Quantisation Effects in Distributed Optimisation

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Content

- Background
- Information exchange between nodes
- Distributed optimisation
- Sample problem
- Conclusion

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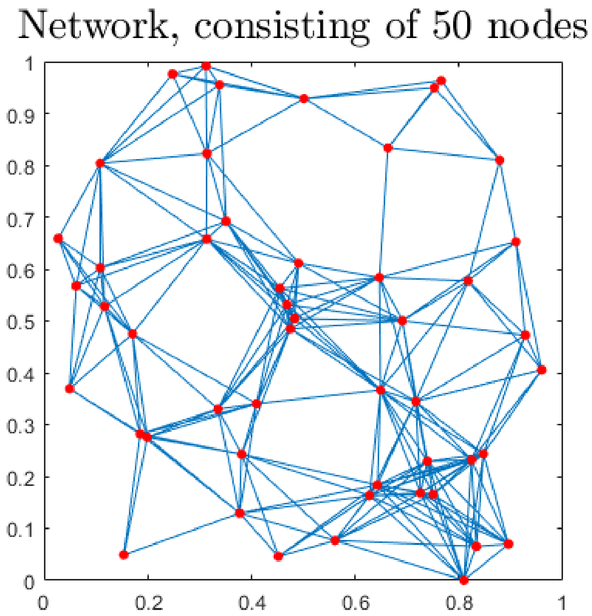
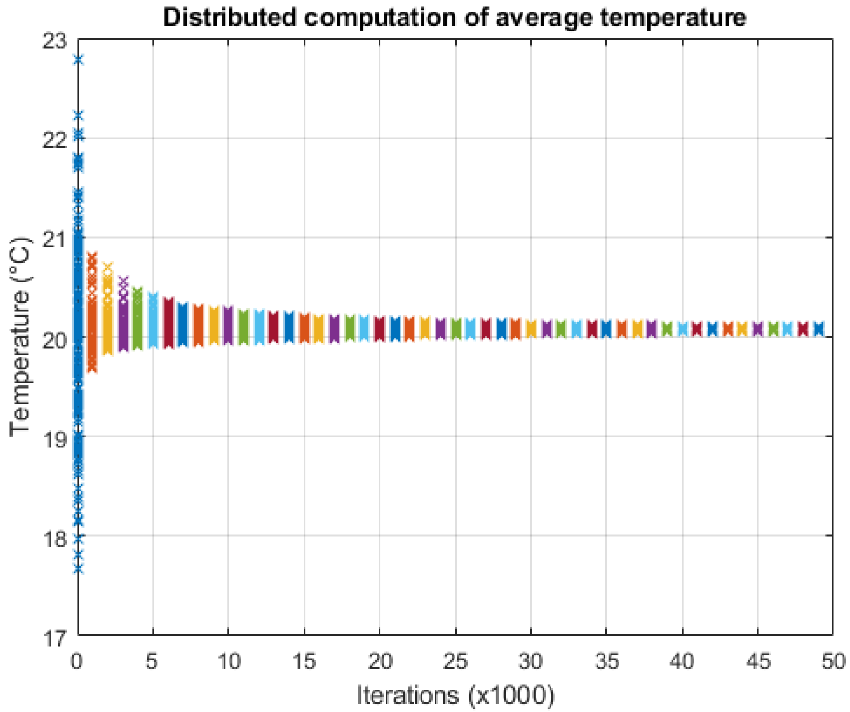
Background

- Graph:



- Quantisation:
  - Additive noise model
  - Variable quantiser stepsize

Inter-node communication



Inter-node communication

- Covariance:

$$\text{cov}\left(v^k\right) = \mathbb{E}\left[v^k {v^k}^H\right] - \mathbb{E}[v^k]\mathbb{E}\left[{v^k}^H\right]$$

- Entropy:

$$H(v) \leq \frac{1}{2}\log\left(\frac{2\pi e \sigma_v^2}{\Delta^2}\right)$$

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Distributed optimisation

- Monotone operator theory
  - Alternating Direction Method of Multipliers (ADMM)
  - Primal-Dual Method of Multipliers (PDMM)
  - Proximal Point Methods
  - …

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Background: monotone operators

Problem formulation: monotonic inclusion

- Lemma:

$$0 \in T(x) \Leftrightarrow x \in (I + \rho T)(x) \Leftrightarrow$$
$$x = (I + \rho T)^{-1}(x) = J_{\rho T}(x)$$

- Solution to minimisation:

$$\min_x f(x) \Leftrightarrow x^* \text{ iff } 0 \in \partial f(x^*)$$

- Conclusion:
  find solution using fixed point of resolvent

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Background: operator splitting

Operator splitting:

- $0 \in T(x) \Leftrightarrow 0 \in T_1(x) + T_2(x)$

ADMM

- $z^{k+1} = ((1-\alpha)I + \alpha C_{\rho T_2} \circ C_{\rho T_1})(z^k)$,
- $C_{\rho T_2}(z^k) = (2J_{\rho T_2} - I)(z^k)$
- $x^k = J_{\rho T_1}(z^k)$

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Inexact Krasnoselskiĭ-Mann iterations

- Standard iterations:

$$z^{k+1} = z^k + \alpha^k\left(T\left(z^k\right) + \epsilon^k - z^k\right)$$
$$\Leftrightarrow \left(\left(1 - \alpha^k\right)I + \alpha^k T\right)\left(z^k\right) + \alpha^k \epsilon^k$$

- Equal to ADMM for

$$\alpha^k = \frac{1}{2} \forall k, T = C_{\rho T_2} \circ C_{\rho T_1}$$

Convergence

- Quantised update:

$$z^{k+1} = T\left(z^k\right) + n_q^k$$

- Non-expansive property:

$$\left\|z^{k+1} - z^*\right\| \leq \left\|T\left(z^k\right) - z^*\right\| + \|n_q^k\|$$

- Difference:

$$v^{k+1} = T\left(z^k\right) - z^k + n_q^k$$

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Convergence

- Difference:

$$v^{k+1} = T(z^k) - z^k + n_q^k$$

- Entropy:

$$\operatorname{cov}(v^k) \leq \mathbb{E}\left[\|v^k\|^2\right] I$$

$$H(v) \leq \frac{1}{2}\log\left(\frac{2\pi e \sigma_v^2}{\Delta^2}\right)$$

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Convergence

$$\Delta^2 \downarrow \Leftrightarrow n_q^{k+1} \downarrow$$

decreasing n_q^{k+1}

$$v^{k+1} = T\left(z^k\right) - z^k + n_q^k$$

$T\left(z^k\right) - z^k$ converging

$$\mathrm{cov}\left(v^k\right) \leq \mathbb{E}\left[\left\|v^k\right\|^2\right] I$$

decreasing σ_v^2

$$H(v) \leq \frac{1}{2}\log\left(\frac{2\pi e\sigma_v^2}{\Delta^2}\right)$$

decreasing Δ^2

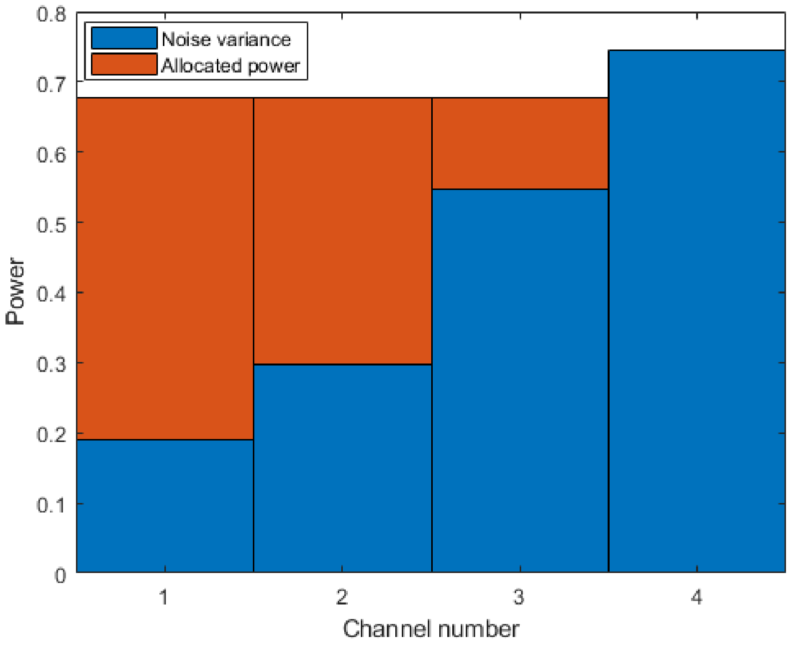
Sample problem

Gaussian Channel Capacity Maximisation Problem:

$$\min_{x} -\sum_{i \in V} \log(\sigma_i^2 + x_i)$$

$$\text{s.t. } x \geq 0,$$

$$\sum_{i \in V} x_i = P_{tot}$$



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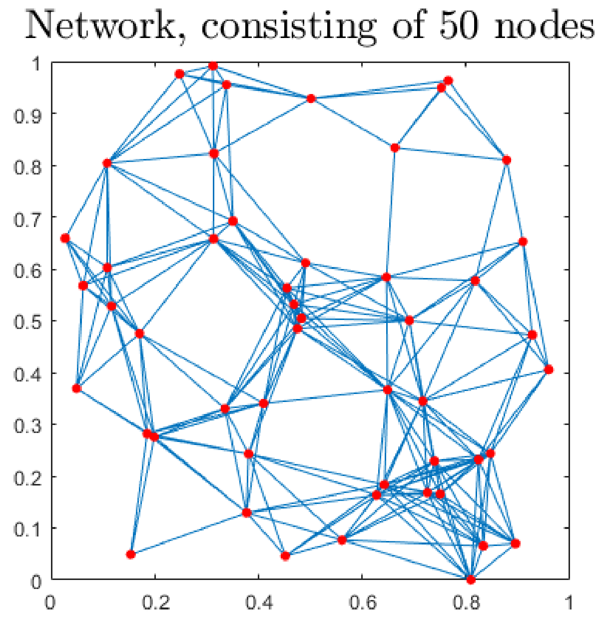
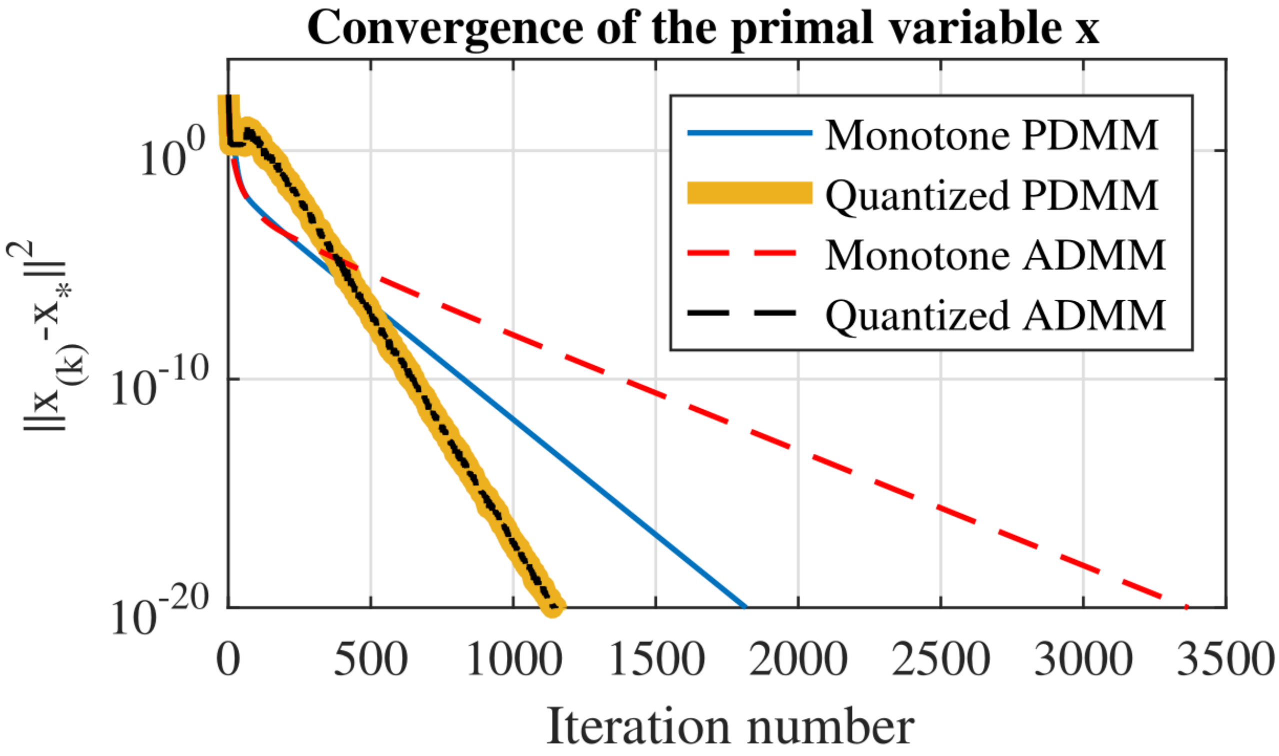
Simulation parameters

- Connected grid, 50 nodes
- ADMM & PDMM
- 1 bit quantiser
- Variable stepsize
- Out of range errors



Simulation results



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Conclusion

- Fixed rate quantization possible without loss of performance
- Out of range errors have effect
- Optimization schemes robust against the quantization errors for finite precision applications

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