OPPORTUNISTIC SYNCHRONISATION OF MULTI-STATIC STARING ARRAY RADARS VIA TRACK-BEFORE-DETECT

Kimin Kim, Murat Üney, and Bernard Mulgrew

Institute for Digital Communications, School of Engineering, The University of Edinburgh, UK

Emails:{K.Kim, M.Uney, B.Mulgrew}@ed.ac.uk

Introduction

This work considers the problem of synchronising separately located transmitters and a staring array receiver for coherent processing. We propose an algorithm that provides the following benefits:

- Use of external reference signals is completely avoided.
- No line-of-sight (LOS) is required between the separated transmitters and the receiver.
- Only local processing is used to estimate the time reference shifts (i.e., synchronisation terms).
- Synchronisation is based on simultaneous estimation of trajectories and reflection coefficients of objects in the surveillance region.
- A high accuracy is achieved with errors on the order of small fractions of the pulse width.

Problem Statement

- We consider a scenario in which M transmitters emit N pulses separated by a pulse repetition interval of T towards a surveillance region. A ULA receiver collects the reflected versions of M transmitted signals (see, Fig. 1.).

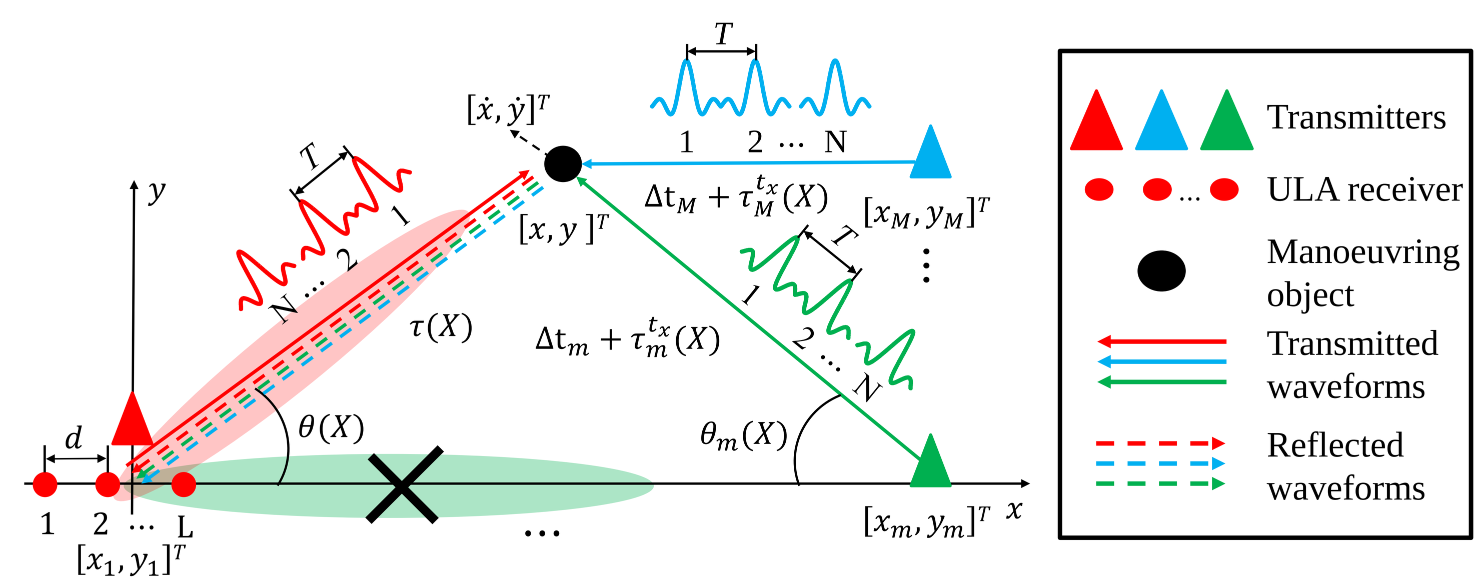


Fig. 1. Geometry of the problem scenario with M transmitters: A ULA (red dots) co-located with one of M transmitters (triangles) collects reflections (dashed lines) from an object at location $[x, y]^T$ with velocity $[\dot{x}, \dot{y}]^T$. Δt_m is a synchronization term in the mth channel. The waveforms used are orthogonal.

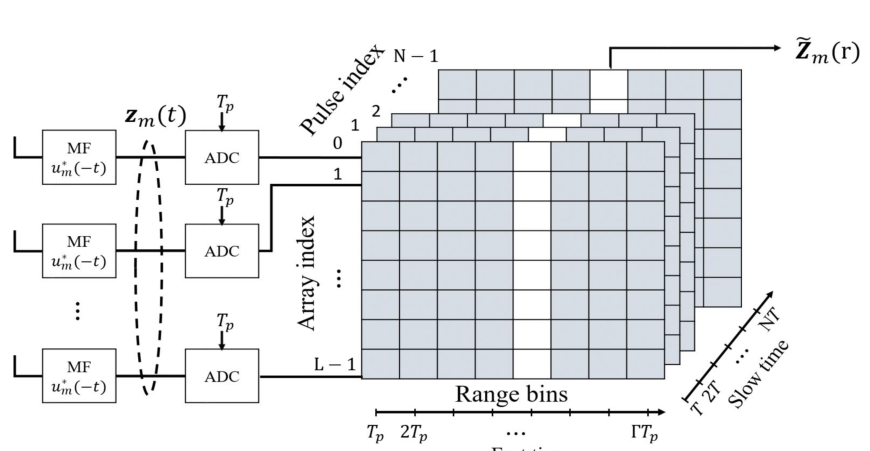


Fig. 2. Data cube acquisition in the mth channel: Sampled versions of the received signals within a coherent processing interval (CPI) as a radar data cube. $\widetilde{\mathbf{Z}}_m(\mathrm{r})$ is a slice along the slow time axis at the rth range bin and forms a L × N matirx.

- We stack columns of $\widetilde{\mathbf{Z}}_m(\mathrm{r})$ and form a LN × 1 vector:

$$\mathbf{Z}_m(\mathrm{r}) = \alpha_m \mathbf{s}_m(\mathrm{r}, \mathrm{X}, \Delta \mathrm{t}_m) + \mathbf{n}(\mathrm{r}) \qquad \mathbf{Z}_m \triangleq [\mathbf{Z}_m(1), \cdots, \mathbf{Z}_m(\Gamma)]$$

- The mth channel parameter likelihood at the kth CPI is found as

$$l\left(\mathbf{Z}_{m,k} \middle| X_k, \alpha_{m,k}, \Delta \mathrm{t}_m\right) \propto \mathcal{CN}\left(\mathbf{Z}_{m,k}(\mathrm{r}); \alpha_{m,k} \mathbf{s}_m(\mathrm{r}, X_k, \Delta \mathrm{t}_m), \Sigma_m\right)$$

- The problem is to estimate a vector of synchronization terms in M channels :

$$\Delta\hat{\mathbf{t}} = \arg\max_{\Delta \mathbf{t}} \prod_{k=1}^{t} l\left(\mathbf{Z}_{1,k}, \cdots, \mathbf{Z}_{M,k} \middle| \mathbf{Z}_{1,1:k-1}, \cdots, \mathbf{Z}_{M,k-1}, \Delta \mathbf{t}\right)$$

$$l\left(\mathbf{Z}_{1,k}, \cdots, \mathbf{Z}_{M,k} \middle| \mathbf{Z}_{1,1:k-1}, \cdots, \mathbf{Z}_{M,k-1}, \Delta \mathbf{t}\right) = \int\int l\left(\mathbf{Z}_{1,k}, \cdots, \mathbf{Z}_{M,k} \middle| X_{1:t}, \boldsymbol{\alpha}_{1:t}, \Delta \mathbf{t}\right) \mathrm{p}\left(X_{1:t}, \boldsymbol{\alpha}_{1:t} \middle| \mathbf{Z}_{1,1:k-1}, \cdots, \mathbf{Z}_{M,1:k-1}\right) \mathrm{d}X_{1:t} \mathrm{d}\boldsymbol{\alpha}_{1:t}$$

$$\Delta \mathbf{t} \triangleq [\Delta \mathrm{t}_1 = 0, \Delta \mathrm{t}_2, \cdots, \Delta \mathrm{t}_M] \text{ and } \boldsymbol{\alpha}_k \triangleq [\alpha_{1,k}, \cdots, \alpha_{M,k}]$$

Research Outcomes

Proposed algorithm:

- Our algorithm uses a parameter estimation in a state space approach to find the vector of M synchronisation terms ($\Delta\mathbf{t}$) associated with the object's trajectory ($X_{1:t}$) and the reflection coefficients ($\boldsymbol{\alpha}_{1:t}$) :

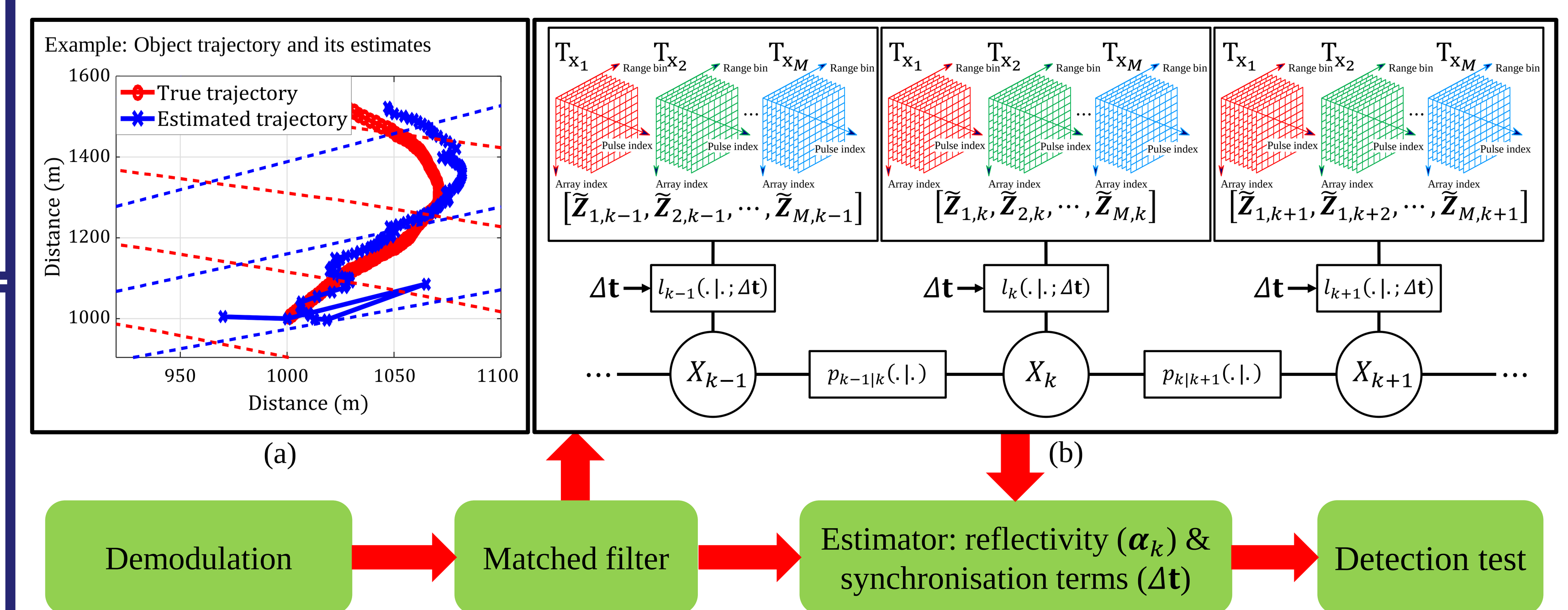


Fig. 3. Block diagram of the proposed detector: (a) Inference on the object's trajectory $X_{1:t} = [x_{1:t}, y_{1:t}, \dot{x}_{1:t}, \dot{y}_{1:t}]^T$, (b) Markov model for the radar data cube measurements

Maximum likelihood estimator: Synchronisation term ($\Delta\mathbf{t}$) estimation using EM approach

- Expectation (E) step:

$$Q(\Delta\mathbf{t}, \Delta\mathbf{t}^{(j)}) \propto \sum_{k=1}^{t} \int \log l(\mathbf{Z}_k | X_k, \widehat{\boldsymbol{\alpha}}_k, \Delta\mathbf{t}) \times p\left(X_k \middle| \mathbf{Z}_{1:k}, \Delta\mathbf{t}^{(j)}\right) \mathrm{d}X_k$$

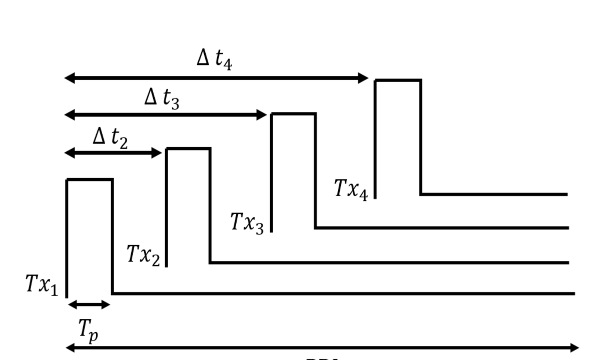
- Maximization (M) step:

$$\Delta\mathbf{t}^{(j)} = \arg\max_{\Delta\mathbf{t}} Q(\Delta\mathbf{t}, \Delta\mathbf{t}^{(j)})$$

- We provide explicit formulae for the gradients used in the maximization step for chirp waveforms:

$$\Delta\mathbf{t}^{(j,i)} = \Delta\mathbf{t}^{(j,i-1)} + \mu \nabla Q(\Delta\mathbf{t}, \Delta\mathbf{t}^{(j)})|_{\Delta\mathbf{t} = \Delta\mathbf{t}^{(j,i-1)}}$$

Example with M = 4 transmitters:



Parameter	Value
Carrier frequency f_c	10e+9 Hz
Bandwidth B	1e+6 Hz
Pulse duration T_p	1e-6 s
Pulse repetition interval PRI	100e-6 s
Number of pulses during a CPI	20
Number of elements on ULA	20
Number of transmitters (Tx)	4

Fig. 4. Transmitted signal parameters for M = 4 transmitters used in this example: Δt_2, Δt_3, and Δt_4 are the synchronisation terms in remote channels.

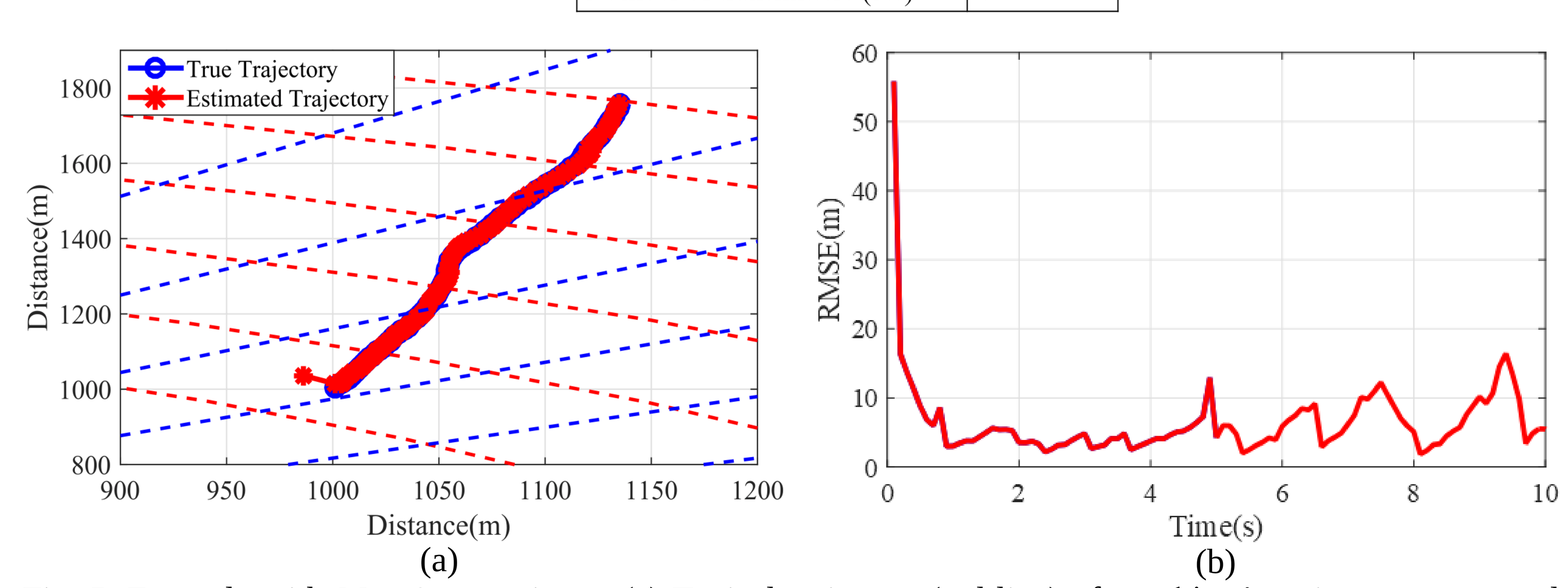


Fig. 5. Example with M = 4 transmitters: (a) Typical estimates (red line) of an object's trajectory compared to the ground truth of the object's trajectory (blue line), (b) RMSE of the estimated trajectory in (a). All for an 10dB SNR object results.

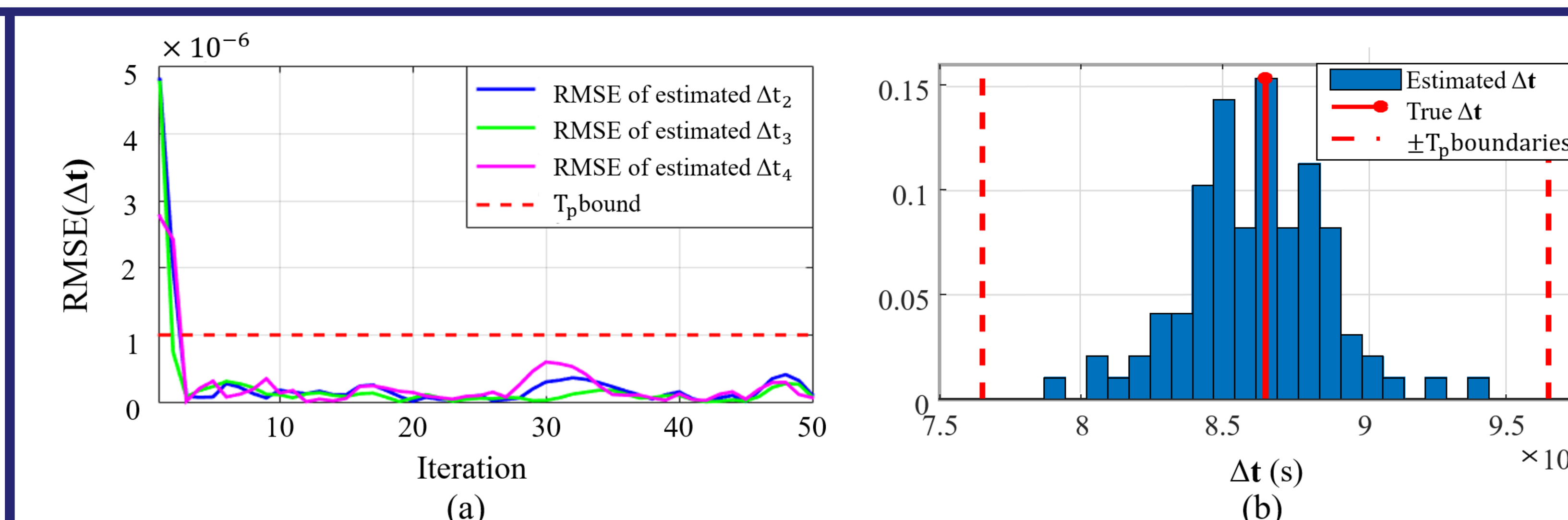


Fig. 6. Example with M = 4 transmitters: (a) RMSE of estimated $\Delta\mathbf{t}$ for $M-1 = 3$ remote channels, (b) Histogram of $\Delta\hat{\mathbf{t}}$ for 100 experiments in comparison with the ground truth value of $\Delta\mathbf{t}$ (solid red line) and $\pm$ $\mathrm{T_p}$boundaries (dashed red line).

Example with M = 2 transmitters:

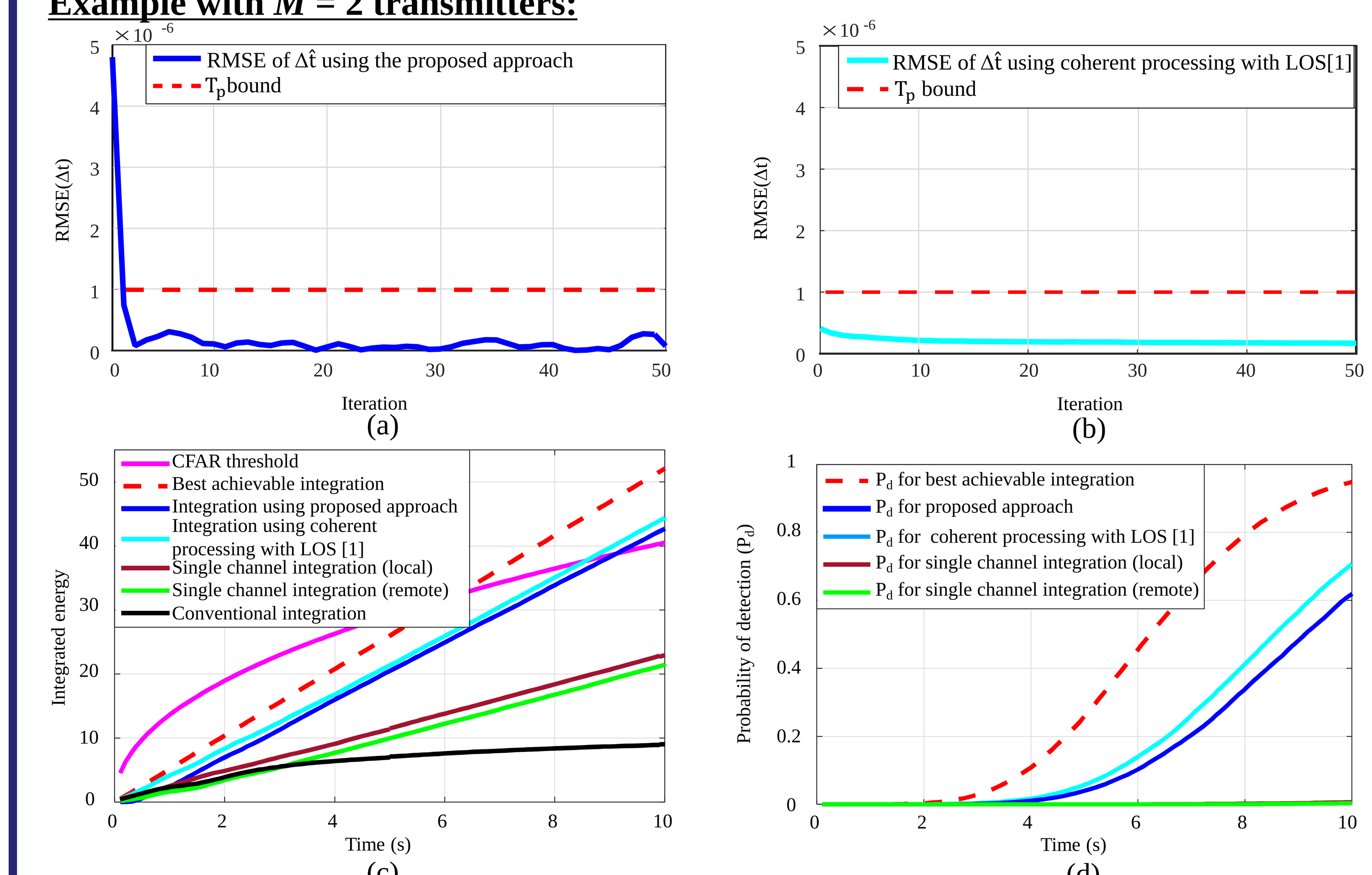


Fig. 7. Example with M = 2 transmitters: (a) RMSE of $\Delta\hat{\mathbf{t}}$ by using the proposed synchronisation approach, (b) RMSE of $\Delta\hat{\mathbf{t}}$ by using coherent processing with line-of-sight (LOS) in [1], (c) Their pulse integration used in [2], (d) Probabilities of detection versus time in (c), All for an -6dB SNR object with a given constant false alarm rate (CFAR) $P_{fa}=10^{-8}$ results averaged over 100 Monte Carlo simulations.

Summary

- The proposed algorithm provides explicit formulae for the maximisation step of the EM for chirp waveforms in the state-space model to find the synchronisation terms.
- This algorithm is capable of finding the synchronisation terms which achieve high accuracy with errors on the order of small fractions of the pulse width.
- The proposed synchronisation scheme enables us to estimate the synchronisation terms close to the values estimated by using coherent processing with line-of-sight between the transmitters and the receiver.

Reference:

[1] K. Kim, M. Üney, and B. Mulgrew, "Simultaneous tracking and long time integration for detection in collaborative array radars," *Proceedings of 2017 Radar conference*, May 2017.

[2] K. Kim, M. Üney, and B. Mulgrew, "Detection via simultaneous trajectory estimation and long time integration," *IEEE transactions on Aerospace and Electronic Systems*, under a review.

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