

$$\hat{G}_{\ell+1} = \left(Y_{\text{p}} S^H_{\text{pilot}} + Y_d \mathbb{E}\left(S_{\text{data}} \middle| \hat{G}_\ell, Y\right)^H\right) \times \left(S_{\text{pilot}} S^H_{\text{pilot}} + Y_d \mathbb{E}\left(S_{\text{data}} S^H_{\text{data}} \middle| \hat{G}_\ell, Y\right)^H\right)^{-1}$$

- Computational complexity of E-step for discrete constellations grows exponentially with K.
- To reduce complexity we consider Gaussian and GMM priors on data symbols.

EM Algorithm with Gaussian Prior:

- For tractability assume S_{data} is Gaussian, which results in a closed form solution for E-step:

E-Step :

$$\boldsymbol{\mu}^\ell_n = \left(\hat{G}^H_\ell \hat{G}_\ell + \sigma^2_v I_K\right)^{-1} \hat{G}^H_\ell \boldsymbol{y}[n]$$

$$\Sigma^\ell = \sigma^2_v \left(\hat{G}^H_\ell \hat{G}_\ell + \sigma^2_v I_K\right)^{-1}$$

M-Step:

$$\hat{G}_{\ell+1} = \left(Y_{\text{pilot}} S^H_{\text{pilot}} + \sum_{n=L}^{N-1} \boldsymbol{y}[n] (\boldsymbol{\mu}^\ell_n)^H\right) \times \left(S_{\text{pilot}} S^H_{\text{pilot}} + \sum_{n=L}^{N-1} \left(\boldsymbol{\mu}^\ell_n (\boldsymbol{\mu}^\ell_n)^H + \Sigma^\ell\right)\right)^{-1}$$

- Channel estimation is improved even when data symbols are from discrete constellation.

Heuristic Semi-blind Algorithm:

- Improving channel estimates by assigning the conditional mean of data symbols to the closest constellation point (heuristic approach):

E-Step :

$$\boldsymbol{\mu}^\ell_n = F\left(\left(\hat{G}^H_\ell \hat{G}_\ell + \sigma^2_v I_K\right)^{-1} \hat{G}^H_\ell \mathbf{y}[n]\right)$$

$$\Sigma^\ell = \sigma^2_v \left(\hat{G}^H_\ell \hat{G}_\ell + \sigma^2_v I_K\right)^{-1}$$

where $F(.)$ is the element-wise constellation demmaping function.

- M-step remains the same.

Semi-Blind Estimation

EM Algorithm with GMM Prior:

- To provide analytical support for the heuristic semi-blind estimation, we consider a Gaussian mixture model (GMM) for data symbols

$$\mathbf{s}[n] \sim \mathcal{CN}\left(\mathbf{c}_n, \sigma^2_s I_K\right), \quad n = L, \cdots, N-1,$$

where $\mathbf{c}_n \in \mathbb{C}^{K \times 1}$ is the transmitted constellation vector at time n.

- Unknown variables $\Theta = [G, \mathbf{c}_L, \cdots, \mathbf{c}_{N-1}]$ at $(\ell+1)$th iteration are estimated as

$$\hat{\Theta}_{\ell+1} = \underset{G}{\text{argmax}}\ \mathbb{E}_{p(S_{\text{data}}|Y, \hat{\Theta}_\ell)} \left(\log p(Y, S_{\text{data}} | \Theta)\right)$$

E-Step:

$$\boldsymbol{\mu}^\ell_n = \left(\hat{G}^H_\ell \hat{G}_\ell + \sigma^2_v \left(\hat{\mathbf{c}}^\ell_n (\hat{\mathbf{c}}^\ell_n)^H + \sigma^2_s I_K\right)^{-1}\right)^{-1} \hat{G}^H_\ell \mathbf{y}[n]$$

$$\Sigma^\ell_n = \sigma^2_v \left(\hat{G}^H_\ell \hat{G}_\ell + \sigma^2_v \left(\hat{\mathbf{c}}^\ell_n (\hat{\mathbf{c}}^\ell_n)^H + \sigma^2_s I_K\right)^{-1}\right)^{-1}$$

M-Step:

$$\hat{G}_{\ell+1} = \left(Y_{\text{pilot}} S^H_{\text{pilot}} + \sum_{n=L}^{N-1} \mathbf{y}[n] (\boldsymbol{\mu}^\ell_n)^H\right) \times \left(S_{\text{pilot}} S^H_{\text{pilot}} + \sum_{n=L}^{N-1} \left(\boldsymbol{\mu}^\ell_n (\boldsymbol{\mu}^\ell_n)^H + \Sigma^\ell_n\right)\right)^{-1}$$

$$\hat{\mathbf{c}}^{\ell+1}_n = F\left(\boldsymbol{\mu}^\ell_n\right)$$

- Heuristic semi-blind and EM algorithm with GMM prior outperform EM algorithm with Gaussian prior at high SNRs.

Cramer-Rao Bound

Deterministic CRB: unknown deterministic S_{data}

- With unlimited number of antennas at BS

$$\text{CRB} \xrightarrow{M \to \infty} \Lambda^{-1},$$

where Λ corresponds to the CRB of channel estimation with *Full Data*.

Stochastic CRB: S_{data} is Gaussian

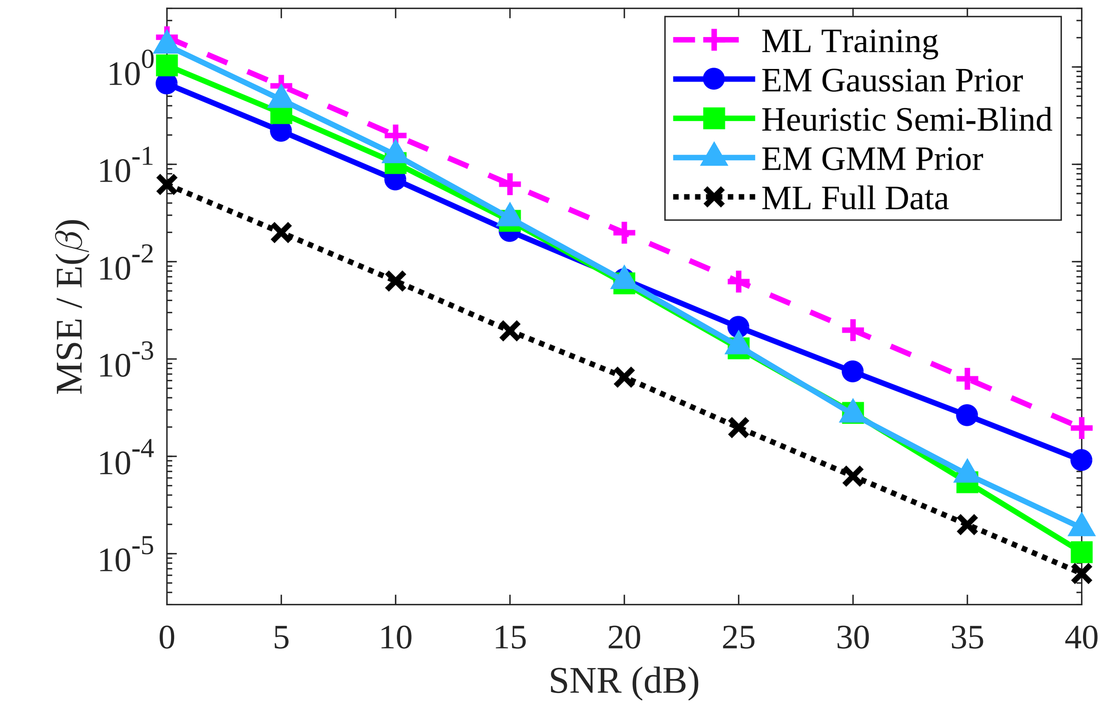
- With unlimited number of antennas at BS

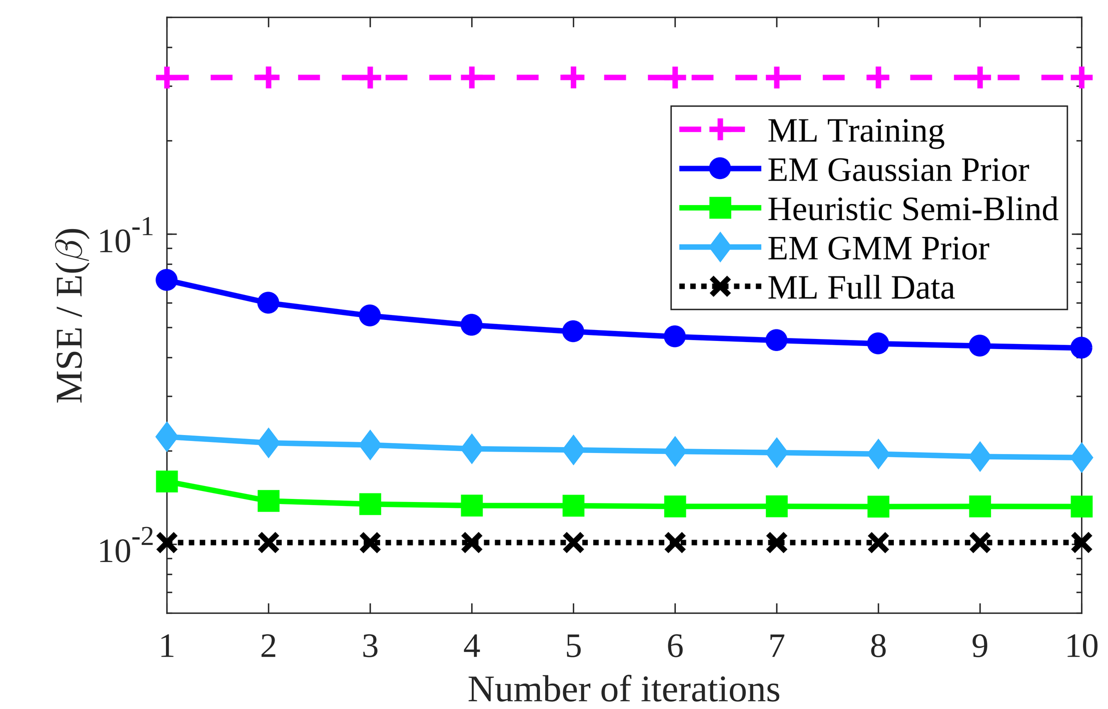
$$\text{CRB} \xrightarrow{M \to \infty} \frac{\sigma^2_v}{2N} I,$$

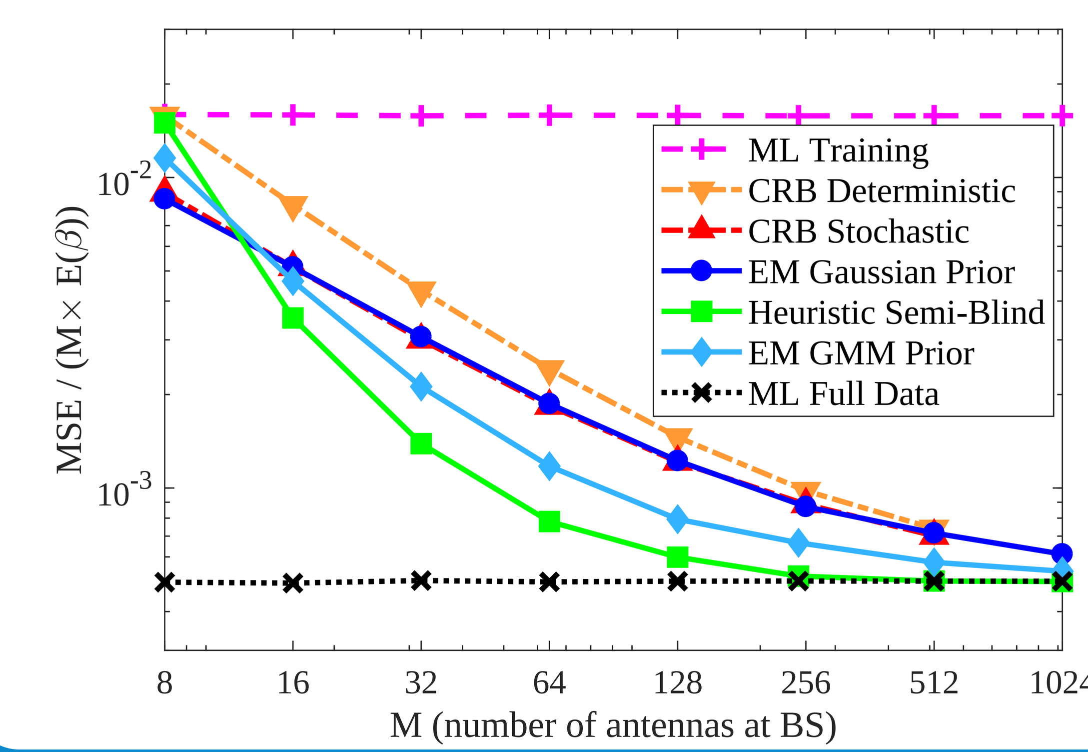
which is equivalent to having orthogonal pilot sequences of length N (pilots+data).

Numerical Results

- Single cell of radius 500m
- QPSK data and orthogonal pilots
- COST-231 Hata model for β_k, $k = 1, \cdots, K$







References

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