Dictionary learning algorithm for Multi-Subject fMRI analysis via temporal and spatial concatenation

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Introduction

- Dictionary learning (DL) methods have been successfully extended to multi-subject fMRI data analysis using spatially or temporally concatenated datasets.
- Spatial concatenation allows for the extraction of group-level temporal dynamics and sub-specific spatial maps.
- Temporal concatenation lets us extract sub-specific dynamics and group-level spatial maps.
- Here we propose a hybrid dictionary learning framework which can extract both group and sub-specific dynamics and spatial maps simultaneously which are of particular interest in task-based fMRI analysis.

Background

Given a set of signals $\mathbf{Y} = [\mathbf{y}_1, \mathbf{y}_2, ..., \mathbf{y}_N]$, DL methods aims at finding a linear representation for the set of signals $\mathbf{Y}$ by solving

$$\{\mathbf{D}, \mathbf{X}\} = \arg\min_{\mathbf{D},\mathbf{X}} \| \mathbf{Y} - \mathbf{D}\mathbf{X} \|_F^2$$

With an overcomplete $\mathbf{D}$, this problem is ill-posed. Extra constraints are imposed on both $\mathbf{D}$ and $\mathbf{X}$ to solve this problem, which are

- Columns of $\mathbf{X} \in \mathbb{R}^{K\times N}$ should be sparse.
- Columns of $\mathbf{D} \in \mathbb{R}^{n\times K}$ should have unit ℓ_2 norm.

The resulting dictionary $\mathbf{D}$ contains K dense temporal dynamics and the sparse matrix $\mathbf{X}$ has the respective K spatial maps.

Multi-subject extensions of DL methods use spatially concatenated datasets $\mathbf{Y}_{sp} = [\mathbf{Y}_1, \mathbf{Y}_2, \dots, \mathbf{Y}_p]$ leading to group-level dynamics or temporally concatenated datasets $\mathbf{Y}_{te} = \left[\mathbf{Y}_1^\top, \mathbf{Y}_2^\top, \dots, \mathbf{Y}_p^\top\right]^\top$ which generates group-level spatial maps. Here p denotes the number of subjects.

The Proposed Algorithm

Goal of the algorithm is to represent each voxels' time course from $\mathbf{Y}_i$ as a linear combination of a few atoms from $\mathbf{D}_0$ (shared) and $\mathbf{D}_i$ (sub-specific) dictionaries such that $\forall\, i = 1, 2, \cdots, p$

$$\mathbf{Y}_i \simeq \tilde{\mathbf{D}}_i \tilde{\mathbf{X}}_i = [\mathbf{D}_0, \mathbf{D}_i] \begin{bmatrix} \mathbf{X}_0 \\ \mathbf{X}_i \end{bmatrix} = \mathbf{D}_0\mathbf{X}_0 + \mathbf{D}_i\mathbf{X}_i \quad (1)$$

To achieve this goal, we solve the following minimization problem:

$$\min_{\mathbf{D}_i, \mathbf{X}_i} \sum_{i=1}^{p} \left\{ \frac{1}{2}\|\mathbf{Y}_i - \mathbf{D}_0\mathbf{X}_0 - \mathbf{D}_i\mathbf{X}_i\|_F^2 + \frac{\eta}{2}\|\mathbf{D}_i^\top \mathbf{A}_i\|_F^2 \right\} \quad (2)$$
$$\text{s.t.}\ \|\mathbf{x}_i^m\|_0 \le s_i, \|\mathbf{x}_0^m\|_0 \le s_0, \|\mathbf{d}_k\|_2 = 1$$
$$\forall \quad i = 1, 2, \dots, p \text{ and } m = 1, 2, \dots, N$$

Here $\mathbf{A}_i = [\mathbf{D}_0, \mathbf{D}_1, \dots, \mathbf{D}_{i-1}, \mathbf{D}_{i+1}, \dots, \mathbf{D}_p]$ is the concatenation of all except currently updating dictionary. We propose to solve 2 in an alternating optimization fashion, i.e. solving for one variable with others fixed.

1. Sparse Coding: With dictionaries $(\mathbf{D}_0, \mathbf{D}_i)$ and sub-specific sparse codes $\mathbf{X}_i$ fixed, we first update $\mathbf{X}_0$, by minimizing

$$\hat{\mathbf{X}}_0 = \min_{\mathbf{X}_0} \frac{1}{2}\|\mathbf{E}_{te} - \mathbf{D}_{te}\mathbf{X}_0\|_F^2; \ \text{s.t.} \|\mathbf{x}_0^m\|_0 \le s_0 \quad (3)$$

where $\mathbf{E}_{te} = \frac{1}{\sqrt{p}}\left[\mathbf{E}_1^\top, \mathbf{E}_2^\top, \dots, \mathbf{E}_p^\top\right]^\top$, $\mathbf{E}_i = \mathbf{Y}_i - \mathbf{D}_i\mathbf{X}_i$, and $\mathbf{D}_{te} \in \mathbb{R}^{np\times K_0}$. Similarly, we find $\mathbf{X}_i$ by minimizing

$$\hat{\mathbf{X}}_i = \min_{\mathbf{X}_i} \frac{1}{2}\|\mathbf{B}_i - \mathbf{D}_i\mathbf{X}_i\|_F^2; \ \text{s.t.}\ \|\mathbf{x}_i^m\|_0 \le s_i \quad (4)$$

where $\mathbf{B}_i = \mathbf{Y}_i - \mathbf{D}_0\mathbf{X}_0$.

2. Dictionary Updates: To solve for $\mathbf{D}_0$, we solve:

$$\hat{\mathbf{D}}_0 = \min_{\mathbf{D}_0} \frac{1}{2}\|\mathbf{E}_{sp} - \mathbf{D}_0\mathbf{X}_{sp}\|_F^2 + \frac{\eta}{2}\|\mathbf{D}_0^\top \mathbf{A}_0\|_F^2 \quad (5)$$

where $\mathbf{E}_{sp} = [\mathbf{E}_1, \mathbf{E}_2, \dots, \mathbf{E}_p]$, $\mathbf{E}_i = \mathbf{Y}_i - \mathbf{D}_i\mathbf{X}_i$. Similarly, we find $\mathbf{D}_i$ by solving:

$$\hat{\mathbf{D}}_i = \min_{\mathbf{D}_i} \frac{1}{2}\|\mathbf{B}_i - \mathbf{D}_i\mathbf{X}_i\|_F^2 + \frac{\eta}{2}\|\mathbf{D}_i^\top \mathbf{A}_i\|_F^2 \quad (6)$$

where $\mathbf{B}_i = \mathbf{Y}_i - \mathbf{D}_0\mathbf{X}_0$.

Algorithm Overview

Input: fMRI datasets $\mathbf{Y}_i$, K_0, K_i, s_0, s_i, η

Initialization: Initialize $\mathbf{D}_0$, $\mathbf{D}_i$, $\mathbf{X}_0$ and $\mathbf{X}_i$

for $t = 1 : noIt$ **do**

Fix $\mathbf{D}_0$, $\mathbf{D}_i$ and use OMP to solve (3) for $\mathbf{X}_0$ and (4) for $\mathbf{X}_i\ \forall\, i = 1, \dots, p$.

Fix $\mathbf{X}_0$, $\mathbf{X}_i$ and sequentially update $\mathbf{D}_0$ by solving (5) and $\mathbf{D}_i$ by solving (6) $\forall\, i = 1, \dots, p$.

Output: $\mathbf{D}_0, \mathbf{X}_0, \mathbf{D}_i, \mathbf{X}_i$

Simulation Results

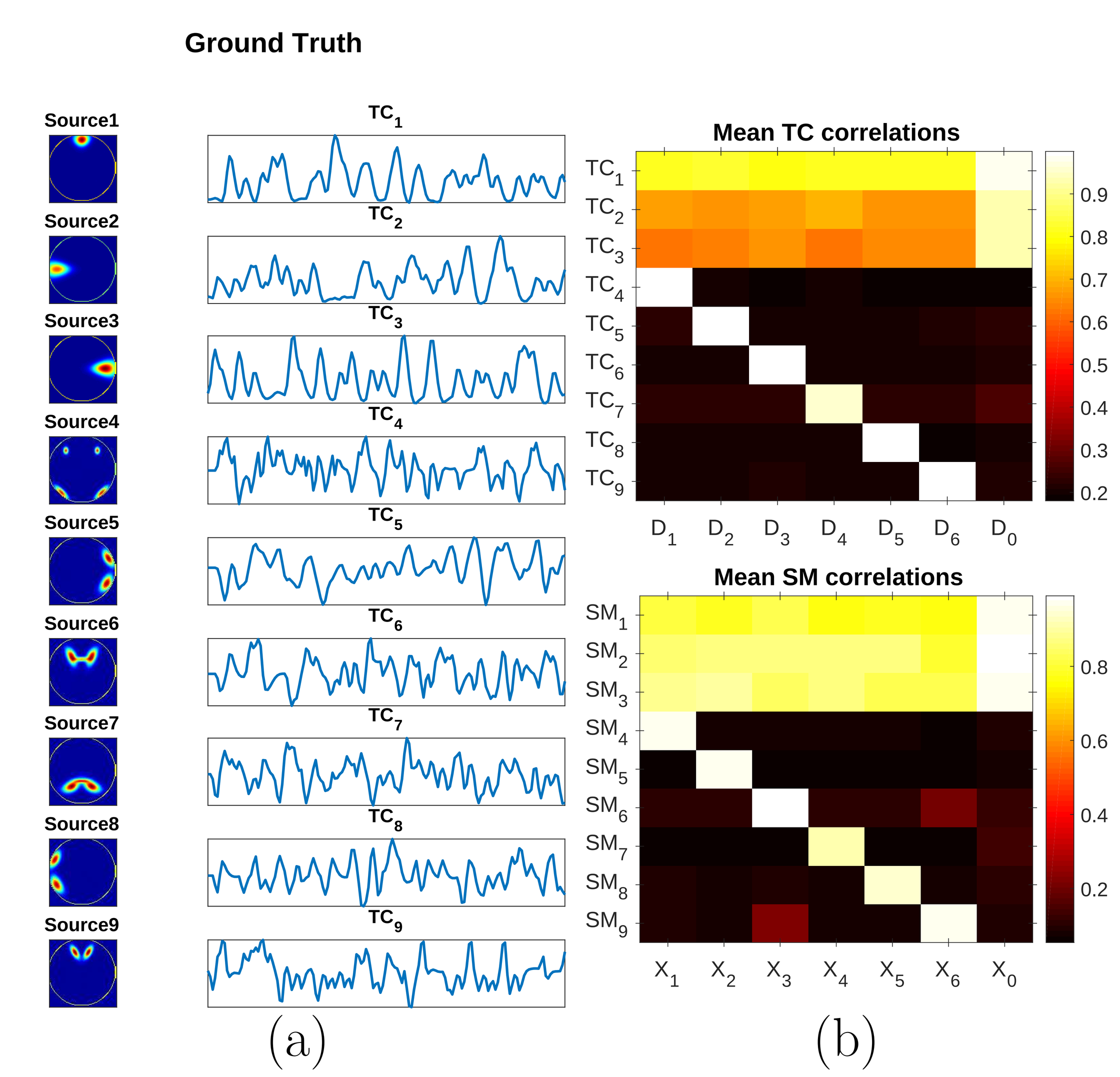

Figure 1: a) The simulated ground truth TC/SMs and their b) mean correlation coefficients w.r.t. $\mathbf{D}_0, \mathbf{D}_i$ and $\mathbf{X}_0, \mathbf{X}_i$ over 100 trials for SNR $= 0$ dB.

Table 1: Mean, median, and standard deviation of most correlated TCs and SMs w.r.t. GrTr over 100 trials.

SNR dB	Algo	TCs			SMs		
		Mean	Median	STD	Mean	Median	STD
-10	Proposed	**0.98**	**0.98**	**0.02**	**0.87**	**0.88**	**0.05**
	CODL	0.95	0.95	0.03	0.79	0.82	0.14
-15	Proposed	**0.92**	**0.96**	**0.08**	**0.69**	**0.66**	**0.18**
	CODL	0.68	0.68	0.23	0.44	0.27	0.34

Real fMRI Results

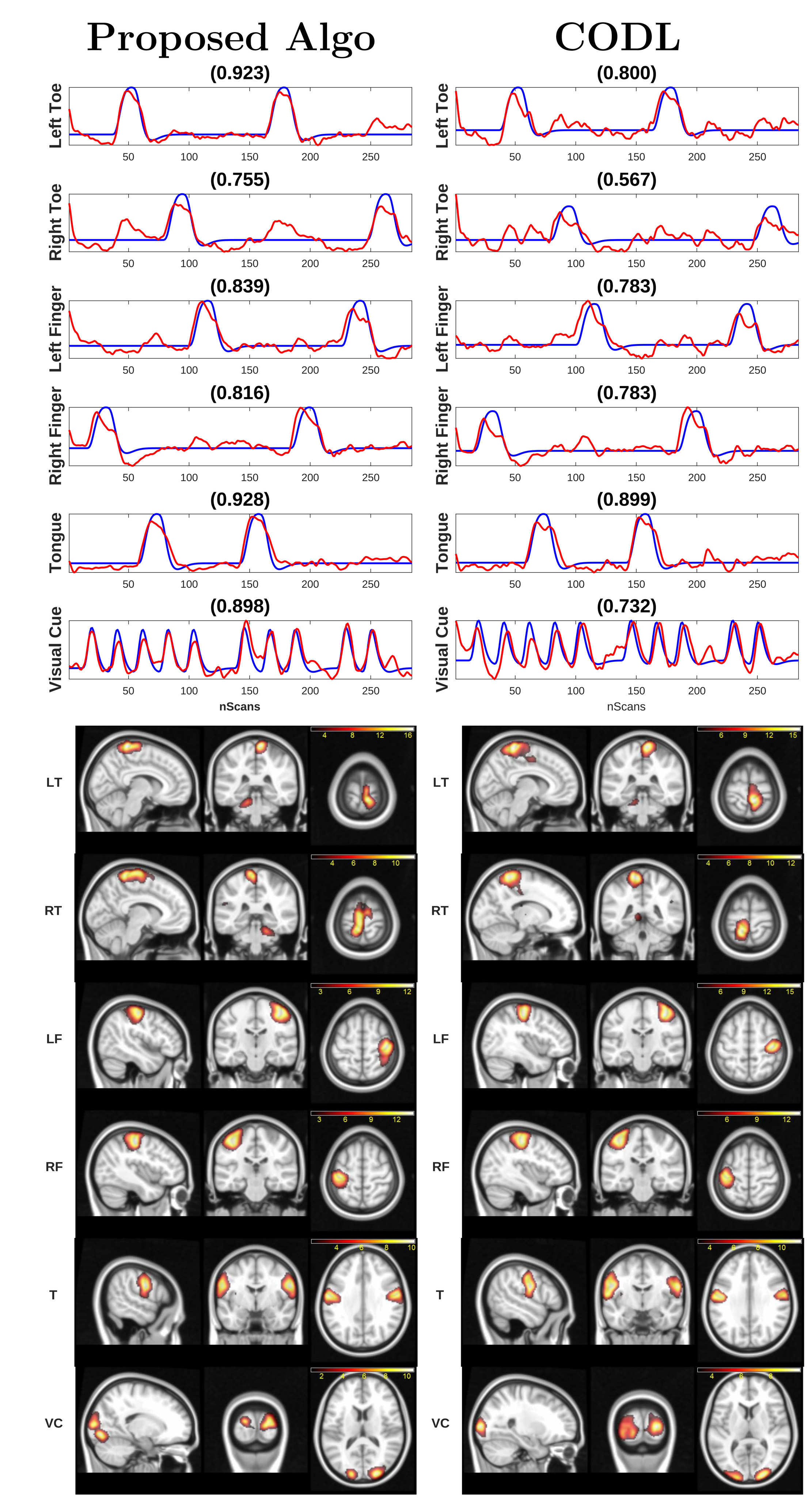

Table 2: Correlation coefficients of most correlated spatial maps w.r.t. the RSN templates as recovered by proposed algorithm and CODL.

RSN	1	2	3	4	5	6	7	8	9	10	Mean
Proposed	0.55	0.48	**0.57**	**0.60**	**0.41**	**0.44**	**0.47**	**0.41**	**0.55**	**0.57**	**0.51**
CODL	**0.72**	**0.71**	0.43	0.47	0.31	0.34	0.36	0.31	0.49	0.37	0.45