Precoding Matrix Design In Linear Video Coding

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Outline

1. Linear Video Coding Scheme: SoftCast
2. Optimal precoding matrix design under per-subchannel power constraint
3. Lower complexity design approaches
4. Simulations
5. Conclusions
6. Appendix

SoftCast [1]

- Joint source-channel video coder
- Linear source coder and channel precoder
  - 3D-DCT on Group of Pictures (GoP) of video
  - Linear scaling for each DCT-coefficients for error protection
  - No motion compensation, no entropy coding, no quantization
- Analog-like modulation
- LMMSE decoder

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Performance of SoftCast [2]

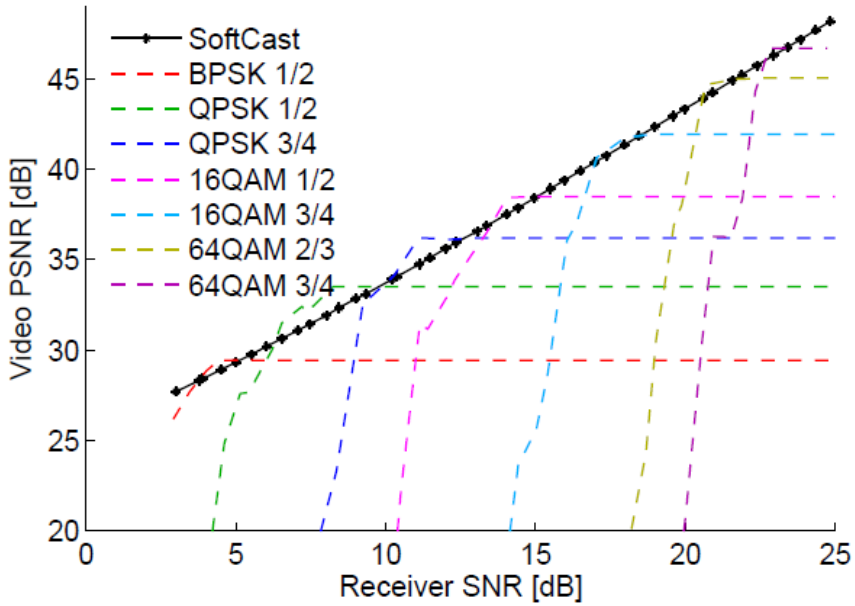


Figure: Performance of SoftCast vs.single-layer MPEG4

Video quality commensurates with channel quality

[2] Jakubczak, S. and Katabi, D., "SoftCast: one-size-fits-all wireless video", ACM (2010), 449–450.

SoftCast Transmitter[3]

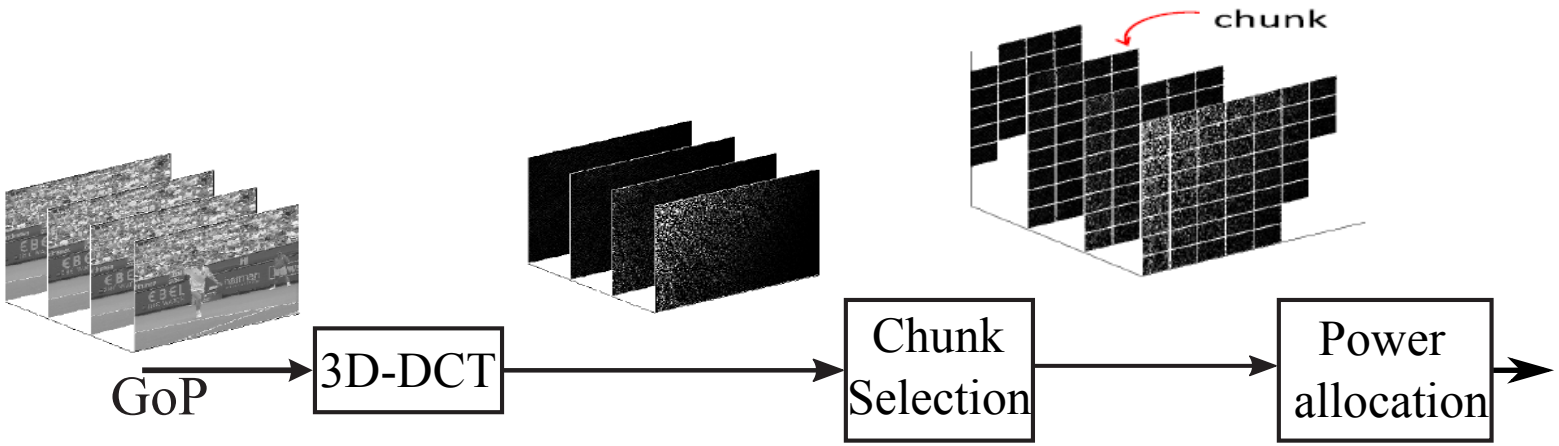


Figure: Softcast coding and power allocation

- Chunk: $n_r \times n_c$ spatial DCT coefficients assumed uncorrelated.
- Low-variance chunks discarded when bandwidth limited.
- Power allocation: each chunk is scaled to meet transmission power constraint.

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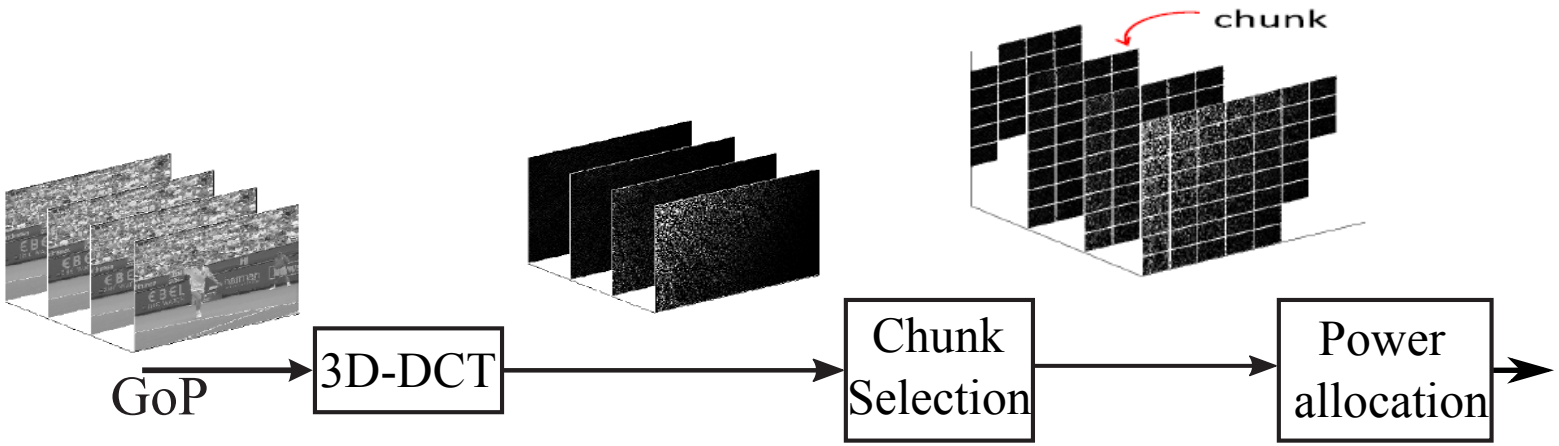


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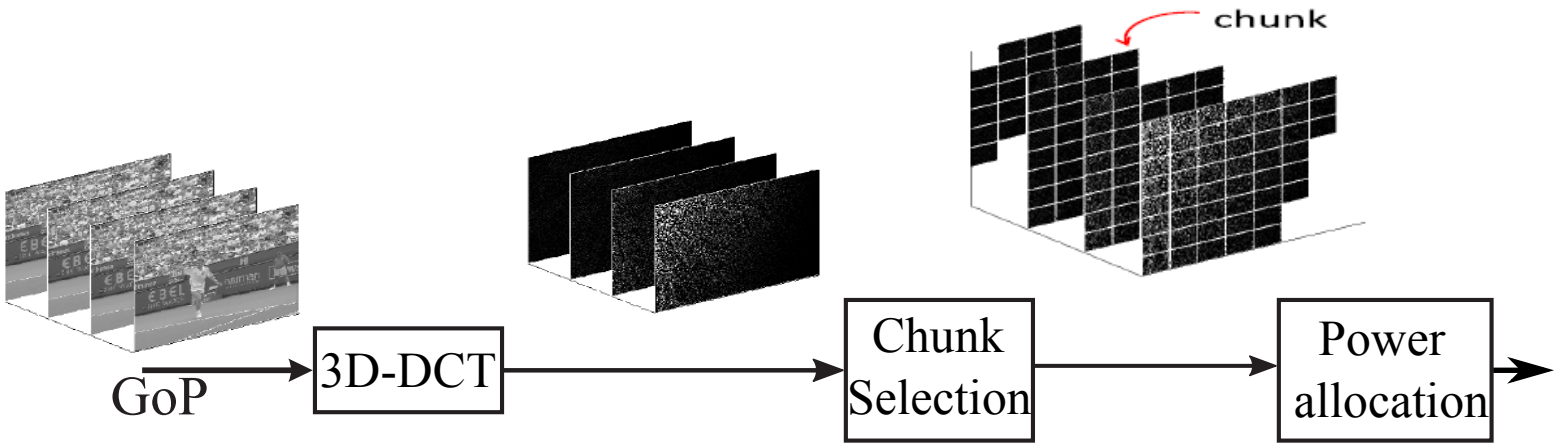


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Total power constraint

Power allocation assumes in most of the cases under total power constraint P_{T}.

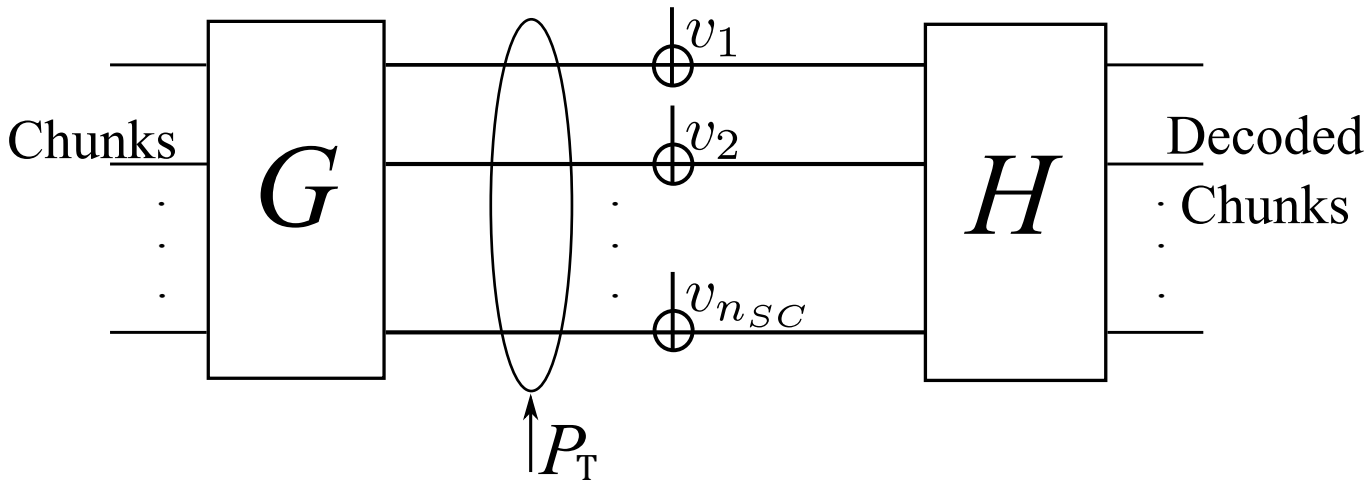


Figure: AWGN channel and total power constraint

G : Precoding matrix

H : LMMSE decoder

Per-subchannel power constraints

Per-subchannel power constraints in

- DSL or powerline telecommunication (PLT) channels[4]
- Multi-antenna transmission

[4] *Yonge et al.*, "An overview of the HomePlug AV2 technology," Journal of Electrical and Computer Engineering, vol. 2013, pp. 1–20, 2013.

Per-subchannel power constraints

Here: power allocation with per-subchannel power constraints.

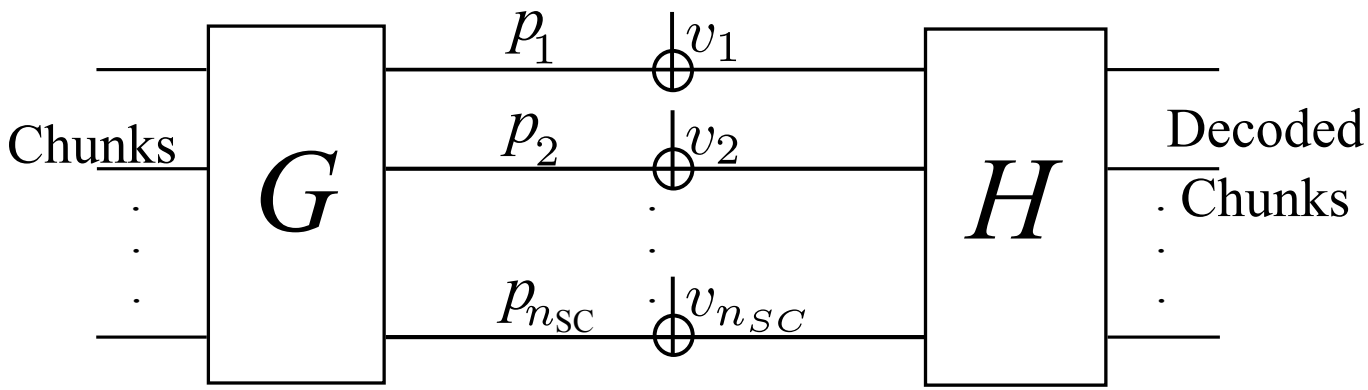


Figure: AWGN channel and per-subchannel constraint

Chunk vector

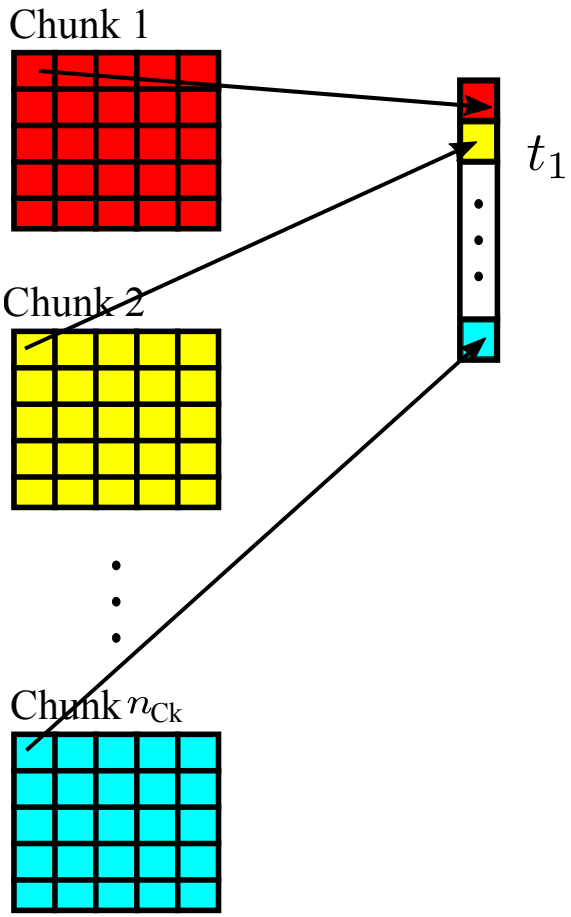


Figure: First components in each chunk form first chunk vector.

Chunk vector

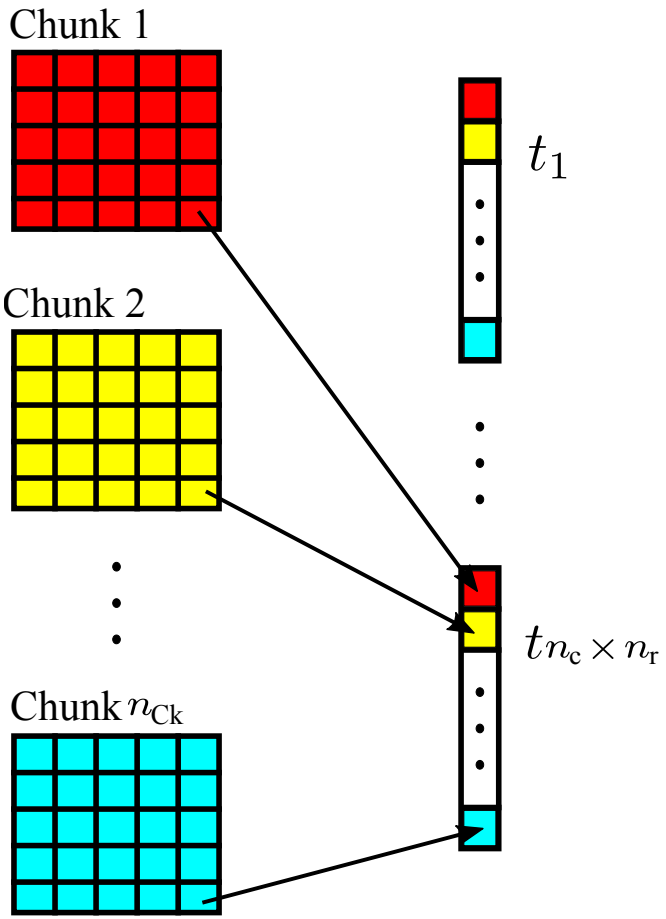


Figure: Last components in each chunk form last chunk vector.

Transmitted vector

- Sequence of $n_r \times n_c$ chunk vectors $\mathbf{t}$, with

$$E\left(\mathbf{t}\mathbf{t}^{\mathsf{T}}\right) = \Lambda = \mathrm{diag}\left(\lambda_1 \ldots \lambda_{n_{\mathrm{Ck}}}\right),$$

  λ_i in decreasing order.
- Transmitted vector: $\mathbf{x} = G\mathbf{t}$. Precoding matrix: $G \in \mathbb{R}^{n_{\mathrm{SC}} \times n_{\mathrm{Ck}}}$[5].
- Per-subchannel power constraint:

$$E\left(\mathbf{x}\mathbf{x}^{\mathsf{T}}\right)_{i,i} = \left(G\Lambda G^{\mathsf{T}}\right)_{i,i} \leqslant p_i, \ i = 1, \ldots, n_{\mathrm{SC}}$$

[5] After chunk selection, we assume $n_{\mathrm{Ck}} = n_{\mathrm{SC}}$ and over each subchannel one chunk is transmitted during one GoP.

Transmission Model

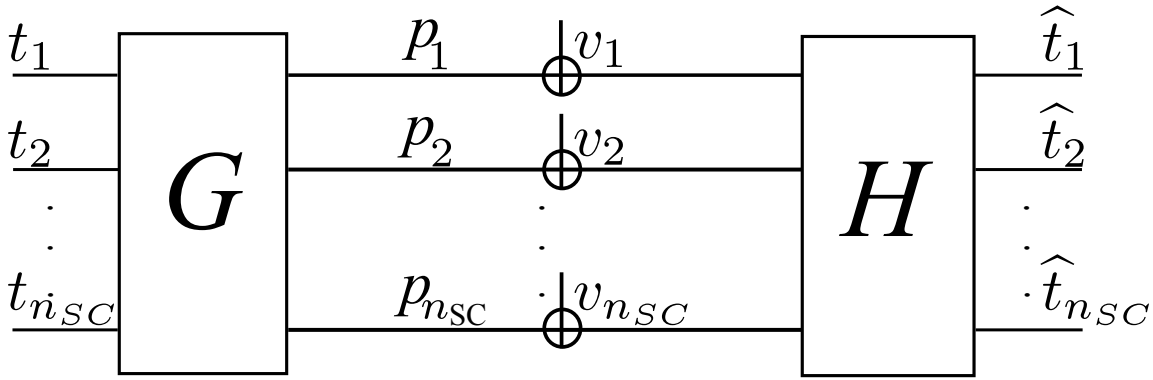


Figure: Channel Model

- Received vector

$$\mathbf{y} = G\mathbf{t} + \mathbf{v} \tag{1}$$

where $\mathbf{v} \sim \mathcal{N}(0, N)$.

- Linear decoding: $\widehat{\mathbf{t}} = H\mathbf{y}$.
- Mean square reconstruction error (MSE)

$$\varepsilon = E\left[\left(\mathbf{t} - \widehat{\mathbf{t}}\right)^{\mathrm{T}}\left(\mathbf{t} - \widehat{\mathbf{t}}\right)\right] \tag{2}$$

Compute the optimal precoding matrix G

- Decoding matrix minimizing ε

$$\overline{H} = \Lambda G^T \left(G \Lambda G^T + N \right)^{-1}. \tag{3}$$

- With $\overline{H}$, ε becomes

$$\varepsilon = \mathrm{tr} \left(\left(I + \left(G \Lambda^{\frac{1}{2}} \right)^T N^{-1} \left(G \Lambda^{\frac{1}{2}} \right) \right)^{-1} \Lambda \right), \tag{4}$$

to be minimized under

$$\left(G \Lambda G^{\mathsf{T}} \right)_{i,i} = p_i, \ i = 1, \ldots, n_{\mathsf{SC}} \tag{5}$$

Optimal precoding matrix G [6]

First, compute an optimal precoding matrix $\widetilde{G}$ under total power constraint.

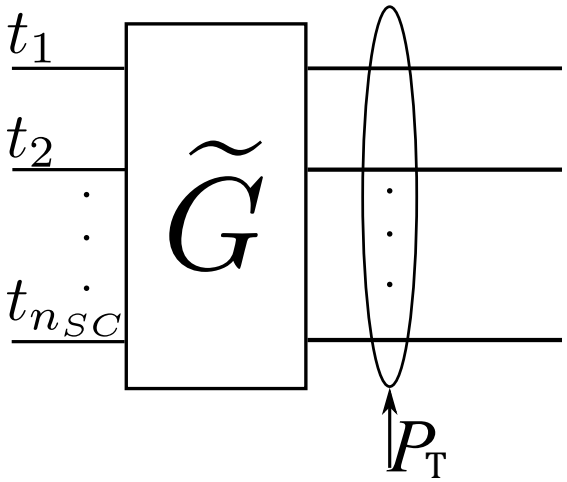


Figure: Total power constraint

[6] Palomar, D. P. and Lagunas, M. A. and Cioffi, J. M., "Optimum Linear Joint Transmit-Receive Processing for MIMO Channels with QoS Constraints", IEEE trans. Signal Processing (2004), 1179-1197.

Optimal precoding matrix G

Then compute an orthogonal transform matrix Z to match per-subchannel power constraints.

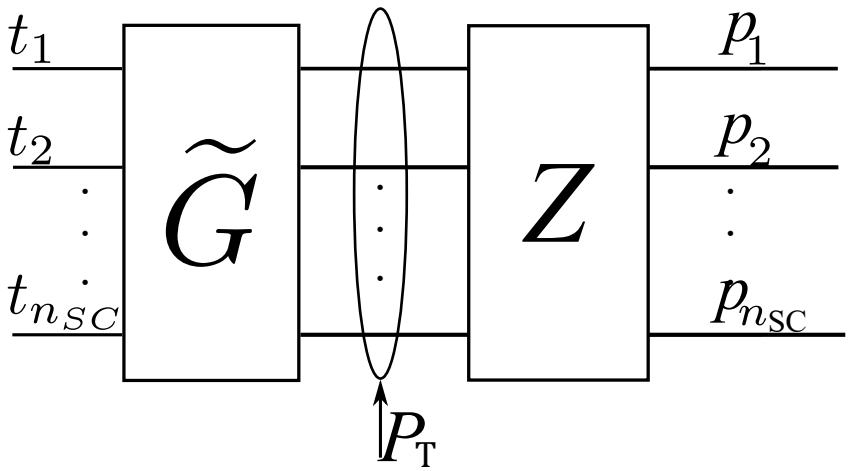


Figure: $G = Z\widetilde{G}$

Therefore $G = Z\widetilde{G}$.

Optimal precoding matrix G

How to compute Z

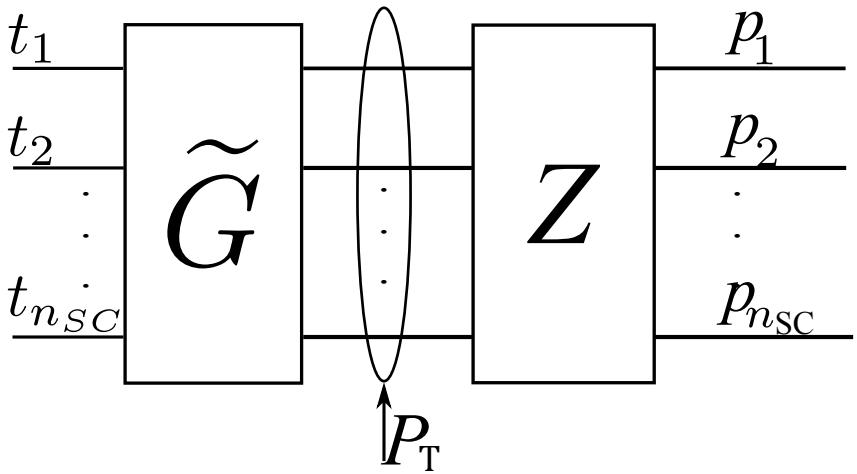


Figure: $G = Z\widetilde{G}$

Conditions for the existence of Z

- Let $m = E\left(\text{diag}\left(\widetilde{G}\mathbf{t}\mathbf{t}^{\mathsf{T}}\widetilde{G}^{T}\right)\right) = \text{diag}\left(\widetilde{G}\Lambda\widetilde{G}^{T}\right)$ with $m_1 \geq \cdots \geq m_{n_{\mathsf{SC}}}$, and $p = \left(p_i, \ldots, p_{n_{\mathsf{SC}}}\right)$ with $p_1 \geq \cdots \geq p_{n_{\mathsf{SC}}}$.
- If

$$\sum_{i=1}^{k} p_i \leq \sum_{i=1}^{k} m_i \tag{6}$$

for all $k = 1, 2, \ldots, n_{\mathsf{SC}}$, then an orthonornal matrix Z such that

$$\text{diag}\left(ZMZ^{T}\right) = p$$

can be found[7].
- What if the conditions do not hold?

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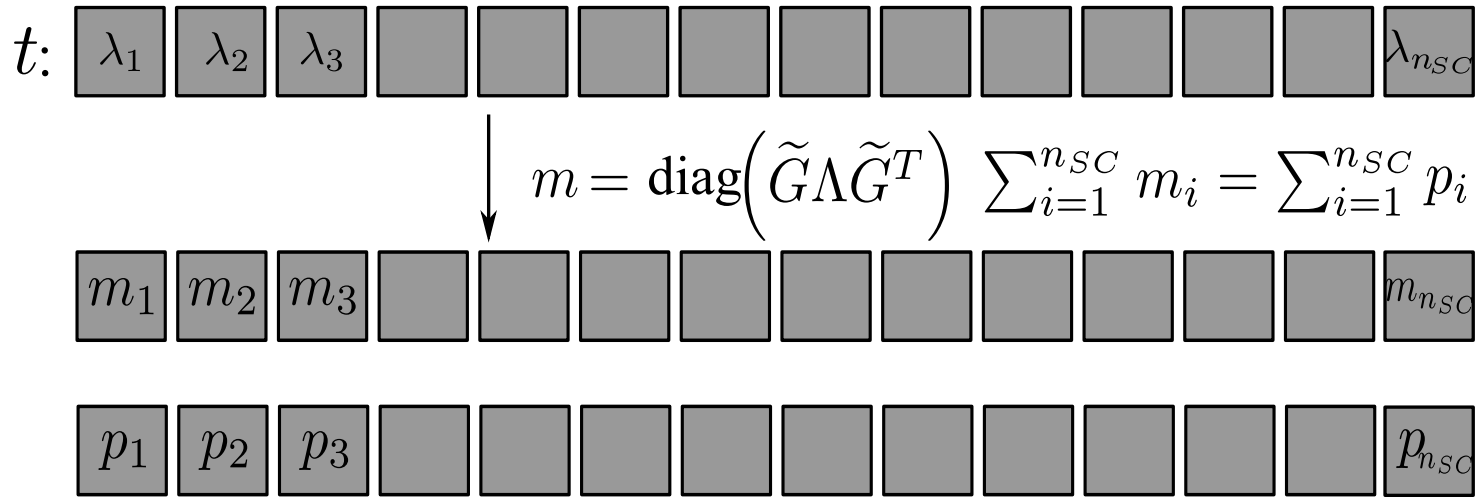
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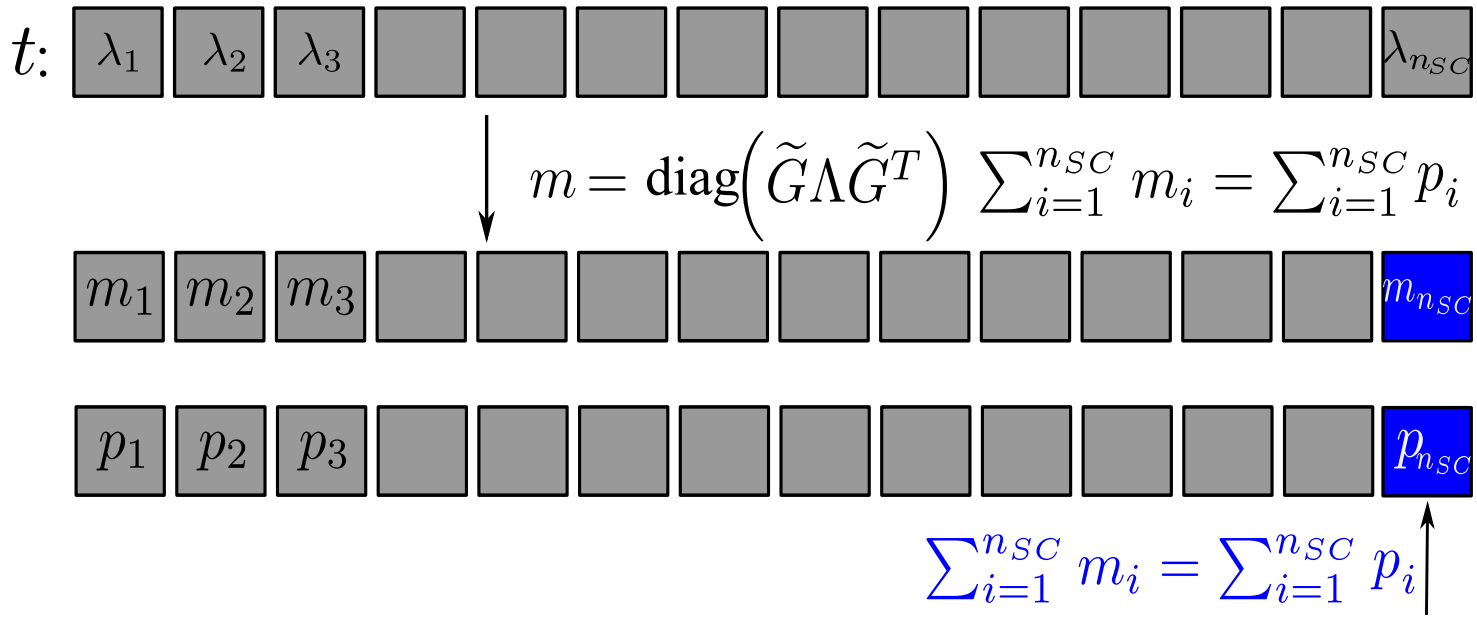
Multi-Level Water-Filling

At first, compute a $\widetilde{G}$ under total power constraint $\sum_{i=1}^{nsc} p_i$.



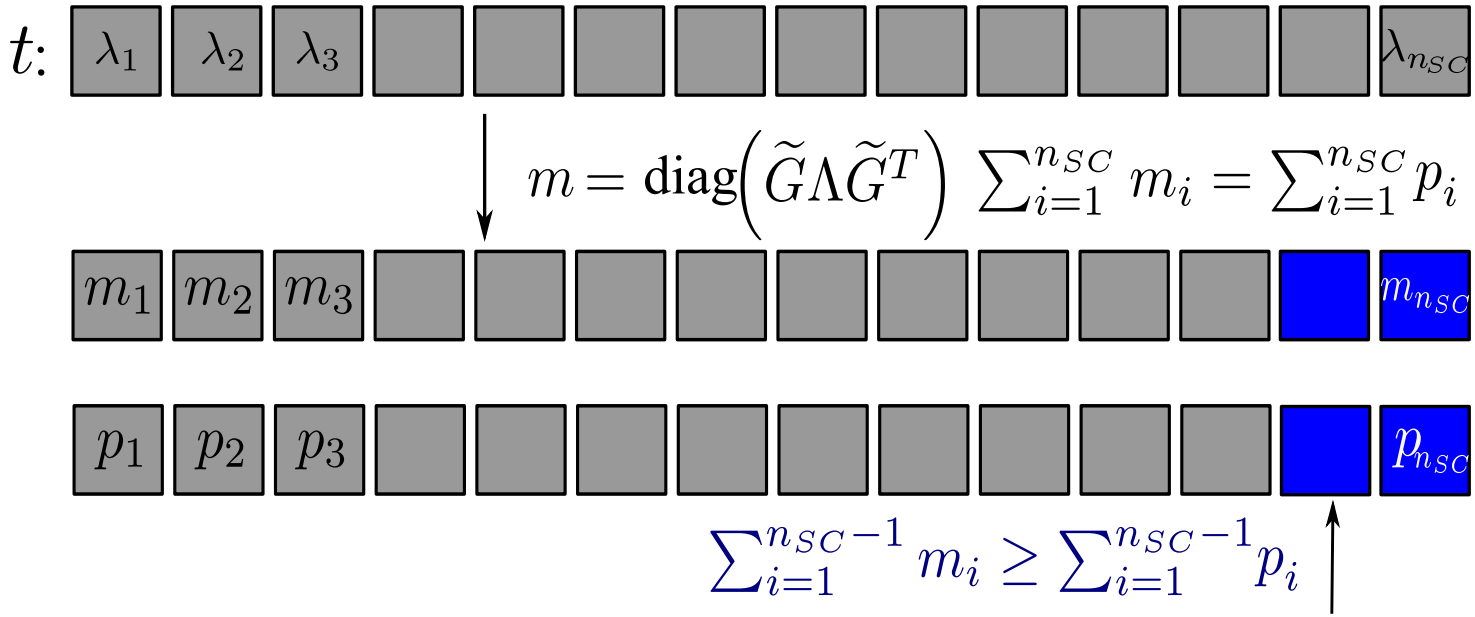
Multi-Level Water-Filling

Then, check the conditions of existence of Z.



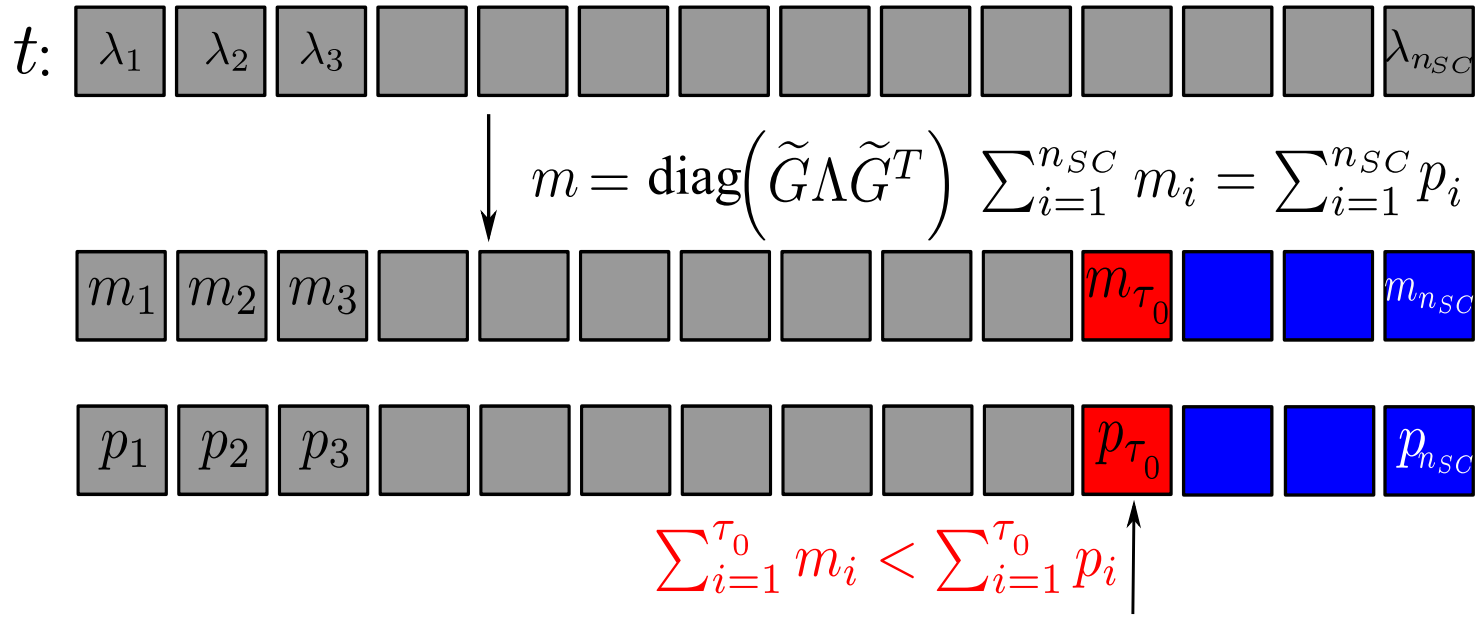
Multi-Level Water-Filling

Check the conditions of existence of Z.



Multi-Level Water-Filling

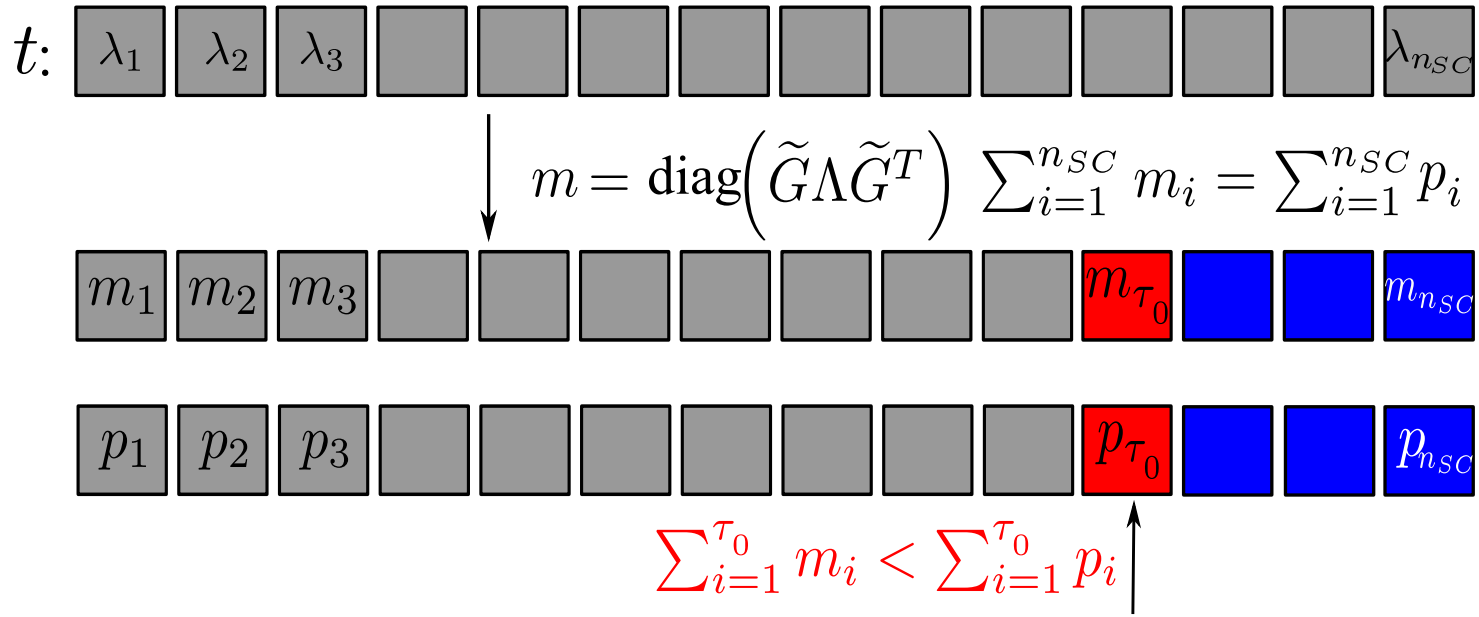
- Check the conditions of existence of Z.



- When conditions are not satisfied, recompute $\widetilde{G}$ for components from 1 to τ_0 of t under total power constraint $\sum_{i=1}^{\tau_0} p_i$.

Multi-Level Water-Filling

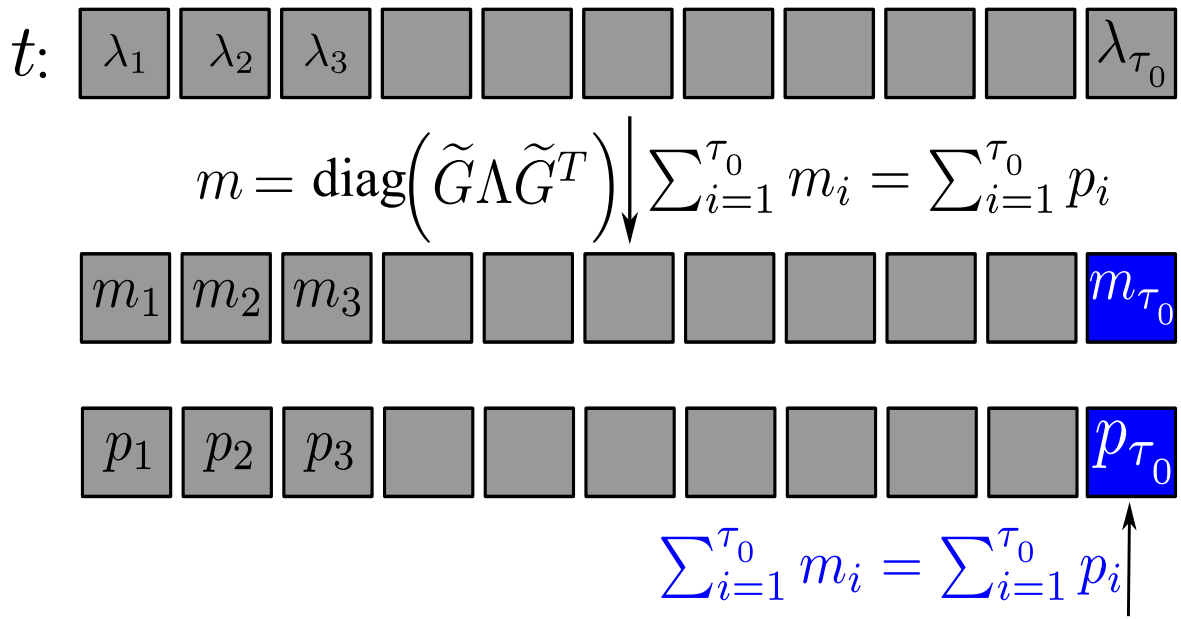
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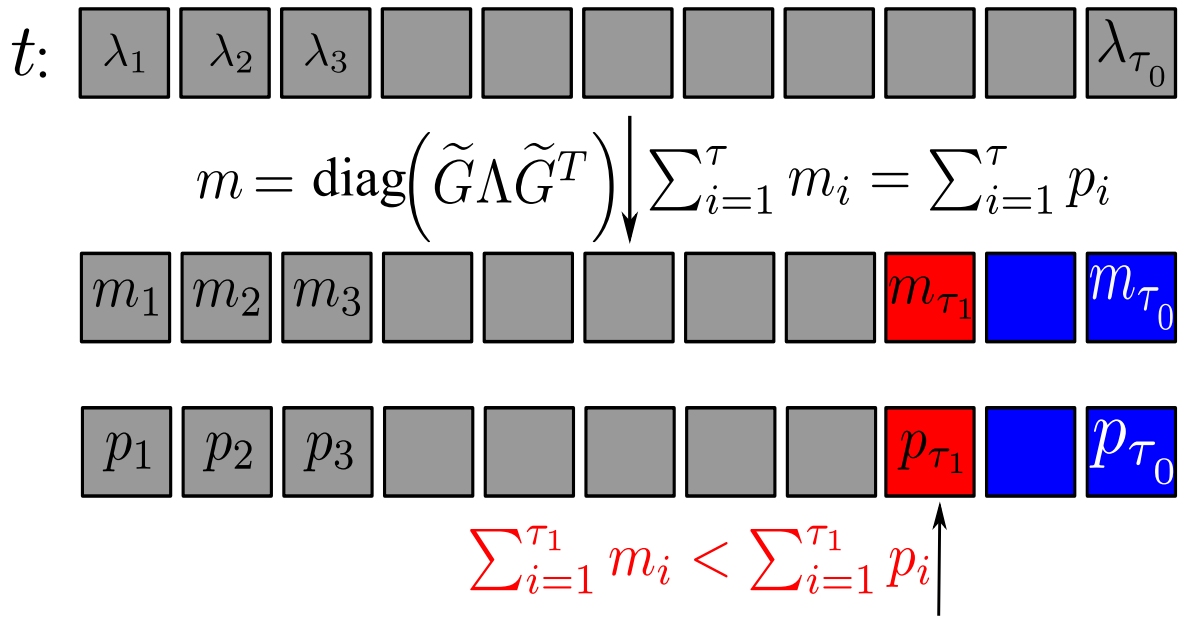
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Multi-Level Water-Filling

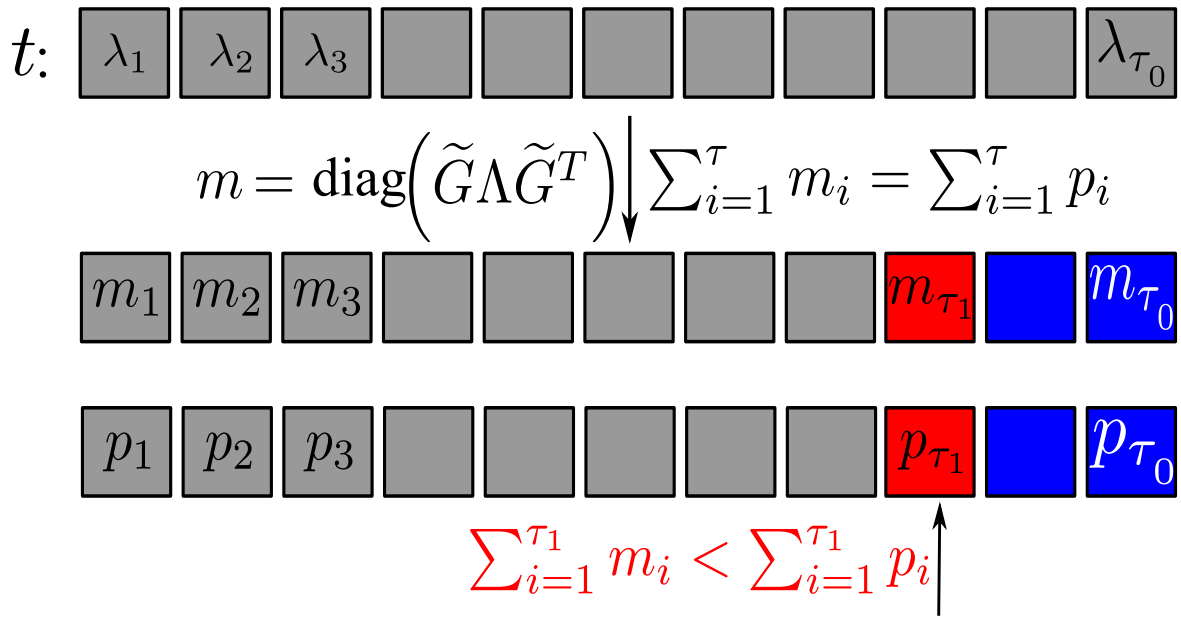
- Check the conditions of existence of Z.



- When conditions are not satisfied, recompute $\widetilde{G}$ for components from 1 to τ_1 of t under total power constraint $\sum_{i=1}^{\tau_1} p_i$.

Multi-Level Water-Filling

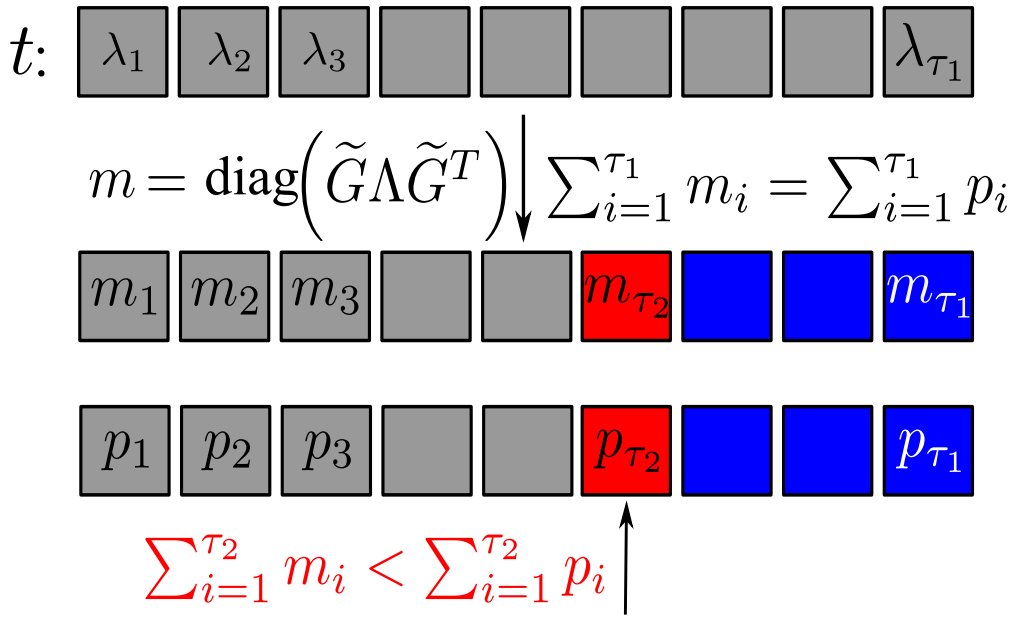
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Multi-Level Water-Filling

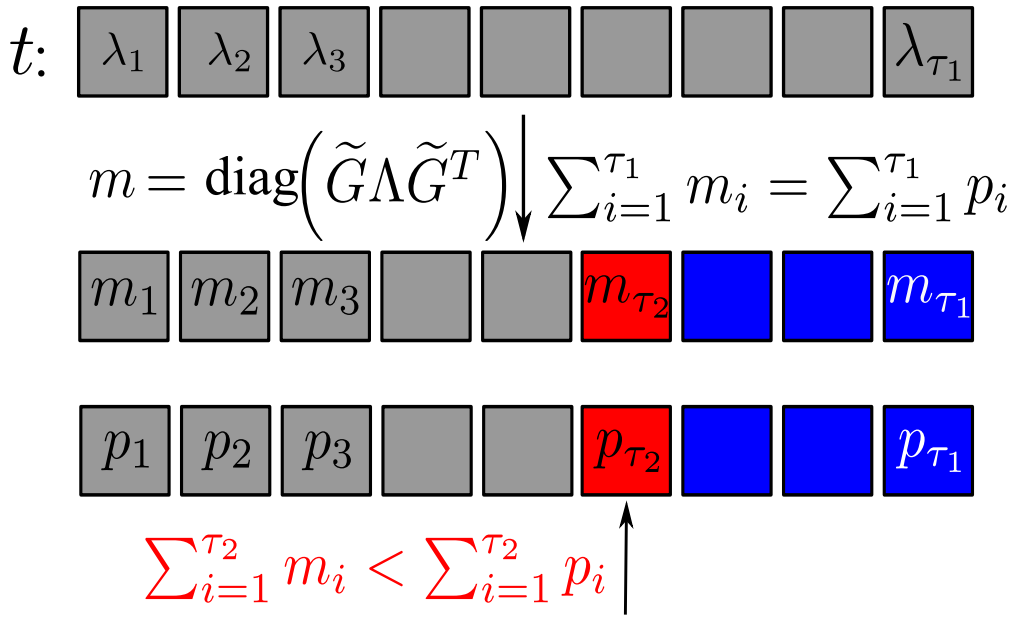
- Compute a $\widetilde{G}$ for components from 1 to τ_1 of t and check the conditions of existence of Z.



- When conditions are not satisfied, recompute $\widetilde{G}$ for components from 1 to τ_2 of t under total power constraint $\sum_{i=1}^{\tau_2} p_i$ and continue.

Multi-Level Water-Filling

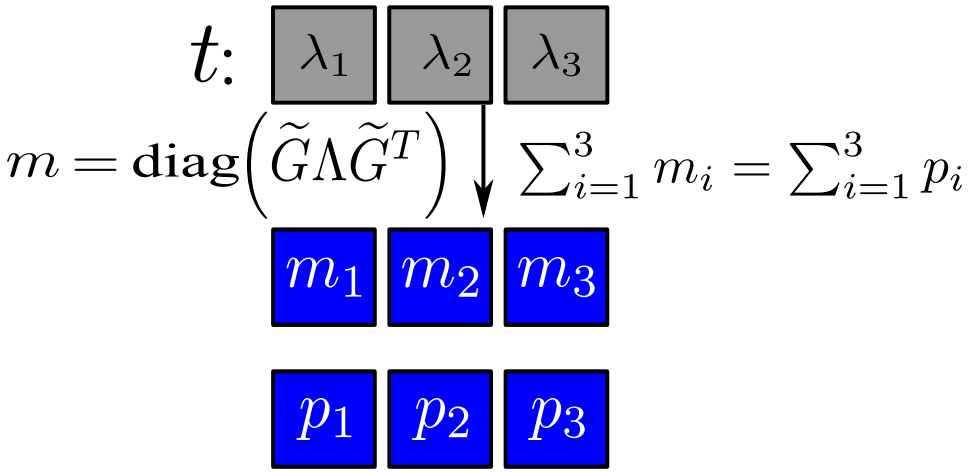
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Multi-Level Water-Filling

At the end, conditions of existence of Z are satisfied for $\lambda_1, \lambda_2, \lambda_3$ and p_1, p_2, p_3.



$$\sum_{i=1}^{j} m_i \geq \sum_{i=1}^{j} p_i, 1 \leq j \leq 3$$

Multi-Level Water-Filling

First subblock of $Z\widetilde{G}$ has been found

$$\left(Z_1\widetilde{G}_1\right)\begin{bmatrix}\lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3\end{bmatrix}\left(Z_1\widetilde{G}_1\right)^{\mathsf{T}}=\begin{bmatrix}p_1 & * & * \\ * & p_2 & * \\ * & * & p_3\end{bmatrix}$$

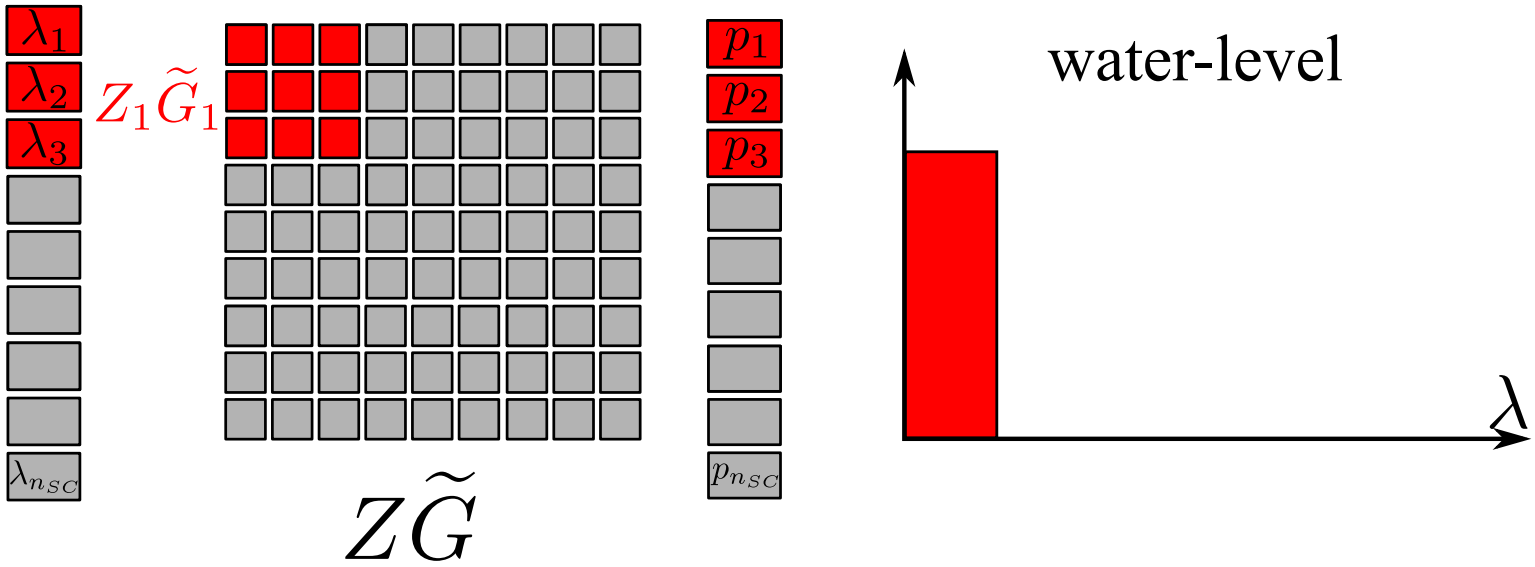


Figure: First subblock of $Z\widetilde{G}$ and associated water-level under total power constraint $\sum_{i=1}^{3} p_i$.

Multi-Level Water-Filling

Second subblock of $Z\widetilde{G}$

$$\left(Z_2\widetilde{G}_2\right)\begin{bmatrix}\lambda_4 & & \\ & \ddots & \\ & & \lambda_k\end{bmatrix}\left(Z_2\widetilde{G}_2\right)^{\mathsf{T}}=\begin{bmatrix}p_4 & * & * \\ * & \ddots & * \\ * & * & p_k\end{bmatrix}$$

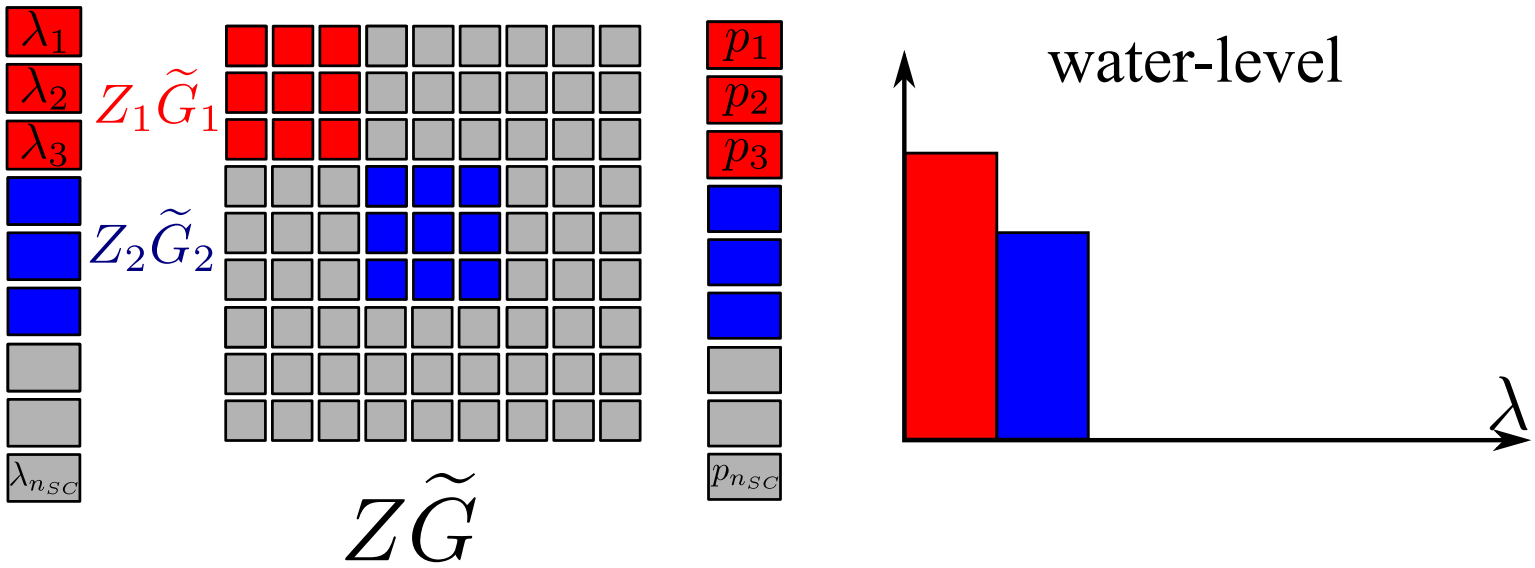


Figure: Second subblock of $Z\widetilde{G}$ and associated water-level under total power constraint $\sum_{i=4}^{k} p_i$.

Multi-Level Water-Filling compute $Z\widetilde{G}$

Last subblock of $Z\widetilde{G}$

$$\left(Z_3\widetilde{G}_3\right)\begin{bmatrix}\lambda_{k+1} & & \\ & \ddots & \\ & & \lambda_{n_{SC}}\end{bmatrix}\left(Z_3\widetilde{G}_3\right)^T=\begin{bmatrix}p_{k+1} & * & * \\ * & \ddots & * \\ * & * & p_{n_{SC}}\end{bmatrix}$$

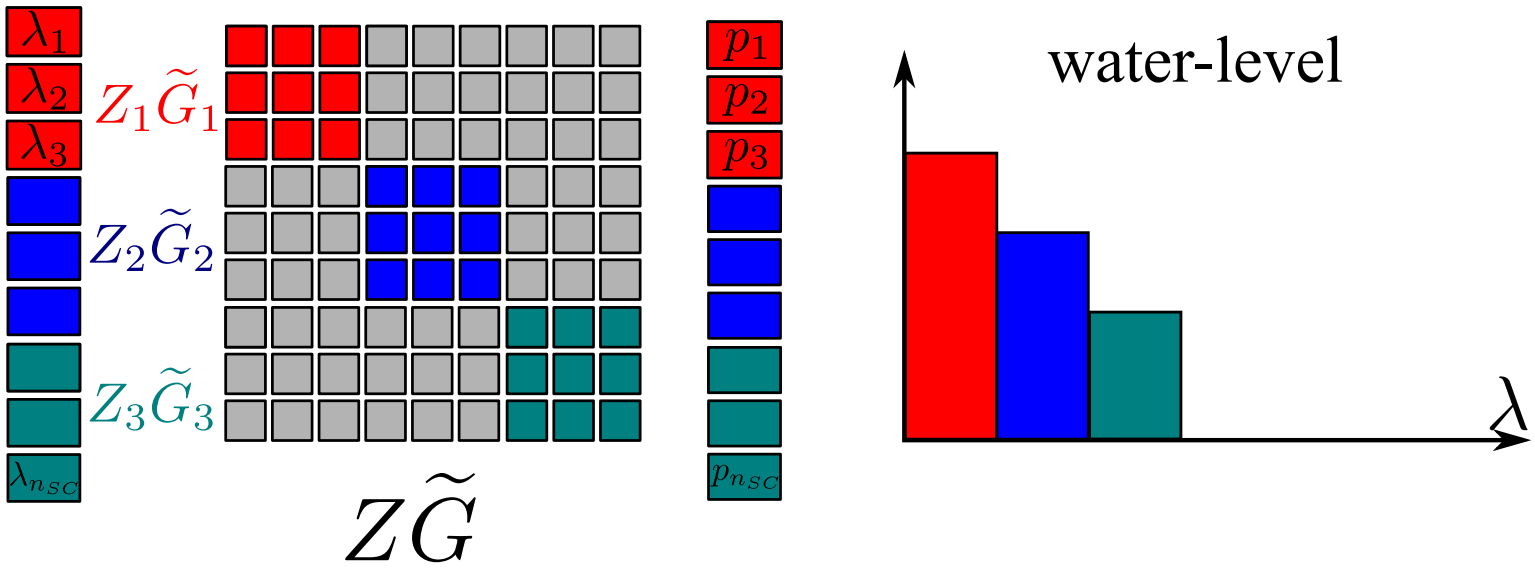


Figure: Last subblock of $Z\widetilde{G}$ and associated water-level under total power constraint $\sum_{i=k+1}^{nsc} p_i$.

Result

- Structure of optimal precoding matrix

$$G = \begin{pmatrix} G_1 & 0 & \cdots & 0 & 0 \\ 0 & G_2 & 0 & \cdots & 0 \\ 0 & 0 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & G_{s-1} & 0 \\ 0 & 0 & \cdots & 0 & G_s \end{pmatrix} = \begin{pmatrix} Z_1\widetilde{G}_1 & 0 & \cdots & 0 & 0 \\ 0 & Z_2\widetilde{G}_2 & 0 & \cdots & 0 \\ 0 & 0 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & Z_{s-1}\widetilde{G}_{s-1} & 0 \\ 0 & 0 & \cdots & 0 & Z_s\widetilde{G}_s \end{pmatrix}$$

- Worst-case complexity to find the first subblock: $\sum_{k=n_{SC}}^{1} k = O\left(n_{SC}^2\right)$.
- Worst-case total complexity: $\sum_{i=n_{SC}}^{1} n_i^2 = O\left(n_{SC}^3\right)$.

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- Worst-case complexity to find the first subblock: $\sum_{k=n_{SC}}^{1} k = O\left(n_{SC}^2\right)$.
- Worst-case total complexity: $\sum_{i=n_{SC}}^{1} n_i^2 = O\left(n_{SC}^3\right)$.

Result

- Structure of optimal precoding matrix

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Lower complexity design approaches

- Power Allocation with Inferred Split Position (PAISP)
  - Infer split position by dichotomy
  - Less iterations required to recompute $\widetilde{G}$ compared to optimal approach
- Power Allocation with Local Power Adjustement (PALPA)
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Power line channel

- Power line channel considered for transmission[8]. Unit noise variance assumed on each channel.

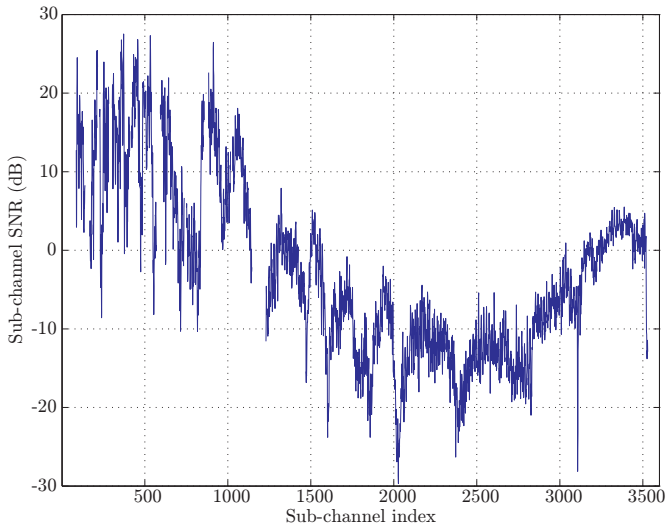


Figure: SNR as a function of the subchannel index for the considered PLT channel

[8] ETSI, "Powerline telecommunications (PLT); powerline HDMI analysis for very short range link HD and UHD applications," ETSI, Technical Report 103 343 V1.1.1, december 2015.

Reference: Simple Chunk Scaling (SCS)

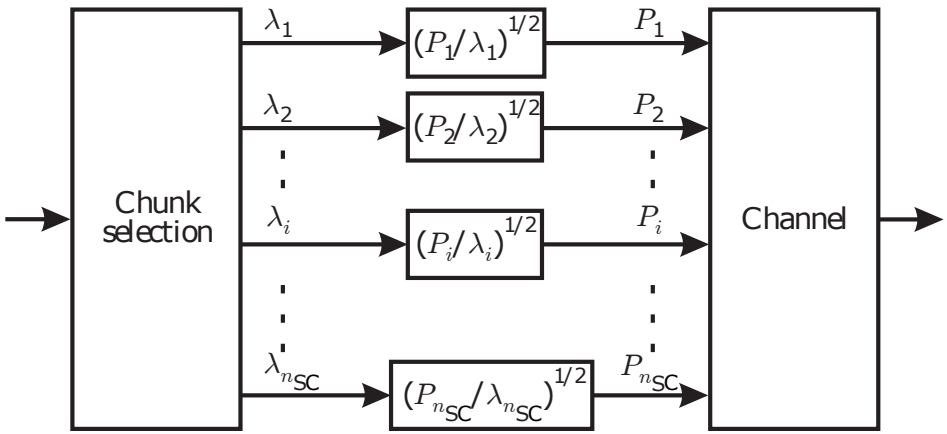
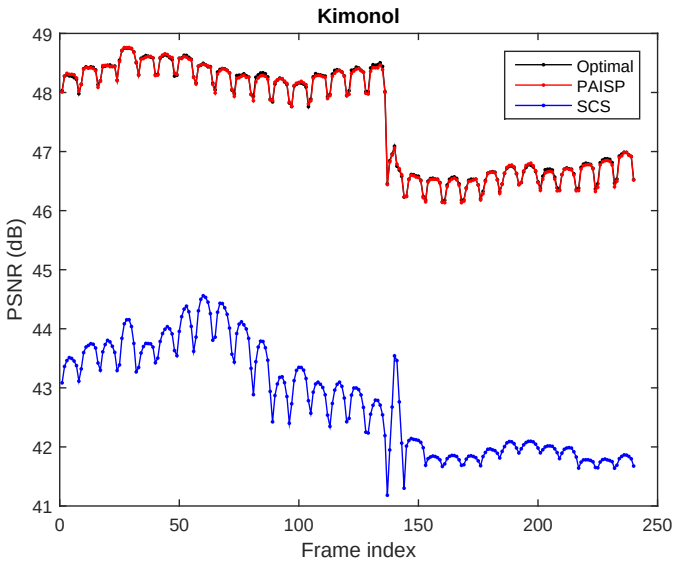
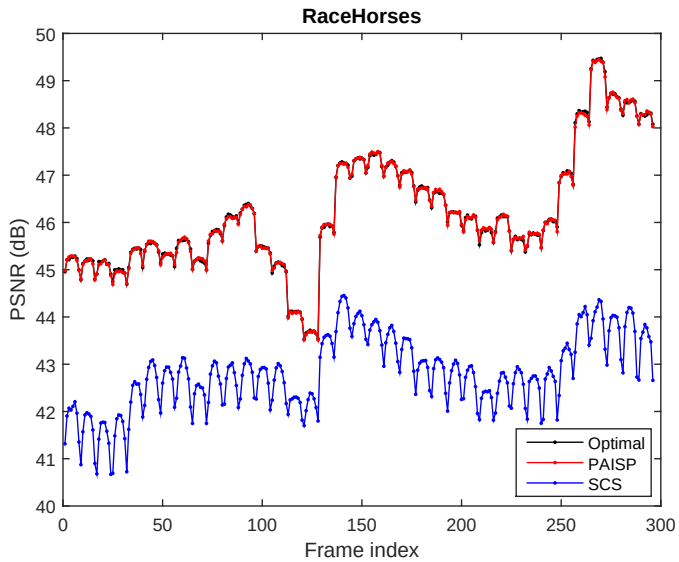


Figure: Simple chunk scaling

Simulation results



(a) Kimonol (Class B)



(b) RaceHorses (Class C)

Figure: PSNRs for optimal, PAISP, and SCS precoding matrix design techniques and the associated decoding matrix.

Simulation results

Cl.	Name	PSNR (dB)			
		SCS	Opt.Alloc/ PAISP	PALPA	Gain
B	Kimonol	42.79	47.57	47.56	4.78
	BasketballDrive	38.83	39.54	39.53	0.71
	BQ Terrace	34.83	35.86	35.85	1.03
	Cactus	36.53	38.47	38.44	1.94
	ParkScene	41.83	44.06	44.03	2.23
C	PartyScene	40.89	42.94	42.94	2.05
	BQMall	41.24	44.91	44.91	3.67
	BasketballDrill	44.96	47.32	47.31	2.36
	RaceHorses	42.81	46.21	46.21	3.4

Simulation results

Cl.	Name	PSNR (dB)			
		SCS	Opt.Alloc/ PAISP	PALPA	Gain
D	BQSquare	39.38	44.55	44.55	5.17
	RaceHorses	43.89	49.03	49.03	5.14
	BlowingBubbles	42.26	47.90	47.90	5.64
	BasketballPass	45.03	49.55	49.55	4.52
E	FourPeople	40.74	47.13	47.13	6.39
	Jonny	40.56	48.43	48.43	7.87
	KristenAndSara	39.77	46.95	46.95	7.18
F	SlideShow	35.28	46.83	46.80	11.55

Complexity comparison

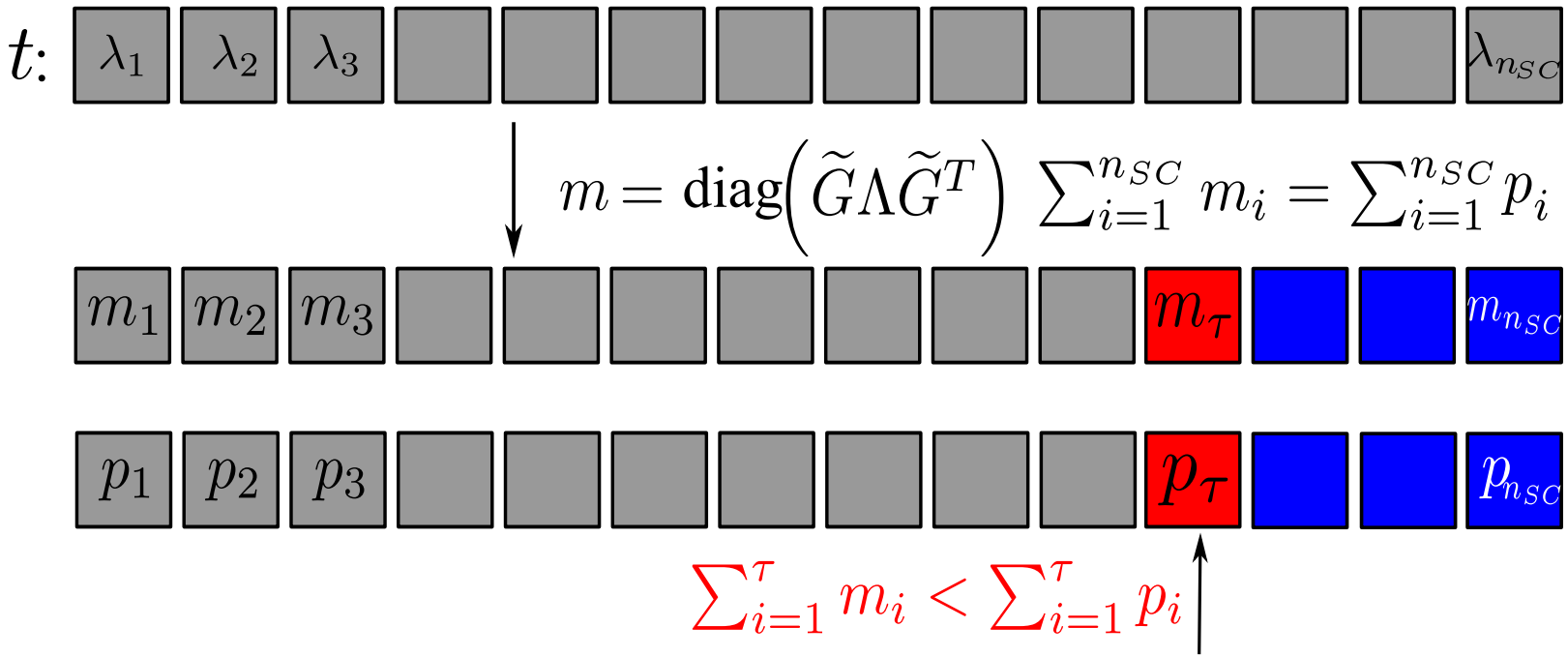
Class	Name	Speed-Up	
		PAISP	PALPA
B	Kimonol	8.6	13
	BasketballDrive	6	14
	BQ Terrace	4	13
	Cactus	3.7	14
	ParkScene	6	12.5
C	PartyScene	3.3	2.6
	BasketballDrill	1.0	1.0
	BQMall	4.5	4.5
	RaceHorses	1.0	1.0
E	FourPeople	3.5	5
	Jonny	5	7
	KristenAndSara	5	5
F	SlideShow	1.0	1.0

Conclusions

- Lower complexity precoding matrix design for SoftCast Scheme under per-channel power constraint
- Average speed up factor of PALPA and PAISP are respectively 13 and 6
- Lower complexity approaches have negligible performance loss compared to optimal precoding matrix design

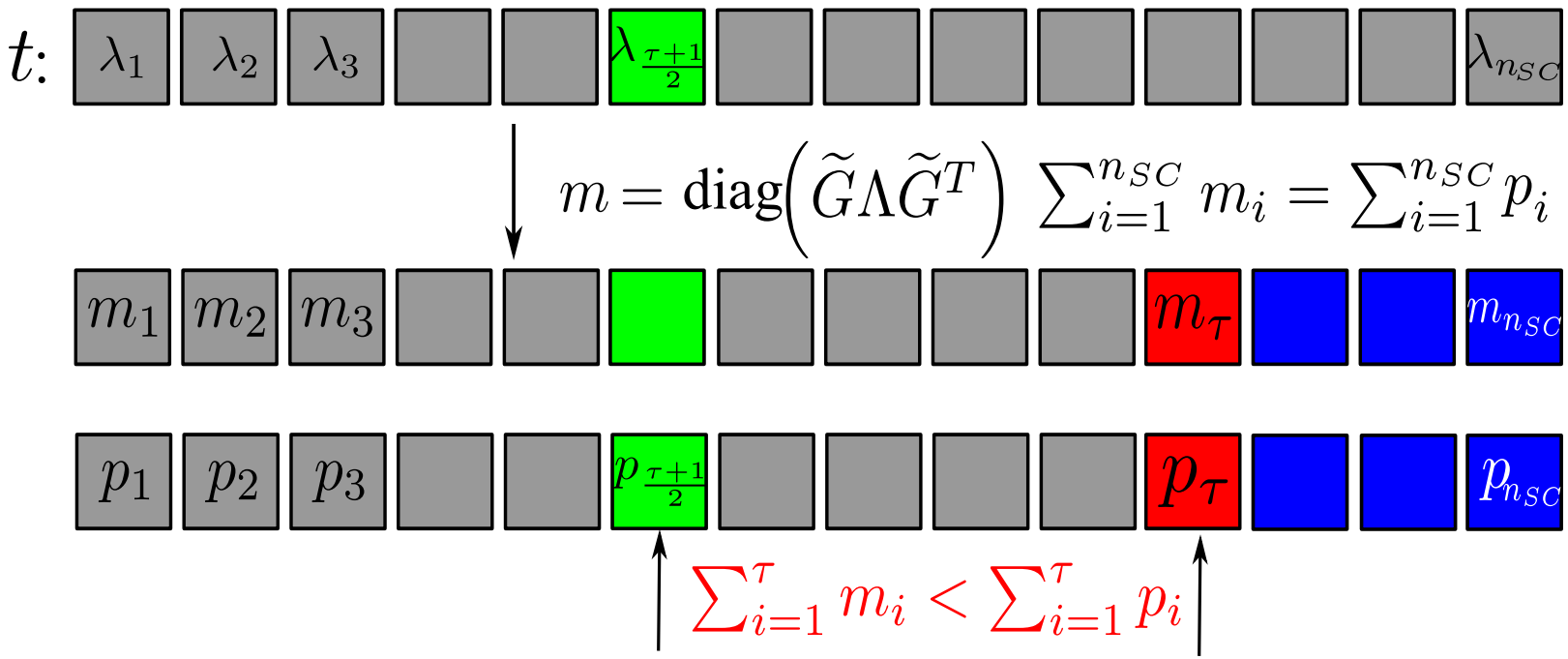
Dichotomy approach

As in optimal method, at first, we compute $\widetilde{G}$ under total power constraint and check the condition of existence of Z.



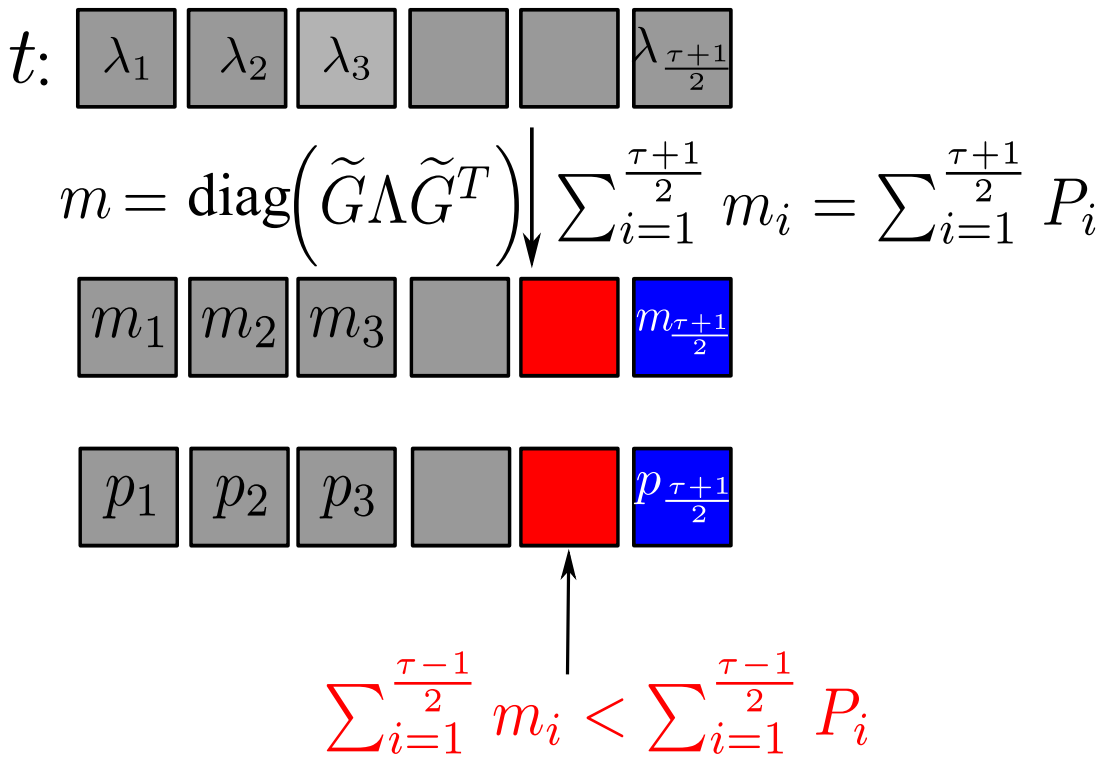
Dichotomy approach

In the next step, we compute a $\widetilde{G}$ for components from 1 to $\frac{\tau+1}{2}$ of t under total power constraint $\sum_{i=1}^{\frac{\tau+1}{2}} p_i$.



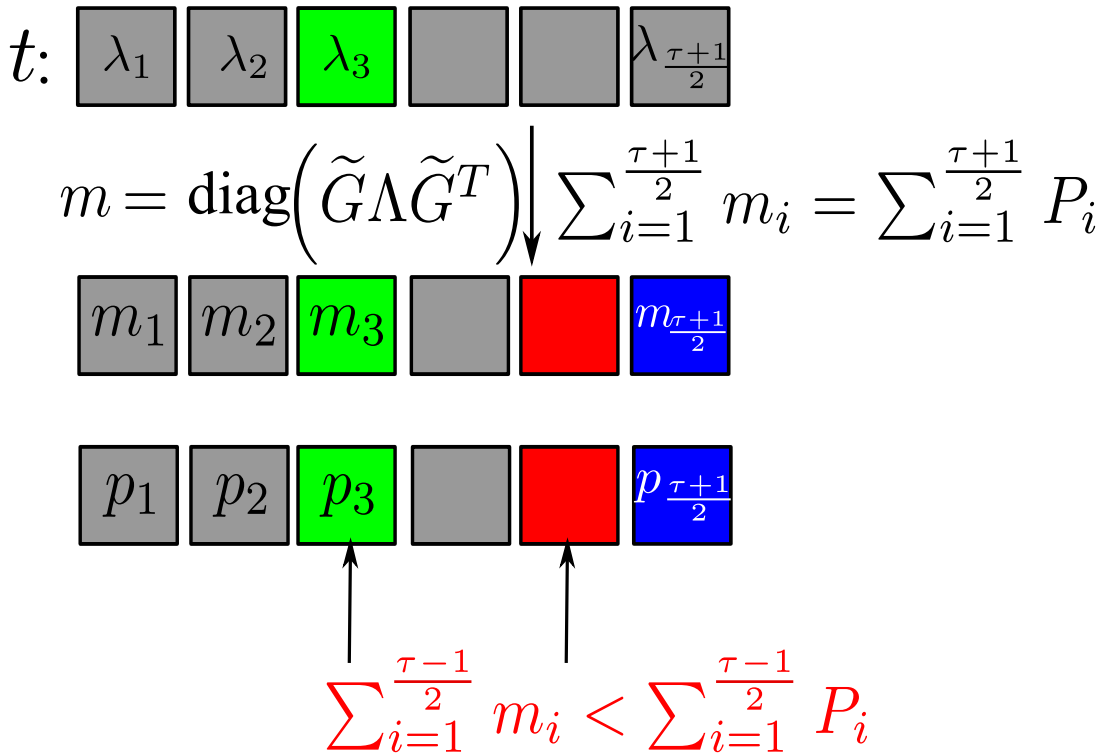
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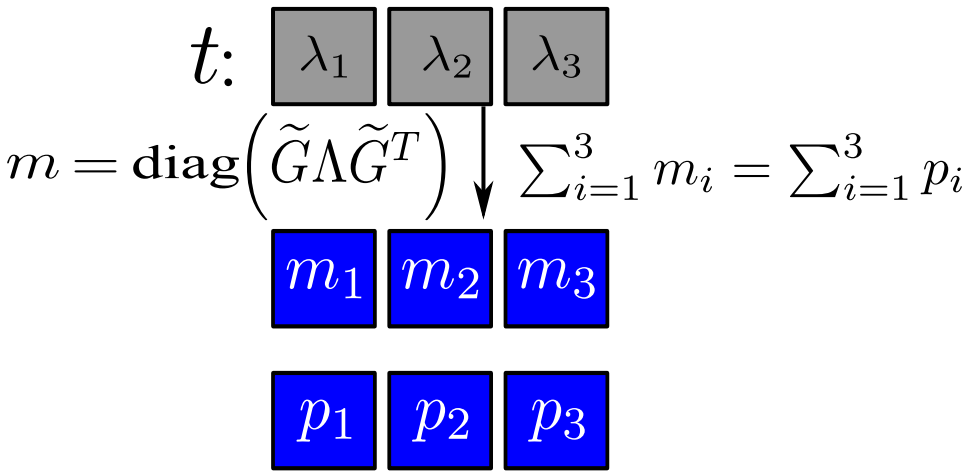
Dichotomy approach

In the next step, compute a $\widetilde{G}$ for components from 1 to 3 of t under total power constraint $\sum_{i=1}^{3} p_i$.



Dichotomy approach

The conditions of existence of Z are satisfied for $\lambda_1, \lambda_2, \lambda_3$ and p_1, p_2, p_3.



$$\sum_{i=1}^{j} m_i \geq \sum_{i=1}^{j} p_i, 1 \leq j \leq 3$$

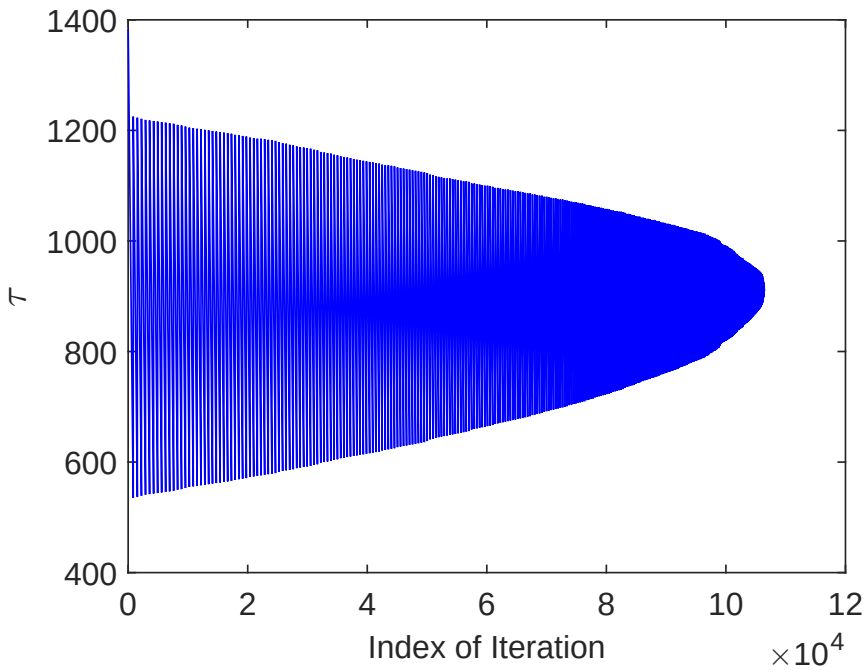
Complexity of PAISP

- The complexity to find the first subblock is $O(n_{SC})$
- The complexity to find all subbloks in worst case (n_{SC} subblocks) is $\sum_{i=n_{SC}}^{1} n_i = O(n_{SC}^2)$

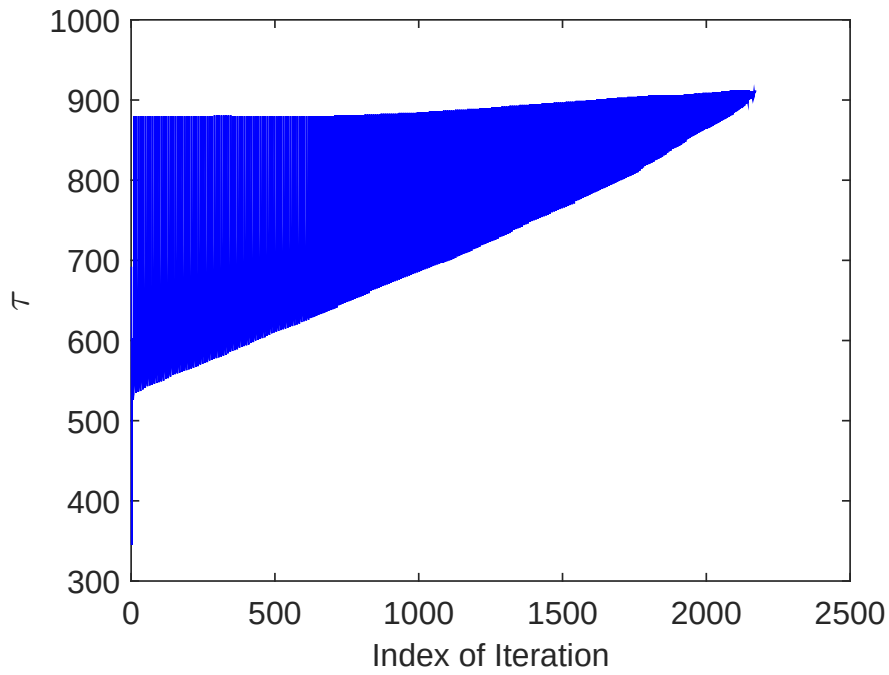
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Computation cost comparison



(a) Optimal



(b) PAISP

Figure: One chunk vector in a GoP of video Kimonol

τ is the position where condition of existence of Z is violated.

Local Power Adjustment

- The allocated power vector under total power constraint $\sum_{i=1}^{n_{SC}} p_i$ are m_i, $i = 1, \ldots, n_{SC}$.
- If at position τ condition is violated:

$$\Delta = \sum_{i=1}^{\tau} p_i - \sum_{i=1}^{\tau} m_i > 0$$

- Since total power constraint implies that

$$\sum_{i=1}^{n_{SC}} p_i = \sum_{i=1}^{n_{SC}} m_i,$$

- therefore too much power has been allocated to the last $n_{SC} - \tau$ subchannels.

$$\sum_{i=\tau+1}^{n_{SC}} m_i - \sum_{i=\tau+1}^{n_{SC}} p_i = \Delta > 0$$

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PALPA

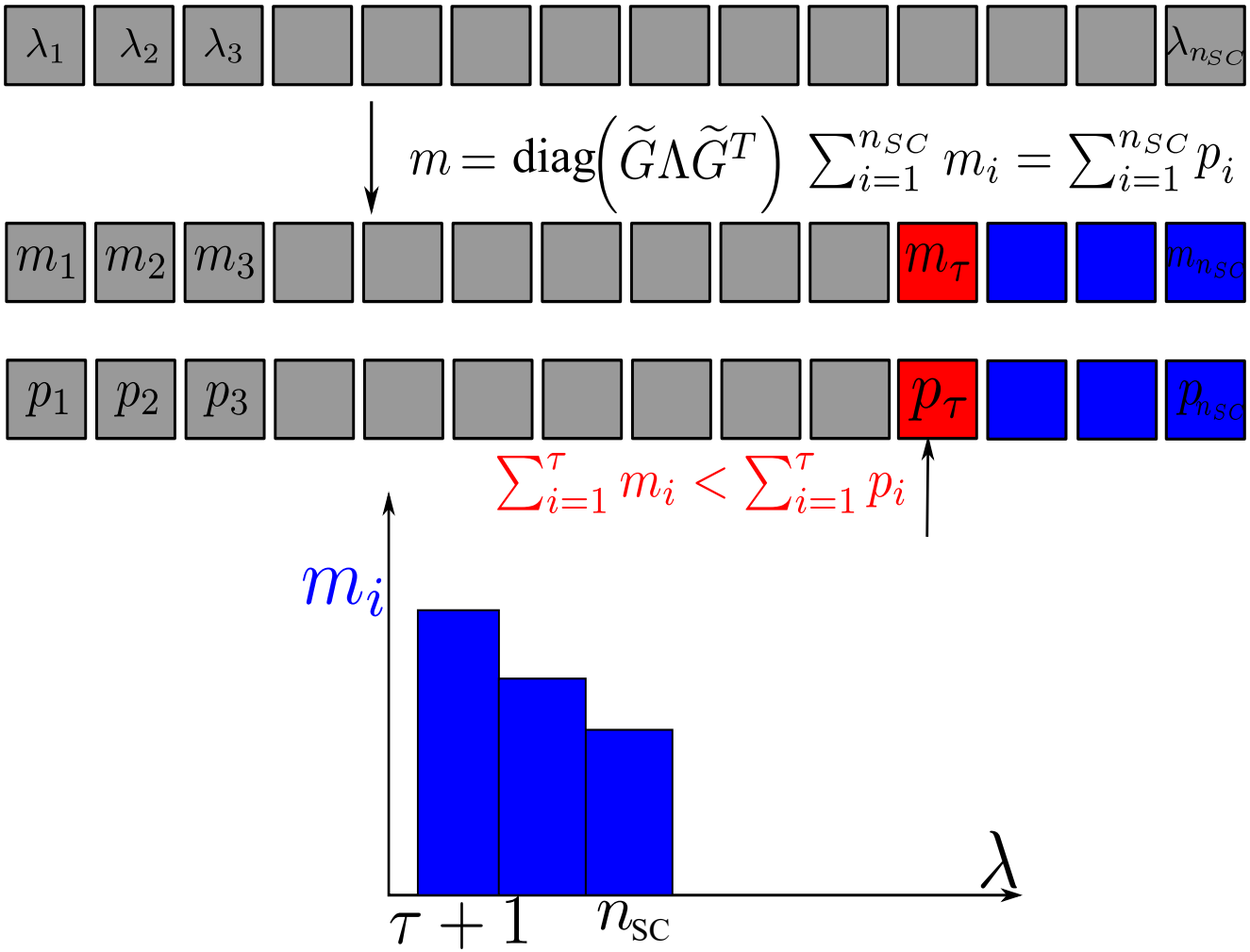


Figure: Power Allocation under Total Power Constraint and Check the condition of

PALPA

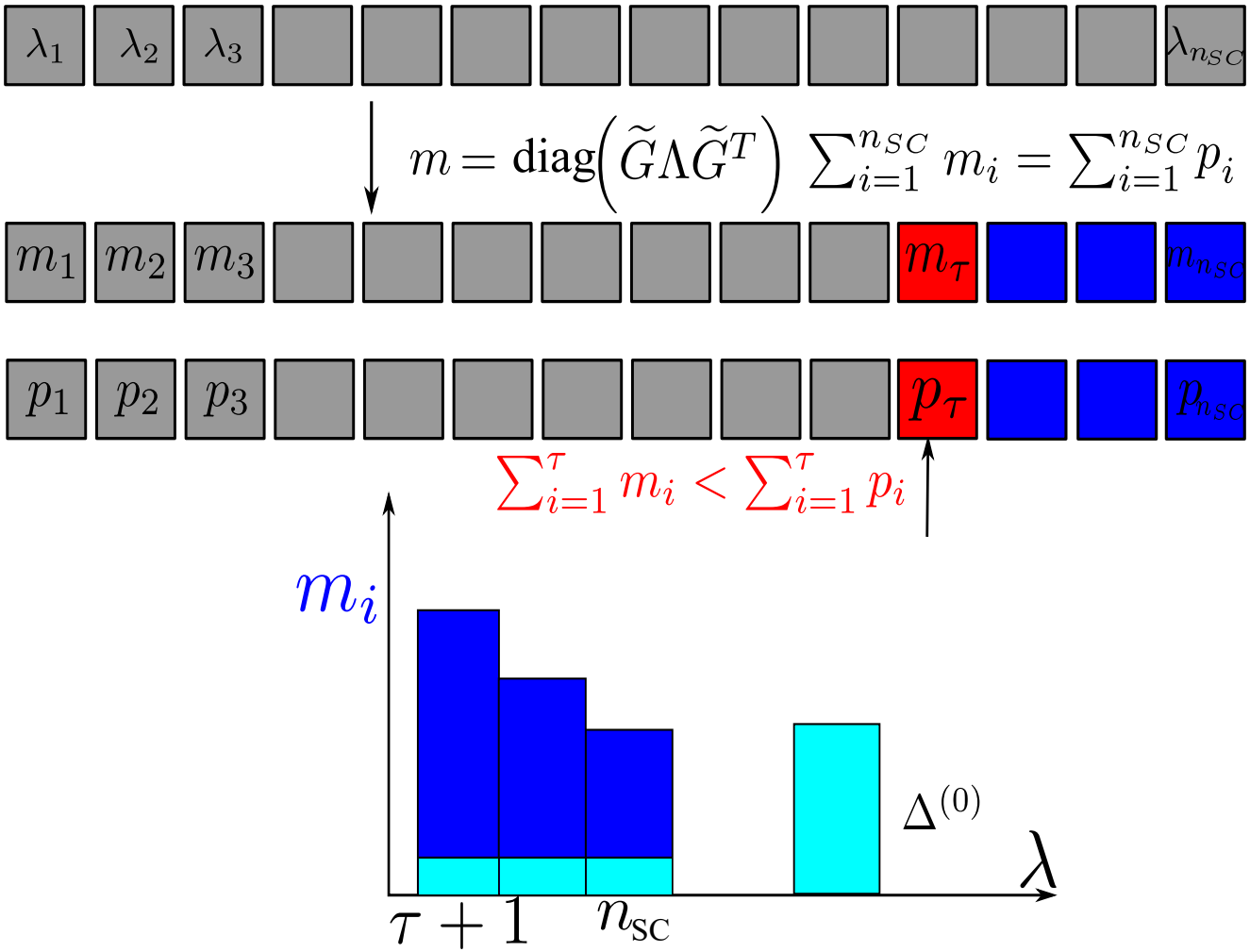


Figure: compute $\Delta^{(0)}$

PALPA

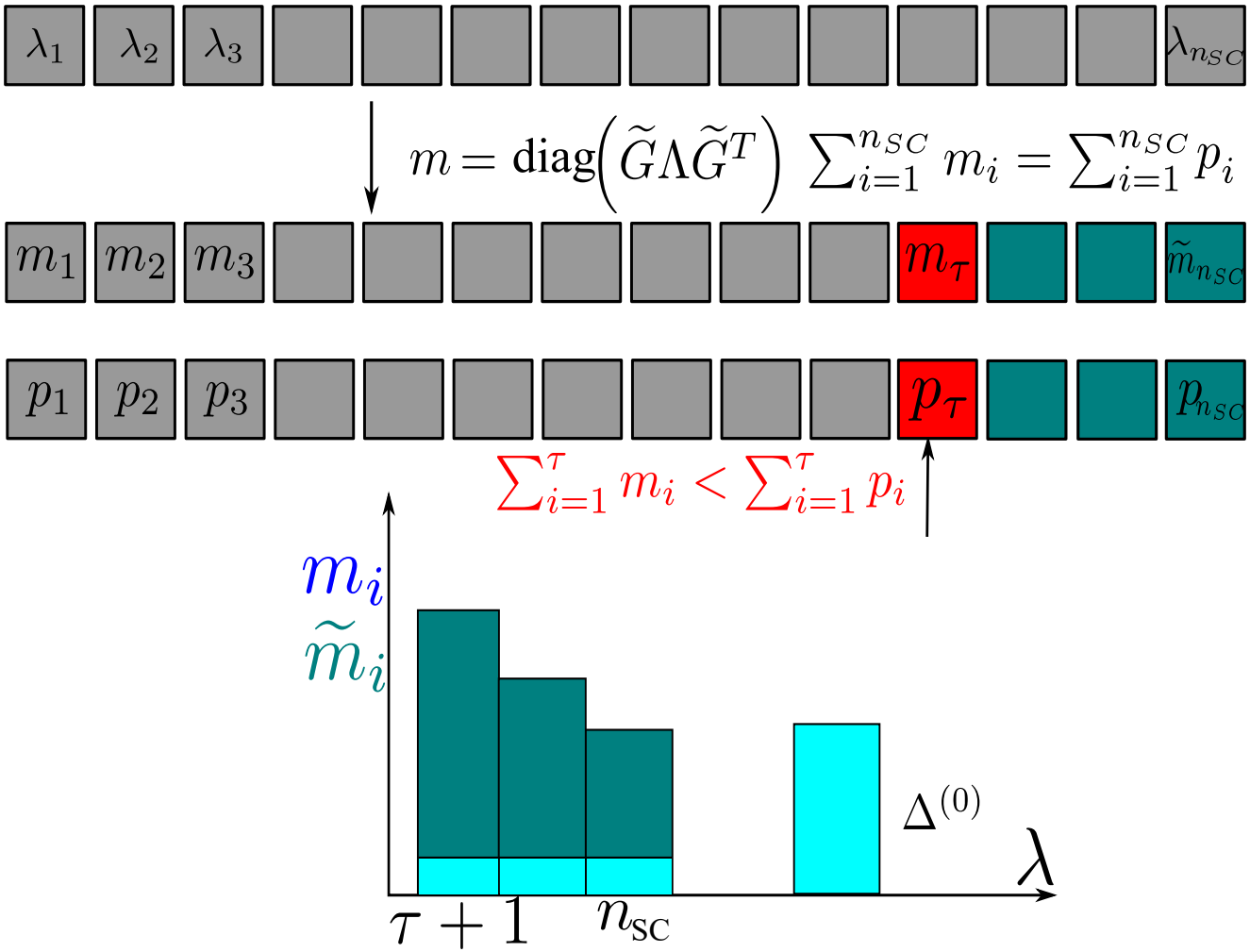


Figure: Local Power Ajustemnt

PALPA

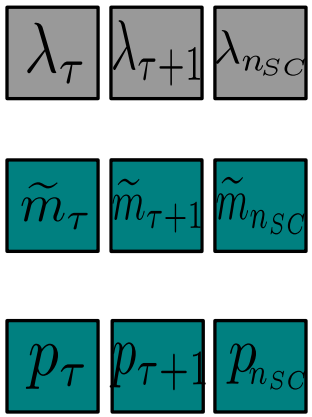


Figure: The conditions of existence of Z satisfied for $\widetilde{m}_1, \widetilde{m}_2, \widetilde{m}_3$ and p_1, p_2, p_3, the first subblock is found.

- Compute $\widetilde{G} = \begin{bmatrix} \sqrt{\frac{\widetilde{m}_\tau}{\lambda_{SC-2}}} & & \\ & \sqrt{\frac{\widetilde{m}_{\tau+1}}{\lambda_{\tau+1}}} & \\ & & \sqrt{\frac{\widetilde{m}_{nSC}}{\lambda_{SC}}} \end{bmatrix}$.
- Compute Z $\left(Z\widetilde{G}\right) \begin{bmatrix} \lambda_\tau & & \\ & \lambda_{\tau+1} & \\ & & \lambda_{nSC} \end{bmatrix} \left(Z\widetilde{G}\right)^T = \begin{bmatrix} p_\tau & * & * \\ * & p_{\tau+1} & * \\ * & * & p_{nSC} \end{bmatrix}$.

PALPA

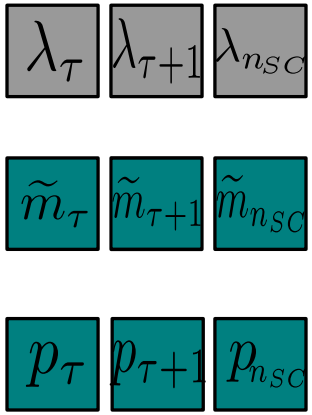


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PALPA

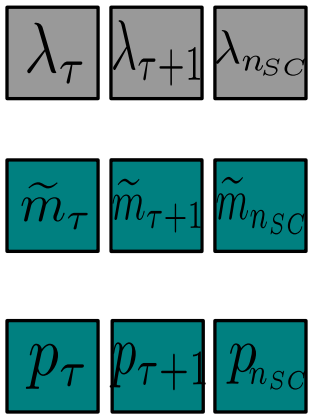


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PALPA

We continue for the remaining parts.

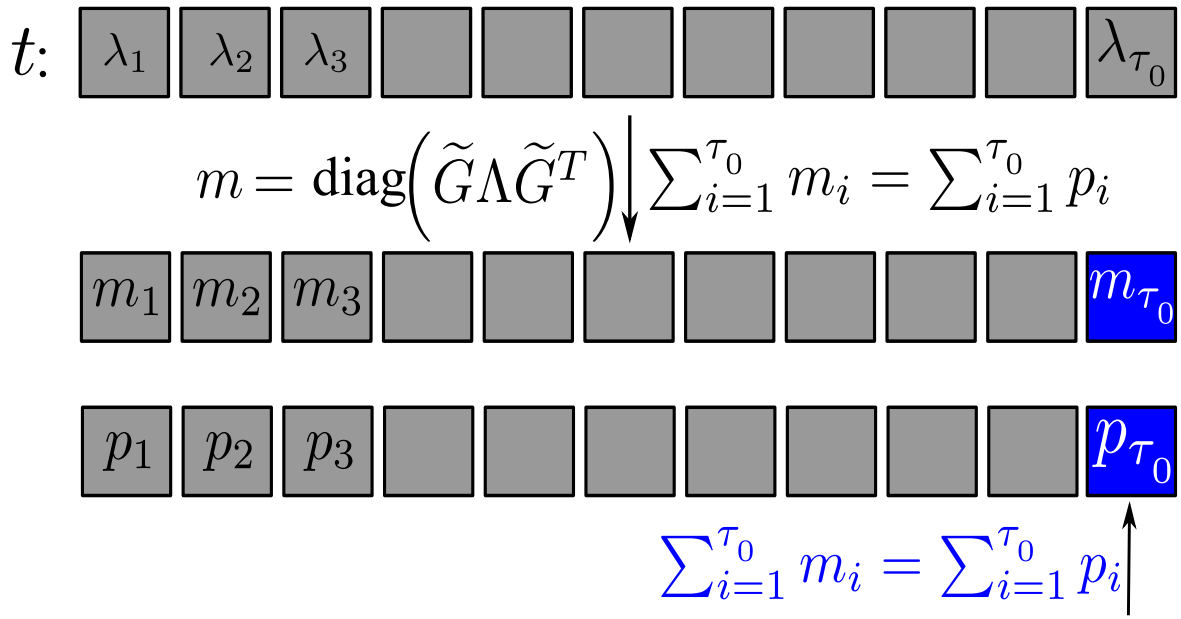


Figure: Power Allocation under Total Power Constraint and Check the condition of existence of Z

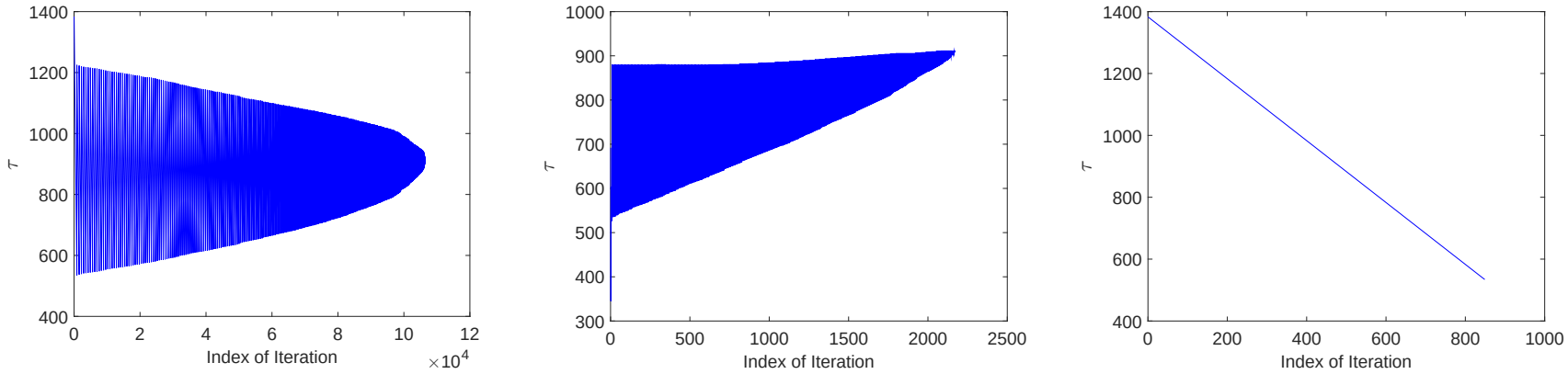
PALPA

- Complexity to find the first subblock in worst case (ajustement the power for m_2 to m_{SC}) $O(n_{SC})$.
- The complexity to find all split positions in worst case (at end $s = n_{SC}$) is $\sum_{i=n_{SC}}^{1} n_i = O(n_{SC}^2)$

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Computation cost comparison[9]



(a) Optimal (b) PAISP (c) PALPA

Figure: One chunk vector in a GoP of video Kimonol

τ is the position where condition of existence of Z is violated.

[9]The complexity of PALPA to find all subblocks in worst case is as same as PAISP $O\left(n_{SC}^2\right)$.