Wireless Power Transfer for Distributed Estimation in Wireless Passive Sensor Networks

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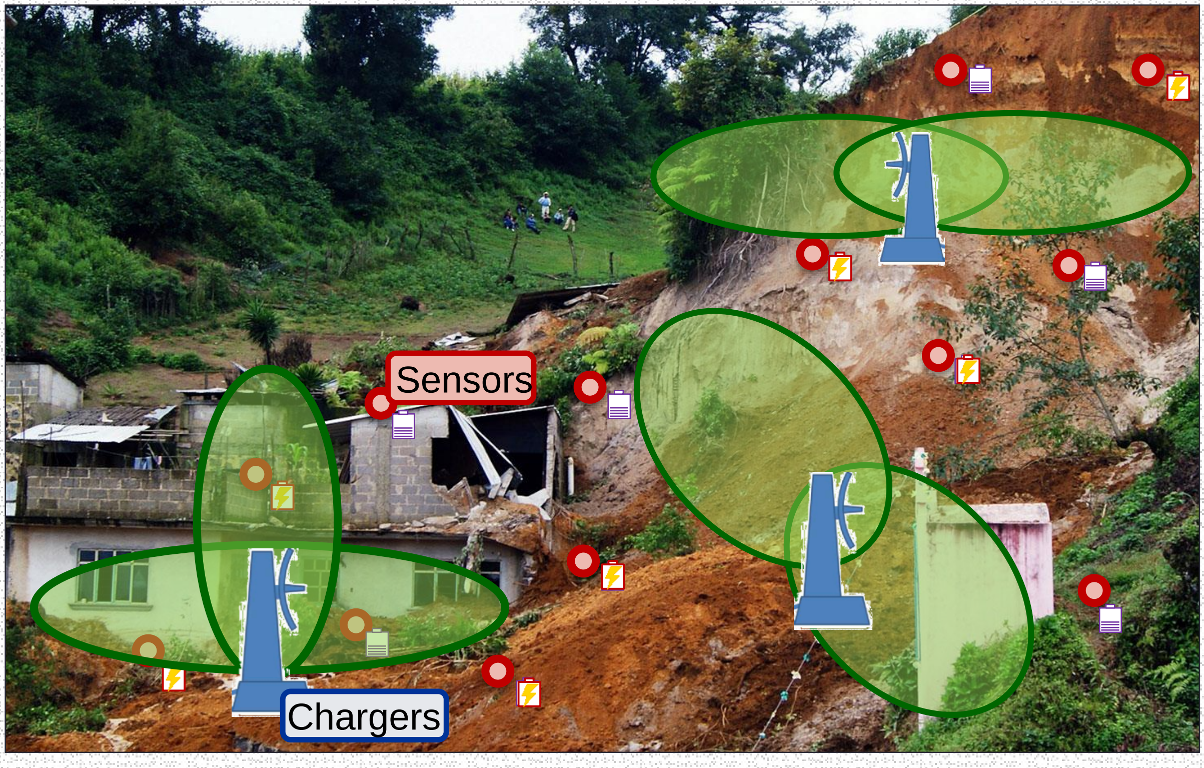
Wireless Power Transfer in WSNs



- Wireless sensor networks (WSNs) have applications in, e.g., environmental monitoring, disaster recovery ... etc.
- Bottleneck: Limited battery capacity!
- **Wireless power transfer (WPT)** allows sensors to be conveniently charged over-the-air whenever needed.

Related Works and Main Contribution

- **Related Works:**

Beamforming designs for WPT:

- ✓ Maximize energy harvested by users [Son & Clerckx '14]
- ✓ Maximize uplink throughput [Liu *et al.*, 2014] [Ju & Zhang, '14]
- ✓ Maximize minimum rate of users [Yang *et al.* '15]

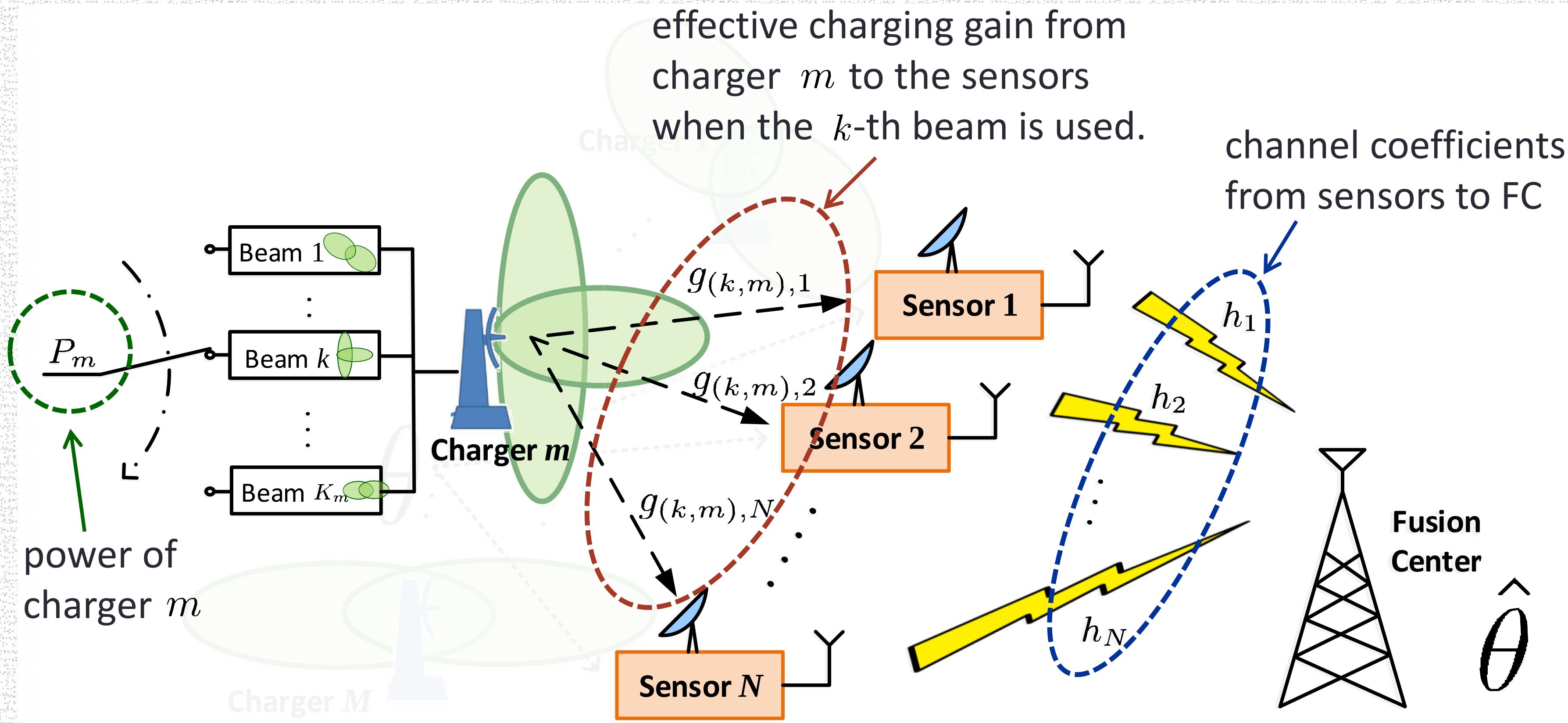
WPT in the context of WSNs:

- ✓ Packet delay and packet loss probability [Wu & Yang '15]
- ✓ Charging scheduling of mobile chargers [Xie *et al.* '12]
- ✓ Charger deployment [He *et al.*, 2013]

➔ Optimize communication-related performance metrics.

Main Objective: Determine the **beam selection** and **power allocation** at the chargers with cross-layer consideration on the ***distributed estimation error***.

WPT for Distributed Estimation



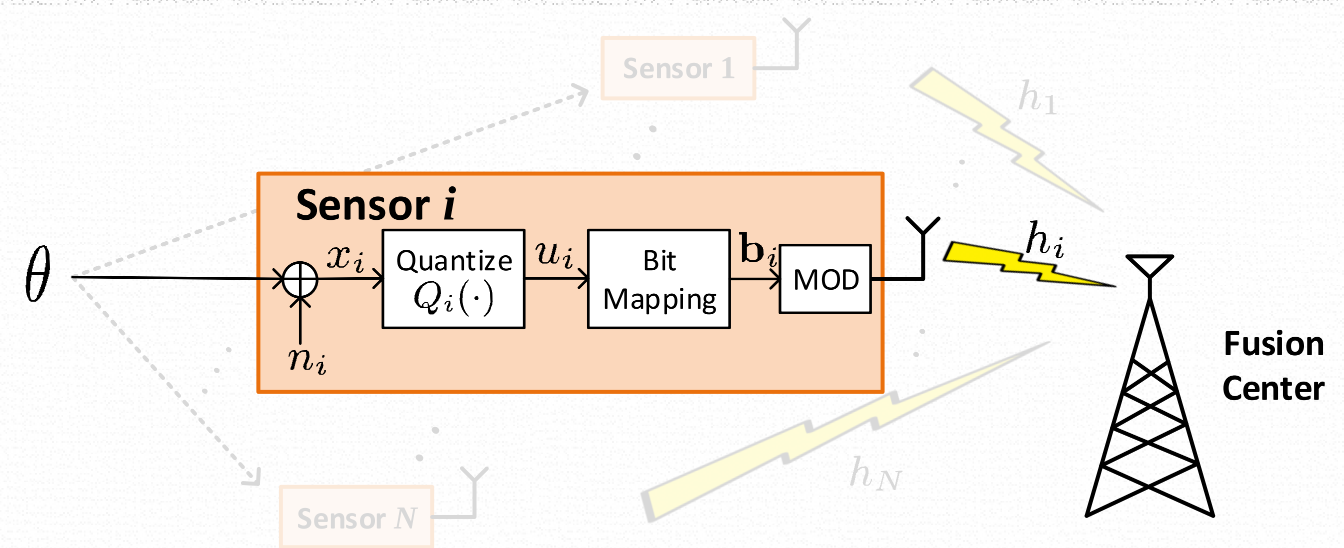
- The beam selection of charger m is described by

$$a_{(m,k)} = \begin{cases} 1, & k\text{-th beam of charger } m \text{ is selected,} \\ 0, & \text{otherwise.} \end{cases} \Rightarrow \sum_{k=1}^{K_m} a_{(m,k)} = 1$$

- The total energy gathered by sensor i over time T_c is

$$\mathcal{E}_i(\mathbf{a}, \mathbf{P}) \triangleq \sum_{m=1}^{M} \left(\sum_{k=1}^{K_m} a_{(m,k)} |g_{(m,k),i}|^2 \right) P_m T_c.$$

Sensor Observation and Data Processing



- The observation at sensor i is

$$x_i = \theta + n_i$$

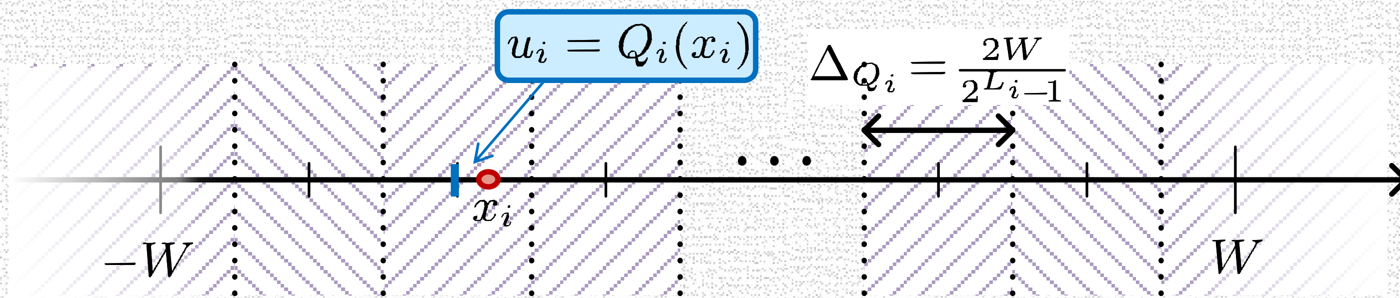
where $\theta \sim \mathcal{N}(0, \sigma_\theta^2)$ is the parameter of interest and $n_i \sim \mathcal{N}(0, \sigma_{n_i}^2)$ is the observation noise.

- Data processing at sensor i:

$$x_i \xrightarrow{\text{quantized}} u_i = Q_i(x_i) \xrightarrow{\text{bit mapping}} \mathbf{b}_i = [b_{i,1}, \ldots, b_{i,L_i}]$$

Digital Forwarding at the Sensor

- **Uniform Quantization** & **Natural Bit Mapping**:



Index = $0 \quad 1 \quad 2 \quad 3 \quad \cdots \quad 2^{L_i}-2 \quad 2^{L_i}-1$

Bit Mapping $\mathbf{b}_i$ = $[0\cdots00][0\cdots01][0\cdots10][0\cdots11] \quad \cdots \quad [1\cdots10]\ [1\cdots11]$

➔ Quantization error is $\epsilon_i \triangleq u_i - x_i \sim \mathcal{U}\left(-\frac{\Delta_{Q_i}}{2}, \frac{\Delta_{Q_i}}{2}\right)$ and, thus, $\sigma^2_{\epsilon_i} = \frac{W^2}{3(2^{L_i}-1)^2}$.

- **BPSK Modulation**: The transmitted signal is

$$\mathbf{s}_i = [s_{i,1}, \ldots, s_{i,L_i}] = \sqrt{\frac{\mathcal{E}_i(\mathbf{a}, \mathbf{P})}{L_i}}(2\mathbf{b}_i - 1)$$

where $\mathcal{E}_i(\mathbf{a}, \mathbf{P})$ is the energy available at sensor i, $\mathbf{a} \triangleq [a_{(1,1)}, a_{(2,1)}, \ldots, a_{(1,K_1)}, \ldots, a_{(M,1)}, \ldots, a_{(M,K_M)}]^T$ is the beam selection vector, and $\mathbf{P} \triangleq [P_1, P_2, \ldots, P_M]^T$ is the power allocation among chargers.

Parameter Estimation at the Fusion Center (FC)

- By considering BPSK modulation, the received signal at FC is

$$\mathbf{r}_i = h_i \mathbf{s}_i + \mathbf{w}_i = h_i \sqrt{\frac{\mathcal{E}_i}{L_i}} (2\mathbf{b}_i - 1) + \mathbf{w}_i,$$

where $\mathbf{w}_i \sim \mathcal{N}(\mathbf{0}, \sigma_{w_i}^2 \mathbf{I}_{L_i})$ is the channel noise vector.

- The reconstruction of sensor i's observation is

$$y_i = [2(\textstyle\sum_{l=1}^{L_i} \hat{b}_{i,l} 2^{L_i - l} + 1) - 1 - 2^{L_i}] \frac{\Delta_{Q_i}}{2}$$

where $\hat{\mathbf{b}}_i = [\hat{b}_{i,1}, \ldots, \hat{b}_{i,L_i}]$ is the detected bit vector.

- By taking the **linear MMSE estimator**, the estimate at FC is

$$\hat{\theta} = \mathbf{C}_{\theta \mathbf{y}} \mathbf{C}_{\mathbf{y}\mathbf{y}}^{-1} \mathbf{y}$$

where $\mathbf{y} \triangleq [y_1, y_2, \ldots, y_K]^T$, $\mathbf{C}_{\theta \mathbf{y}} \triangleq E[\theta \mathbf{y}^T]$, and $\mathbf{C}_{\mathbf{y}\mathbf{y}} \triangleq E[\mathbf{y}\mathbf{y}^T]$.

➔ The corresponding MSE is

$$\mathrm{MSE}(\mathbf{a}, \mathbf{P}) \triangleq E[|\theta - \hat{\theta}|^2] = \sigma_\theta^2 - \mathbf{C}_{\theta \mathbf{y}} \mathbf{C}_{\mathbf{y}\mathbf{y}}^{-1} \mathbf{C}_{\theta \mathbf{y}}^T.$$

Beam Selection and Power Allocation

- The MSE minimization problem can be formulated as

$$\min_{\mathbf{a},\mathbf{P}} \ \mathrm{MSE}(\mathbf{a},\mathbf{P})$$
$$\text{subject to } \sum_{m=1}^{M} P_m \le \bar{P}_{\text{charge}},$$ ← Total power constraint over all chargers.
$$0 \le P_m \le \bar{P}_{m,\text{charge}}, \ \forall m,$$ ← Individual constraints at each charger
$$\textstyle\sum_{k=1}^{K_m} a_{(m,k)} = 1, \ \forall m,$$
$$a_{(m,k)} \in \{0,1\}, \ \forall m,k,$$

- The MSE can be upper bounded by

error due to quantization

error due to transmission channel

$$\mathrm{MSE}(\mathbf{a},\mathbf{P}) \le 2 \min_{\mathbf{k}\in\mathbb{R}^N} \{E[|\theta - \mathbf{k}^T\mathbf{u}|^2] + E[|\mathbf{k}^T\mathbf{u} - \mathbf{k}^T\mathbf{y}|^2]\}$$
$$= 2[\sigma_\theta^2 - \mathbf{C}_{\theta\mathbf{u}}(\mathbf{C}_{\mathbf{uu}} + \mathbf{C}_{(\mathbf{u}-\mathbf{y})(\mathbf{u}-\mathbf{y})})^{-1}\mathbf{C}_{\theta\mathbf{u}}^T]$$

where $\mathbf{u} \triangleq [Q(x_1),\ldots,Q(x_N)]^T$ is vector of quantized observations, $\mathbf{y} \triangleq [y_1,\ldots,y_N]^T$ is the vector of reconstructed values at FC, and $\mathbf{C}_{\mathbf{ab}} \triangleq E[\mathbf{ab}^T]$ is the covariance matrix between vectors $\mathbf{a}$ and $\mathbf{b}$.

Approximations of the MSE Upper Bound

- Assumptions:
  1) Independence of noise n_i and quantization error ϵ_i.
  2) High SNR such that $\Pr(\hat{b}_{i,\ell} \neq b_{i,\ell}) = \mathcal{Q}\left(\sqrt{\frac{|h_i|^2 \mathcal{E}_i(\mathbf{a},\mathbf{P})}{L_i \sigma_{w_i}^2}}\right) \ll 1$
- The MSE upper bound can be approximated as

$$\mathrm{MSE}(\mathbf{a},\mathbf{P}) \leq 2[\sigma_\theta^2 - \mathrm{tr}(\mathbf{C}_{\theta\mathbf{u}}(\mathbf{C}_{\mathbf{uu}} + \mathbf{C}_{(\mathbf{u}-\mathbf{y})(\mathbf{u}-\mathbf{y})})^{-1}\mathbf{C}_{\theta\mathbf{u}}^T)]$$

$$\approx 2\left[\sigma_\theta^2 - \frac{\sum_{i=1}^N \{\mathbf{D}\}_{i,i}^{-1}}{1+\sigma_\theta^2 \sum_{i=1}^N \{\mathbf{D}\}_{i,i}^{-1}}\right] \triangleq \overline{\mathrm{MSE}}(\mathbf{a},\mathbf{P}),$$

where $\mathbf{D}$ is a diagonal matrix with

$$\{\mathbf{D}\}_{i,i} = \sigma_{n_i}^2 + \frac{W^2}{3(2^{L_i}-1)} + \frac{W^2(4^{L_i+1}-1)}{3(2^{L_i}-1)^2}\mathcal{Q}\left(\sqrt{\frac{|h_i|^2 \mathcal{E}_i(\mathbf{a},\mathbf{P})}{L_i \sigma_{w_i}^2}}\right).$$

- Consequently,

$$\min_{\mathbf{a},\mathbf{P}} \mathrm{MSE}(\mathbf{a},\mathbf{P}) \Rightarrow \min_{\mathbf{a},\mathbf{P}} \overline{\mathrm{MSE}}(\mathbf{a},\mathbf{P}) \equiv \max_{\mathbf{a},\mathbf{P}} \sum_{i=1}^N \{\mathbf{D}\}_{ii}^{-1}$$

Approximation Problem Formulation

- The MSE minimization problem can be approximated as

$$\max_{\mathbf{a},\mathbf{P}} \sum_{i=1}^{N} \left(c_{i,1} + c_{i,2} \mathcal{Q} \left(\sqrt{\frac{|h_i|^2 \sum_{m=1}^{M} P_m T_c \sum_{k=1}^{K_m} a_{(m,k)} |g_{(m,k),i}|^2}{L_i \sigma_{w_i}^2}} \right) \right)^{-1}$$

$$\text{subject to } \sum_{m=1}^{M} P_m \leq \bar{P}_{\text{charge}}$$

$$0 \leq P_m \leq \bar{P}_{m,\text{charge}}, \ \forall m,$$

$$\sum_{k=1}^{K_m} a_{(m,k)} = 1, \quad a_{(m,k)} \in \{0, 1\},$$

Relaxed as $0 \leq a_{(m,k)} \leq 1$

where $c_{i,1} = \sigma_{n_i}^2 + \frac{W^2}{3(2^{L_i}-1)}$ and $c_{i,2} = \frac{W^2(4^{L_i+1}-1)}{3(2^{L_i}-1)^2}$.

- Solved by an alternating optimization algorithm, i.e.,
  1) Given $\mathbf{P} = \mathbf{P}^{(\ell)}$, find $\mathbf{a} = \mathbf{a}^{(\ell+1)}$.
  2) Given $\mathbf{a} = \mathbf{a}^{(\ell+1)}$, find $\mathbf{P} = \mathbf{P}^{(\ell+1)}$.
  3) Repeat 1) and 2) until no significant increase in the objective. Take $\mathbf{a}^*$ $a_{m,k}^* = 1$ if $a_{m,k}^{(\infty)} > a_{m,k'}^{(\infty)}, \forall k'$, and 0, otherwise. Solve for $\mathbf{P}^*$ given $\mathbf{a}^*$.

Optimization of $\mathbf{a}$ Given $\mathbf{P} = \mathbf{P}^{(\ell)}$

- Given $\mathbf{P} = \mathbf{P}^{(\ell)}$, we optimize $\mathbf{a}$ by solving

$$\max_{\mathbf{a}} \sum_{i=1}^{N} \left(c_{i,1} + c_{i,2} \mathcal{Q}\left(\sqrt{\frac{|h_i|^2 \sum_{m=1}^{M} (\sum_{k=1}^{K_m} a_{(m,k)} \gamma_{(m,k),i}) P_m^{(\ell)} T_c}{L_i \sigma_{w_i}^2}} \right) \right)^{-1}$$

$$\text{subject to} \sum_{k=1}^{K_m} a_{(m,k)} = 1, \quad 0 \le a_{(m,k)} \le 1, \ \forall m, k.$$

$\mathbf{t} = [t_0, t_1, \ldots, t_N]^T$

equivalent

$\mathcal{Q}(v) \lesssim \frac{1}{2} e^{-\frac{v^2}{2}}, \ v > 0$

$$\max_{\mathbf{a},\mathbf{t}} t_0$$

$$\text{subject to } t_i \ge c_{i,1} + \frac{c_{i,2}}{2} e^{-\frac{|h_i|^2 \sum_{m=1}^{M} \sum_{k=1}^{K_m} a_{(m,k)} \gamma_{(m,k),i} P_m^{(\ell)} T_c}{2 L_i \sigma_{w_i}^2}}, \forall i, \overline{) P_m^{(\ell)} T_c}))^{-1} > 1/t_i, \forall i,$$

$$\sum_{i=1}^{N} 1/t_i \ge t_0,$$

Successive Convex Approximation:
Replace $1/t_i$ by $\frac{1}{t_i^{(n-1)}} - \frac{t_i - t_i^{(n-1)}}{(t_i^{(n-1)})^2}$ in iteration n.

$$\sum_{k=1}^{K_m} a_{(m,k)} = 1, \quad 0 \le a_{(m,k)} \le 1, \ \forall m, k,$$

$$t_i > 0, \ \forall i,$$

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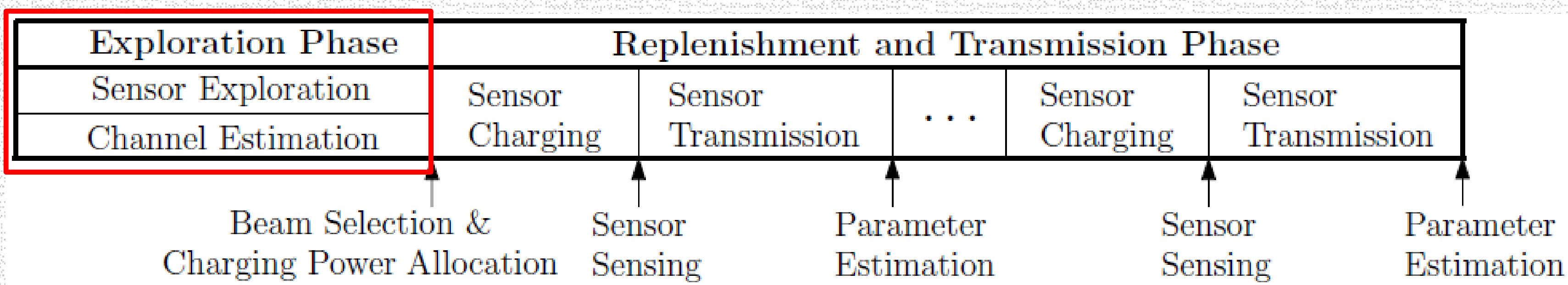
- Similarly, for solving $\mathbf{P}$ with given $\mathbf{a} = \mathbf{a}^{(\ell+1)}$.

Exploration of Channel State Information



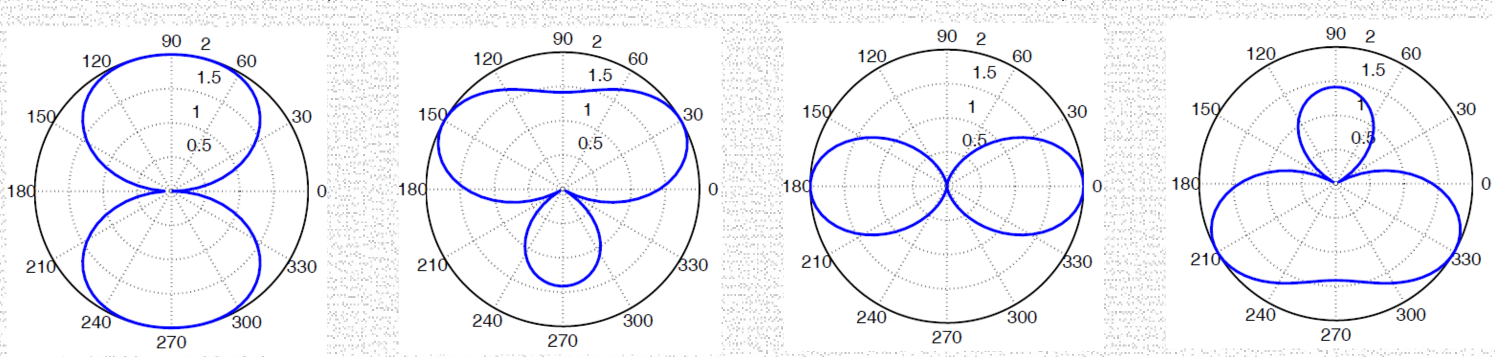
- **Exploration Phase:**
  1) Each charger emits power P_e in turn with all of its beams.
  2) If the harvested energy by a certain sensor exceeds a given threshold $\mathcal{E}_{th}$, it immediately emits a pilot signal to the FC.
  3) The FC estimates the charging gain $|g_{(m,k),i}|^2$ by the arrival time of the pilot signal and estimates the fading coefficient h_i by using the received pilot signal.
  4) CSI of sensors not responding by time T_e are set to 0.

✓ Higher $\mathcal{E}_{th}$ ➔ *less explored sensors*; *smaller estimation error*.

Simulation Environment

- $N = 20$ sensors and $M = 5$ chargers in 50m-by-50m square area
- Let $d_{s,i}$, d_i, and $d_{m,i}$ be the distance between sensor i and the event, FC, and charger m, respectively.
  $\sigma_\theta^2 = 1$, $\sigma_{n_i}^2 = 0.001\max(d_{s,i}^2, 1)$, and $\sigma_{w_i}^2 = -55$dBm.
- $|h_i|^2 = \min(1/d_i^2, 1)$ and $|g_{(m,k),i}|^2 = G_{(m,k),i} G_r \zeta \lambda^2 / (4\pi L_p d_{m,i})^2$ where

$$G_i^{(m,k)} = \left|\sum_{n=0}^{1} \omega_{n,k} \exp(j(n-1)\sin\psi_{m,i})\right|, \ (K_m = 4, \ \forall m)$$

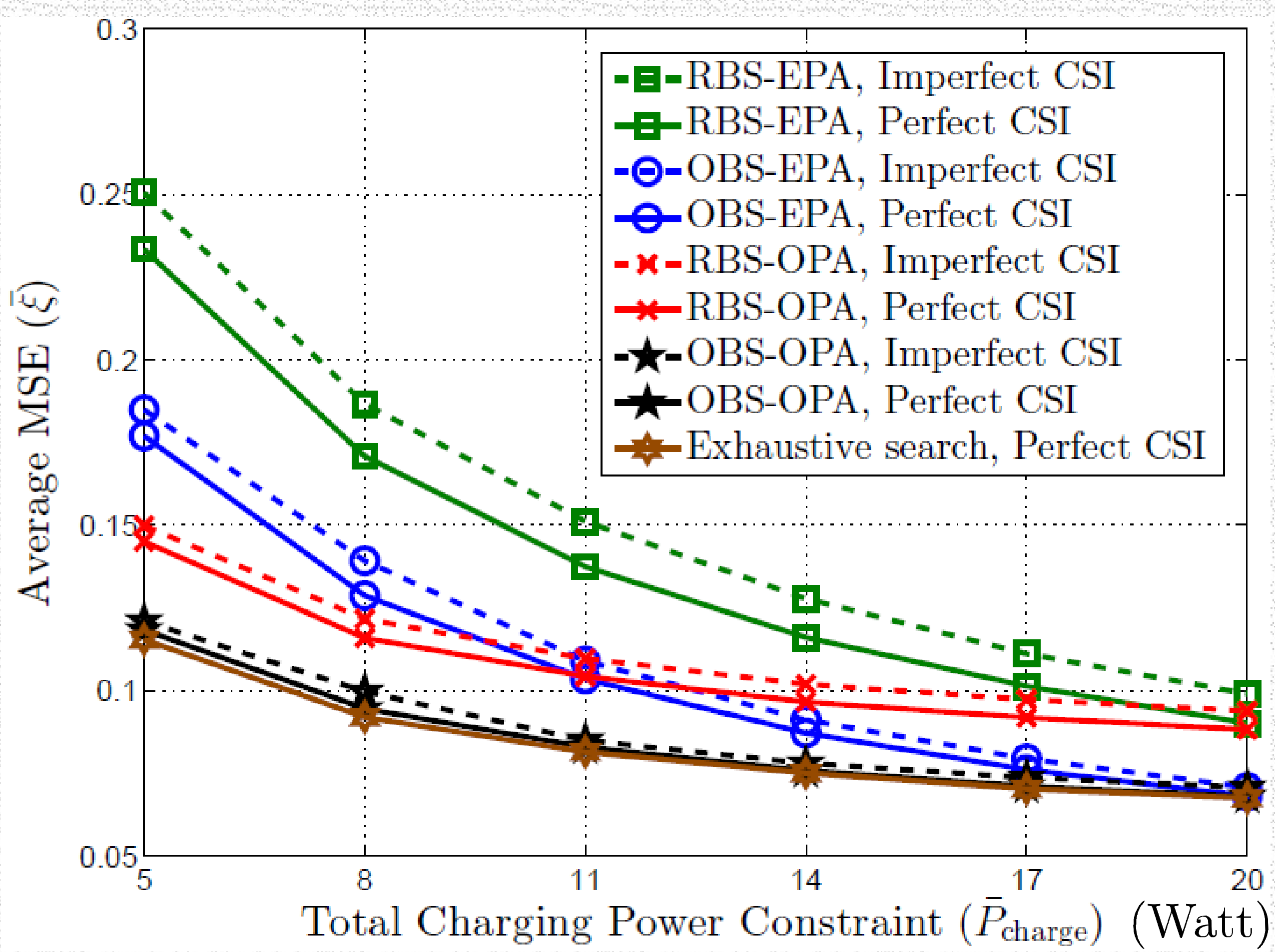


$\boldsymbol{\omega}_1 = [1, -1]^T$ $\boldsymbol{\omega}_2 = [1, -j]^T$ $\boldsymbol{\omega}_3 = [1, 1]^T$ $\boldsymbol{\omega}_4 = [1, j]^T$

$G_r = 8$dBi: receive antenna gain $L_p = 3$dB: polarization loss

$\zeta = 0.1$: rectifier efficiency $\lambda = 0.3$m: wavelength

MSE vs Total Charging Power Constraint



Imperfect CSI

$P_e = 4$Watt

$T_e = 500$ms

$\mathcal{E}_{th} = 1\mu$J

$B = 2$MHz

OBS-OPA : Optimized Beam Selection and Optimized Power Allocation

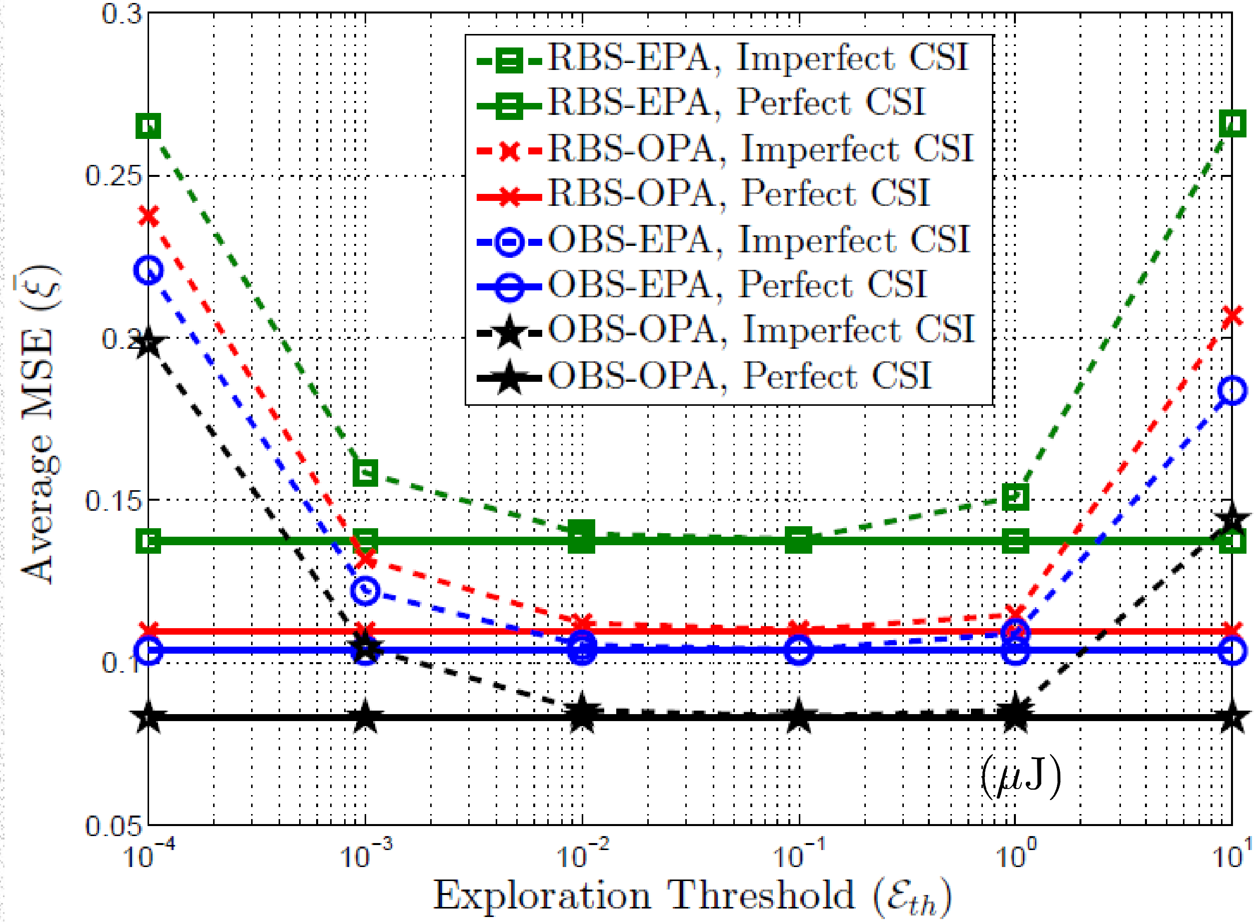
OBS-EPA : Optimized Beam Selection but Equal Power Allocation

RBS-OPA : Random Beam Selection but Optimized Power Allocation

RBS-EPA : Random Beam Selection and Equal Power Allocation

MSE vs Exploration Threshold

- Imperfect CSI: $P_e = 4$ Watt, $T_e = 500$ ms, $B = 2$ MHz.



Conclusion

- Examined the cross-layer impact of the WPT charging strategy on the distributed estimation performance.
- Derived an upper bound for the MSE of the distributed estimation system.
- Proposed an effective beam selection and charging power allocation scheme based on successive convex approximation.
- Furthermore, in a more recent work, we proposed methods to explore the sensors and CSI, and examined the impact of imperfect CSI on the distributed estimation performance.