Radar Precoding for Spectrum Sharing Between Matrix Completion Based MIMO Radars and A MIMO Communication System

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Motivation

- Spectrum is a limited resource. Spectrum sharing can increase the spectrum efficiency.
- Radar and communication systems overlap in the spectrum domain thus causing interference to each other.

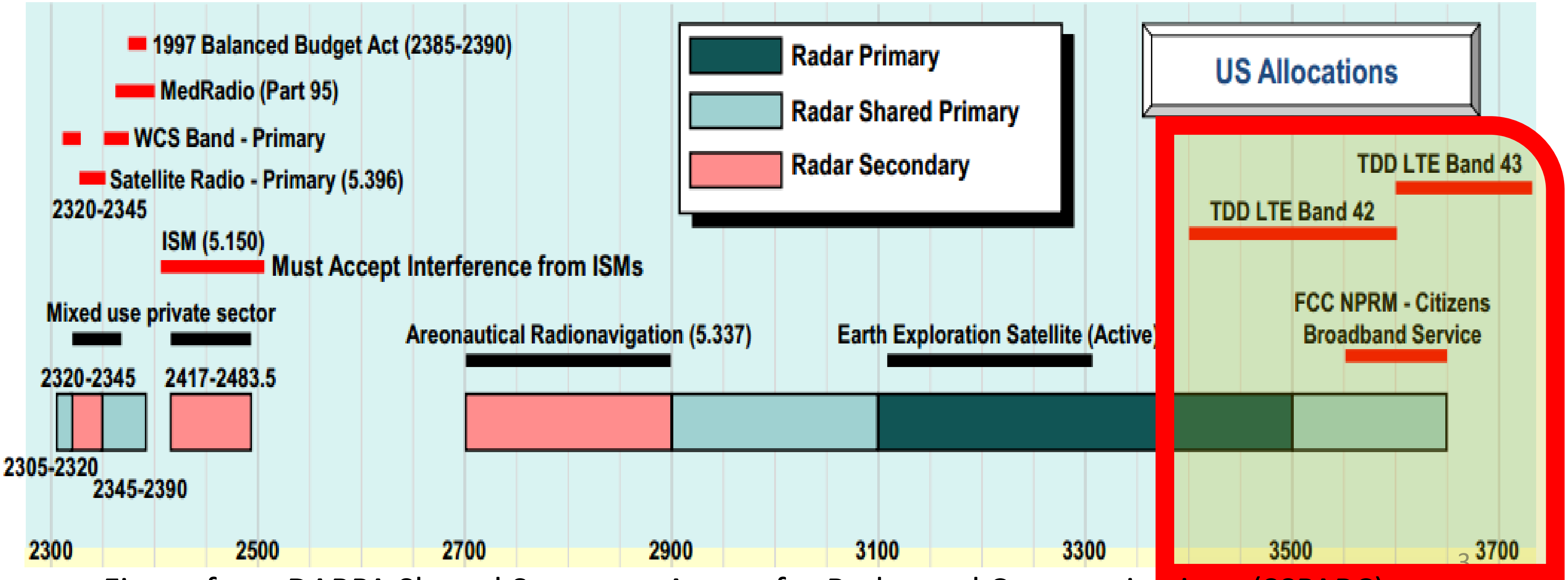


Figure from DARPA Shared Spectrum Access for Radar and Communications (SSPARC)

Background

- Matrix completion based MIMO radars (MIMO-MC) [1] is a good candidate for reducing interference at the radar receiver [2].
  - Traditional MIMO radars transmit orthogonal waveforms from their transmit (TX) antennas, and their receive (RX) antennas forward their measurements to a fusion center to populate a "data matrix" for further processing.
  - Based on the low-rankness of the data matrix, MIMO-MC radar RX antennas forward to the fusion center a small number of pseudo-randomly obtained samples. Subsequently, the full data matrix is recovered using MC techniques.
  - MIMO-MC radars maintain the high resolution of MIMO radars, while requiring significantly fewer data to be communicated to the fusion center, thus enabling savings in communication power and bandwidth.
  - The sub-sampling of data matrix introduces new degrees of freedom for system design enabling additional interference power reduction at the radar receiver [2].

[1] S. Sun, W. U. Bajwa, and A. P. Petropulu, ''MIMO-MC radar: A MIMO radar approach based on matrix completion,'' IEEE Trans. on Aerospace and Electronic Systems, vol. 51, no. 3, pp. 1839-1852, 2015.
[2] B. Li and A. Petropulu, "Spectrum sharing between matrix completion based MIMO radars and a MIMO communication system," in 2015 IEEE Int. Conf. Acoust. Speech Sig. Process., April 2015, pp. 2444–2448.

- Existing Spectrum Sharing Approaches
  - Avoiding interference by large spatial separation;
  - Dynamic spectrum access based on spectrum sensing;
  - Spatial multiplexing: MIMO radar waveforms designed to eliminate the interference at the communication receiver [1].
- Our previous work [2]

  Spectrum sharing between a MIMO-MC radar and a MIMO communication system is achieved by
  - Sharing the radar waveforms with the communication system, and
  - Jointly designing the communication system signals and the radar system sampling scheme.
- In this work

  A new framework for spectrum sharing between a MIMO-MC radar and a MIMO communication system is proposed
  - Radar precoding is jointly designed with the communication codewords to maximized the radar SINR while meeting certain rate and power constraints at the communication system.
  - The radar precoder is shared with the communication system rather than the radar waveforms, which preserves the radar waveform confidentiality.

[1] A. Khawar, A Abdel-Hadi, and T.C. Clancy, "Spectrum sharing between s-band radar and LTE cellular system: A spatial approach," in *2014 IEEE International Symposium on Dynamic Spectrum Access Networks*, April 2014, pp. 7–14.

[2] B. Li and A. Petropulu, "Spectrum sharing between matrix completion based MIMO radars and a MIMO communication system," in 2015 IEEE Int. Conf. Acoust. Speech Sig. Process., April 2015, pp. 2444–2448.

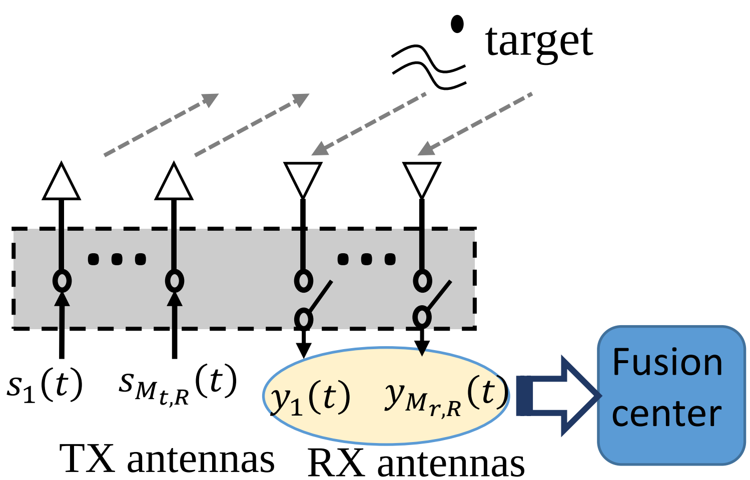
Introduction to the Matrix Completion MIMO radar (MIMO-MC)

- The received data at the radar receivers can be expressed as

$$\mathbf{Y}_R = \mathbf{B\Sigma A}^T\mathbf{PS} + \mathbf{W}_R \triangleq \mathbf{DPS} + \mathbf{W}_R$$

  - $\mathbf{A}$: $\mathbb{C}^{M_{t,R}\times K}$, $\mathbf{B}$: $\mathbb{C}^{M_{r,R}\times K}$, transmit/receive manifold matrices;
  - $\mathbf{\Sigma}$: $\mathbb{C}^{K\times K}$, diagonal matrix contains target reflection coefficients;
  - $\mathbf{S}$: $\mathbb{C}^{M_{t,R}\times L}$, coded MIMO radar waveforms, which are chosen orthonormal;
  - $\mathbf{P}$: $\mathbb{C}^{M_{t,R}\times M_{t,R}}$, the radar precoding matrix; $\mathbf{D} \triangleq \mathbf{B\Sigma A}^T$;

Notation	
$M_{t,R}$	# of radar TX antennas
$M_{r,R}$	# of radar RX antennas
K	# of targets
L	Length of waveform
$\mathbf{W}_R$	Additive noise



- $\mathbf{DPS}$ is low rank if $M_{r,R}$ and $L >> K$.
- Random subsampling is applied to each receive antenna. The matrix formulated at the fusion center can be expressed as:

$$\boldsymbol{\Omega} \circ \mathbf{Y}_R = \boldsymbol{\Omega} \circ (\mathbf{DPS}) + \boldsymbol{\Omega} \circ \mathbf{W}_R$$

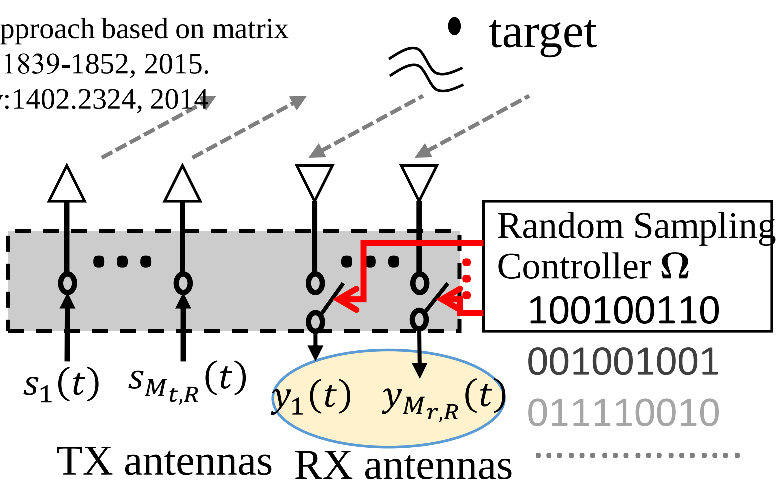
where $\boldsymbol{\Omega}$ is a matrix with binary entries, whose "1"s correspond to sampling times at the RX antennas, and $\circ$ denotes Hadamard product.

- Matrix completion can be applied to recover $\mathbf{DPS}$ using partial entries of $\mathbf{Y}_R$ if [1-2]:
  - $\mathbf{DPS}$ has low coherence;
  - Ω has large spectral gap.

[1] S. Sun, W. U. Bajwa, and A. P. Petropulu, ''MIMO-MC radar: A MIMO radar approach based on matrix completion,'' IEEE Trans. on Aerospace and Electronic Systems, vol. 51, no. 3, pp. 1839-1852, 2015.
[2] S. Bhojanapalli and P. Jain, "Universal matrix completion," arXiv preprint arXiv:1402.2324, 2014

$$\underset{M_{r,R} \times L}{\boldsymbol{\Omega}} \quad \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

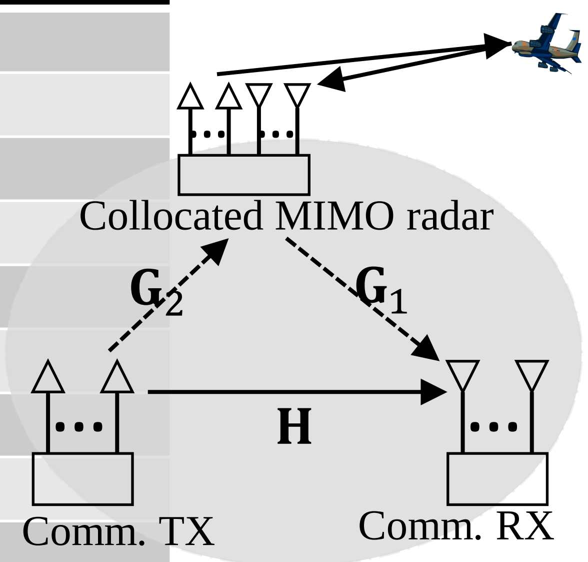
Subsampling rate $p = \|\boldsymbol{\Omega}\|_0 / L M_{r,R}$



The Coexistence Signal Model

- Consider a MIMO communication system which coexists with a MIMO-MC radar system using the same carrier frequency.
- Assumptions:
  - Flat fading, narrow band radar and comm. signals;
  - Block fading: the channels remain constant for L symbols;
  - Both systems have the same symbol rate;

Parameters	Radar System	Communication System
Carrier Freq. (f_c)	3550 MHz	3550 MHz
Baseband Bandwidth (w)	0.5 MHz	0.5 MHz [3]
Sub-pulse/Symbol duration (T_b)	2 us	2 us
Transmit power	750kW [1]	790 W [1]
Range resolution	$c/(2*w)$ = 300m [2]	
Pulse repetition freq. (PRF)	1 kHz	
Unambiguous range	$c/(2*\text{PRF})$= 150 km	
Symbols per pulse (L)	128	
Duty cycle	25%	



[1] F. H. Sanders, R. L. Sole, J. E. Carroll, G. S. Secrest, and T. L. Allmon, "Analysis and resolution of RF interference to radars operating in the band 2700–2900 MHz from broadband communication transmitters," US Dept. of Commerce, Tech. Rep. NTIA Technical Report TR-13-490,2012.
[2] "Radar performance," Radtec Engineering Inc., [online], http://www.radar-sales.com/PDFs/Performancen RDRn%26TDR.pdf, (Accessed:July 2015).
[3] Telesystem Innovations, "LTE in a nutshell: The physical layer," White paper, 2010.

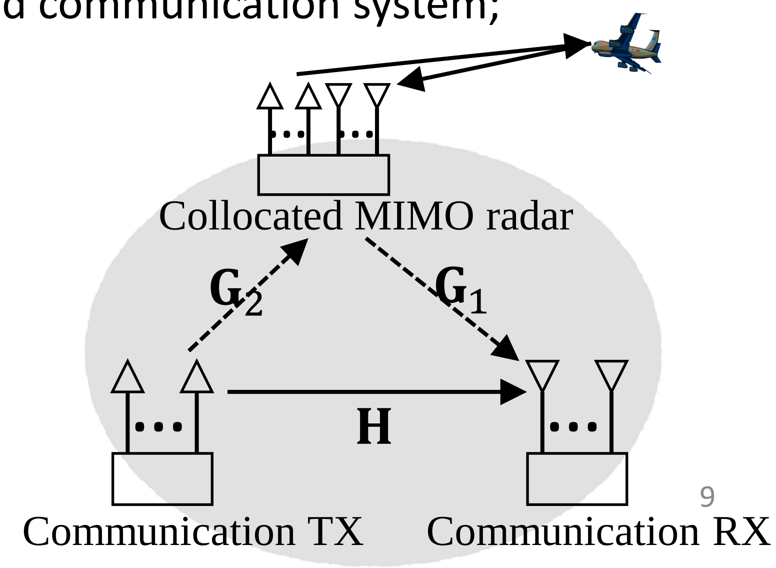
- The received signals at the MIMO-MC radar and communication RX are

$$\boldsymbol{\Omega}_l \circ \mathbf{y}_R(l) = \boldsymbol{\Omega}_l \circ \left[\mathbf{DPs}(l) + e^{j\alpha_{2l}}\mathbf{G}_2\mathbf{x}(l) + \mathbf{w}_R(l)\right],$$

$$\mathbf{y}_C(l) = \mathbf{Hx}(l) + e^{j\alpha_{1l}}\,\mathbf{G}_1\mathbf{Ps}(l) + \mathbf{w}_C(l), \forall l \in \mathbb{N}_L^+,$$

where

- l is the sampling time instance, Ω_l is the l-th column of Ω;
- $\mathbf{H}$: $\mathbb{C}^{M_{t,C}\times M_{r,C}}$, the communication channel;
- $\mathbf{G_1}$: $\mathbb{C}^{M_{t,R}\times M_{r,C}}$, the interference channel from the radar TX to comm. RX;
- $\mathbf{G_2}$: $\mathbb{C}^{M_{t,C}\times M_{r,R}}$, the interference channel from the comm. TX to radar RX;
- $\mathbf{s}(l)$ and $\mathbf{x}(l)$: transmit vector by radar and communication system;
- $e^{j\alpha_{1l}}$ and $e^{j\alpha_{2l}}$: random phase jitters
  - We do not make any assumption on α_{il}.



- Grouping L samples together, we have

$$\boldsymbol{\Omega} \circ \mathbf{Y}_R = \boldsymbol{\Omega} \circ (\mathbf{DPS} + \mathbf{G}_2 \mathbf{X} \boldsymbol{\Lambda}_2 + \mathbf{W}_R),$$

$$\mathbf{Y}_C = \mathbf{HX} + \mathbf{G}_1 \mathbf{PS} \boldsymbol{\Lambda}_1 + \mathbf{W}_C, \quad \text{where } \boldsymbol{\Lambda}_i = \text{diag}\left(e^{j\alpha_{i1}}, \ldots, e^{j\alpha_{iL}}\right), i \in \{0,1\}.$$

- In our previous work [1], $\mathbf{P} = \mathbf{I}$ and we share $\mathbf{S}$ with the comm. system for interference subtraction at the comm. receiver.
  - Sharing of radar waveforms makes the radar vulnerable to adversaries an jammers.
- In this paper, we consider the following system setting
  - Radar precoding is employed and shared with the communication system- jammer could not benefit from the knowledge of $\mathbf{P}$;
  - We take $\mathbf{S}$ to be a random orthonormal matrix [2];
  - Communication codewords are circularly symmetric complex Gaussian $\mathbf{x}(l) \sim \mathcal{CN}(0, \mathbf{R}_x), \forall \boldsymbol{l}$;
- The precoding matrix $\mathbf{P}$ and the communication covariance matrix $\mathbf{R}_x$ are jointly designed to
  - maximize the SINR at the MIMO-MC radar receiver, while maintaining certain communication system rate and power constraints.

[1] B. Li and A. Petropulu, "Spectrum sharing between matrix completion based MIMO radars and a MIMO communication system," in 2015 IEEE Int. Conf. Acoust. Speech Sig. Process., April 2015, pp. 2444–2448.
[2] S. Sun, W. U. Bajwa, and A. P. Petropulu, ''MIMO-MC radar: A MIMO radar approach based on matrix completion,'' IEEE Trans. on Aerospace and Electronic Systems, vol. 51, no. 3, pp. 1839-1852, 2015.

The Proposed Spectrum Sharing Method

- For the MIMO communication system:
  - The total TX power of the communication TX antennas equals

$$\mathbb{E}\{\mathrm{Tr}(\mathbf{X}\mathbf{X}^H)\} = L\mathrm{Tr}(\mathbf{R}_x).$$

  - The interference plus noise covariance is given as

$$\mathbf{R}_w \triangleq \mathbf{G}_1\mathbf{P}\mathbb{E}\{\mathbf{s}(l)\mathbf{s}^H(l)\}\mathbf{P}^H\mathbf{G}_1^H + \sigma_C^2\mathbf{I}$$
$$= \mathbf{G}_1\mathbf{\Phi}\mathbf{G}_1^H + \sigma_C^2\mathbf{I},$$

    where $\mathbf{\Phi} = \mathbf{P}\mathbf{P}^H/L$. The second equality holds because the entries of S can be approximated by i.i.d. Gaussian random variables with distribution $\mathcal{N}(0, 1/L)$, if $M_{t,R} = \mathcal{O}(L/\mathrm{ln}L)$ [1].
  - The communication rate achieved, which is a lower bound of the channel capacity, is given by

$$\underline{\mathrm{C}}(\mathbf{R}_x, \mathbf{\Phi}) \triangleq \log_2|\mathbf{I} + \mathbf{R}_w^{-1}\mathbf{H}\mathbf{R}_x\mathbf{H}^H|.$$

[1] T. Jiang, "How many entries of a typical orthogonal matrix can be approximated by independent normals?," The Annals of Probability, vol. 34, no. 4, pp. 1497--1529, 2006.

- For the MIMO-MC radar:
  - The total interference power (TIP) exerted at the RX antennas equals

$$\mathrm{TIP} \triangleq \mathbb{E}\{\mathrm{Tr}(\mathbf{G}_2\mathbf{X}\boldsymbol{\Lambda}_2\boldsymbol{\Lambda}_2^H\mathbf{X}^H\mathbf{G}_2^H)\} = L\mathrm{Tr}(\mathbf{G}_2\mathbf{R}_x\mathbf{G}_2^H).$$

  - Recall that only partial entries of $\mathbf{Y}_R$ are forwarded to the fusion center, which implies that only a portion of TIP affects the MIMO-MC radar.
  - The *effective* interference power to MIMO-MC radar is given as:

$$\mathrm{EIP} \triangleq \mathbb{E}\left\{\mathrm{Tr}\left(\boldsymbol{\Omega}\circ(\mathbf{G}_2\mathbf{X}\boldsymbol{\Lambda}_2)\left(\boldsymbol{\Omega}\circ(\mathbf{G}_2\mathbf{X}\boldsymbol{\Lambda}_2)\right)^H\right)\right\} = \mathrm{Tr}(\Delta\mathbf{G}_2\mathbf{R}_x\mathbf{G}_2^H)$$

  where $\Delta \triangleq \sum_{l=1}^{L}\Delta_l$ and $\Delta_l = \mathrm{diag}(\boldsymbol{\Omega}_l)$. We note that the EIP is a re-weighted version of the TIP.
  - Similarly, we could derive the effective signal power (ESP) of the target echo signal forwarded to the fusion center

$$\mathrm{ESP} \triangleq \mathrm{Tr}(\Delta\mathbf{D}\boldsymbol{\Phi}\mathbf{D}^H)$$

  - We assume that the information of targets contained in $\mathbf{D}$ is given a *priori*. In practice, such information could be obtained in various ways, e.g., in tracking applications, the parameter estimates obtained from previous tracking cycles.

- If the MIMO-MC radar shares its random sampling scheme with the communication system, the spectrum sharing problem can be formulated as:

$$(\mathbf{P}_1)\ \ \max_{\mathbf{R}_x,\boldsymbol{\Phi}} \text{ESINR} \equiv \frac{\text{Tr}(\Delta \mathbf{D}\boldsymbol{\Phi}\mathbf{D}^H)}{\text{Tr}(\Delta \mathbf{G}_2\mathbf{R}_x\mathbf{G}_2^H)+pP_{wR}}$$

$$\text{s.t. } L\text{Tr}(\mathbf{R}_x) \le P_C,\ L\text{Tr}(\boldsymbol{\Phi}) \le P_R,\ \underline{\text{C}}(\mathbf{R}_x,\boldsymbol{\Phi}) \ge C,\ \mathbf{R}_x \succcurlyeq 0,\ \boldsymbol{\Phi} \succcurlyeq 0.$$

  where $P_{wR} = LM_{r,R}\sigma_R^2$. The first two constraints restrict the total communication and radar transmit power. The third constraint restricts the communication rate to be at least C, in order to support reliable communication and avoid service outage.
- Problem (P_1) is non-convex w.r.t. both $\mathbf{R}_x$ and $\boldsymbol{\Phi}$. A solution can be obtained via alternating optimization.
- Fixing $\boldsymbol{\Phi}$, the $\mathbf{R}_x$ sub-problem is given as

$$(\mathbf{P}_\text{R})\ \ \min_{\mathbf{R}_x\succcurlyeq 0}\text{Tr}(\mathbf{G}_2^H\Delta\mathbf{G}_2\mathbf{R}_x)\ \ \text{s.t. } \underline{\text{C}}(\mathbf{R}_x,\boldsymbol{\Phi}) \ge C,\ L\text{Tr}(\mathbf{R}_x) \le P_C$$

  - The above problem is convex w.r.t. $\mathbf{R}_x$ and can be solved efficiently using the interior point method.

- Fixing $\mathbf{R}_x$, the $\mathbf{\Phi}$ sub-problem is given as

$$(\mathrm{P}_\Phi)\ \ \min_{\mathbf{\Phi}\succcurlyeq 0} \mathrm{Tr}(\mathbf{D}^H \Delta \mathbf{D}\mathbf{\Phi})$$
$$\text{s.t. } \underline{C}(\mathbf{R}_x, \mathbf{\Phi}) \geq C, L\mathrm{Tr}(\mathbf{\Phi}) \leq P_R$$

  - We can express $\underline{C}(\mathbf{R}_x, \mathbf{\Phi})$ as follows:

$$\underline{C}(\mathbf{R}_x, \mathbf{\Phi}) = \log_2|\mathbf{G}_1\mathbf{\Phi}\mathbf{G}_1^H + \widetilde{\mathbf{R}}_x| - \log_2|\mathbf{G}_1\mathbf{\Phi}\mathbf{G}_1^H + \sigma_C^2\mathbf{I}|,$$

  where $\widetilde{\mathbf{R}}_x \triangleq \sigma_C^2\mathbf{I} + \mathbf{H}\mathbf{R}_x\mathbf{H}^H$. $\underline{C}(\mathbf{R}_x, \mathbf{\Phi}) \geq C$ is a non-convex constraint.
  - To overcome the non-convexity, an auxiliary $\mathbf{\Psi}$ is introduced by transforming (P_Φ) into the following problem:

$$(\mathbf{P}_{\Phi\Psi})\ \ \max_{\mathbf{\Phi}\succeq 0} \mathrm{Tr}(\mathbf{\Delta}\mathbf{D}\mathbf{\Phi}\mathbf{D}^H),\ \ \text{s.t.}\, LTr(\mathbf{\Phi}) \leq P_R,$$
$$\log_2\left|\mathbf{G}_1\mathbf{\Phi}\mathbf{G}_1^H + \widetilde{\mathbf{R}}_x\right| + \max_{\mathbf{\Psi}\succeq 0}\log_2|\mathbf{\Psi}|$$
$$-\mathrm{Tr}((\mathbf{G}_1\mathbf{\Phi}\mathbf{G}_1^H + \sigma_C^2\mathbf{I})\mathbf{\Psi}) + M_{r,C} \geq C$$

  - Again, alternating optimization is applied as an inner iteration. During the n-th outer alternating iteration, let $(\mathbf{\Phi}^{nk}, \mathbf{\Psi}^{nk})$ be the variables at the k-th inner iteration.

- One inner iteration is given as follows

$$\boldsymbol{\Psi}^{nk} = (\mathbf{G}_1 \boldsymbol{\Phi}^{n(k-1)} \mathbf{G}_1^H + \sigma_C^2 \mathbf{I})^{-1}$$

$$(\mathbf{P}'_\Phi) \quad \boldsymbol{\Phi}^{nk} = \underset{\boldsymbol{\Phi}\succeq 0}{\arg max} \operatorname{Tr}(\boldsymbol{\Delta}\mathbf{D}\boldsymbol{\Phi}\mathbf{D}^H), \quad \text{s.t.}\, L\operatorname{Tr}(\boldsymbol{\Phi}) \le P_R,$$

$$\log_2|\mathbf{I} + \mathbf{G}_1^H (\widetilde{\mathbf{R}}_x)^{-1} \mathbf{G}_1 \boldsymbol{\Phi}| - \operatorname{Tr}\big(\mathbf{G}_1^H \boldsymbol{\Psi}^{nk} \mathbf{G}_1 \boldsymbol{\Phi}\big) \ge C',$$

where C' is a constant w.r.t. $\boldsymbol{\Phi}$. $(\mathbf{P}'_\Phi)$ is convex and can be solved efficiently.

- The complete spectrum share algorithm proposed in this section is summarized in Algorithm 1.

Algorithm 1 The proposed spectrum sharing method.

Input: $\mathbf{D}, \mathbf{H}, \mathbf{G}_1, \mathbf{G}_2, \boldsymbol{\Omega}, P_R, P_C, C, \sigma_C^2, \delta_1, \delta_2$
Initialization: $\boldsymbol{\Phi}^0 = P_R / M_{t,R}\mathbf{I}$
repeat
  $\mathbf{R}_x^n \leftarrow$ Solve problem $(\mathbf{P}_\mathbf{R})$ using interior point method or $(\mathbf{P}_\mathbf{R}$-D) using [13, **Algorithm** 1] with fixed $\boldsymbol{\Phi}^{n-1}$;
  $\boldsymbol{\Phi}^{nk} \leftarrow \boldsymbol{\Phi}^{n-1}$ for $k = 0$;
  repeat
    $\boldsymbol{\Psi}^{nk} = \left(\mathbf{G}_1 \boldsymbol{\Phi}^{n(k-1)} \mathbf{G}_1^H + \sigma_C^2 \mathbf{I}\right)^{-1}$;
    $\boldsymbol{\Phi}^{nk} \leftarrow$ Solve problem $(\mathbf{P}'_\Phi)$ using interior point method or $(\mathbf{P}'_\Phi$-D) with fixed $\boldsymbol{\Psi}^{nk}$ and $\mathbf{R}_x^n$;
    $k \leftarrow k + 1$
  until $|\mathrm{ESP}^k - \mathrm{ESP}^{k-1}| < \delta_1$
  $\boldsymbol{\Phi}^n \leftarrow \boldsymbol{\Phi}^{nk}$
  $n \leftarrow n + 1$
until $|\mathrm{ESINR}^n - \mathrm{ESINR}^{n-1}| < \delta_2$
Output: $\mathbf{R}_x = \mathbf{R}_x^n, \mathbf{P} = \sqrt{L}(\boldsymbol{\Phi}^n)^{1/2}$

Simulations

- MIMO-MC radar with half-wavelength uniform linear TX&RX arrays transmit random orthonormal waveforms. Two far-field targets at angles $\pm 60^\circ$.
- Entries in $\mathbf{H}$ are i.i.d. and $\mathbf{H}_{ij} \sim \mathcal{CN}(0,1)$; Entries in $\mathbf{G}_1$ and $\mathbf{G}_2$ are i.i.d. $\mathcal{CN}(0,0.01)$.
- $L = 32, \sigma_R^2 = \sigma_C^2 = .01, C = 20$bits/symbol, $P_C = LM_{t,C}$ (the power is normalized by the power of radar waveform).
- The obtained $\mathbf{R}_x$ is used to generate $x(l) = \mathbf{R}_x^{1/2}\text{randn}(M_{t,C}, 1)$.
- The TFOCUS package is used for matrix completion at the radar fusion center.
- ESINR and MC relative recovery error ($\left\|\mathbf{DPS} - \widehat{\mathbf{DPS}}\right\|_F / \|\mathbf{DPS}\|_F$) are used as the performance metrics.
- Comparing methods include
  - Method #1: no radar precoding, i.e., $\mathbf{P} = \sqrt{LP_R/M_{t,R}}\mathbf{I}$, plus "selfish communication", where the communication system minimizes the TX power to achieve certain rate without any concern about the interferences it exerts to the radar system.
  - Method #2: no radar precoding, but $\mathbf{R}_x$ being designed to minimize the interferences it exerts to the radar system while achieving certain communication rate.
  - Method #3: our previous approach where $\mathbf{S}$ is shared with the communication receiver, and there is no radar precoding.

Simulations

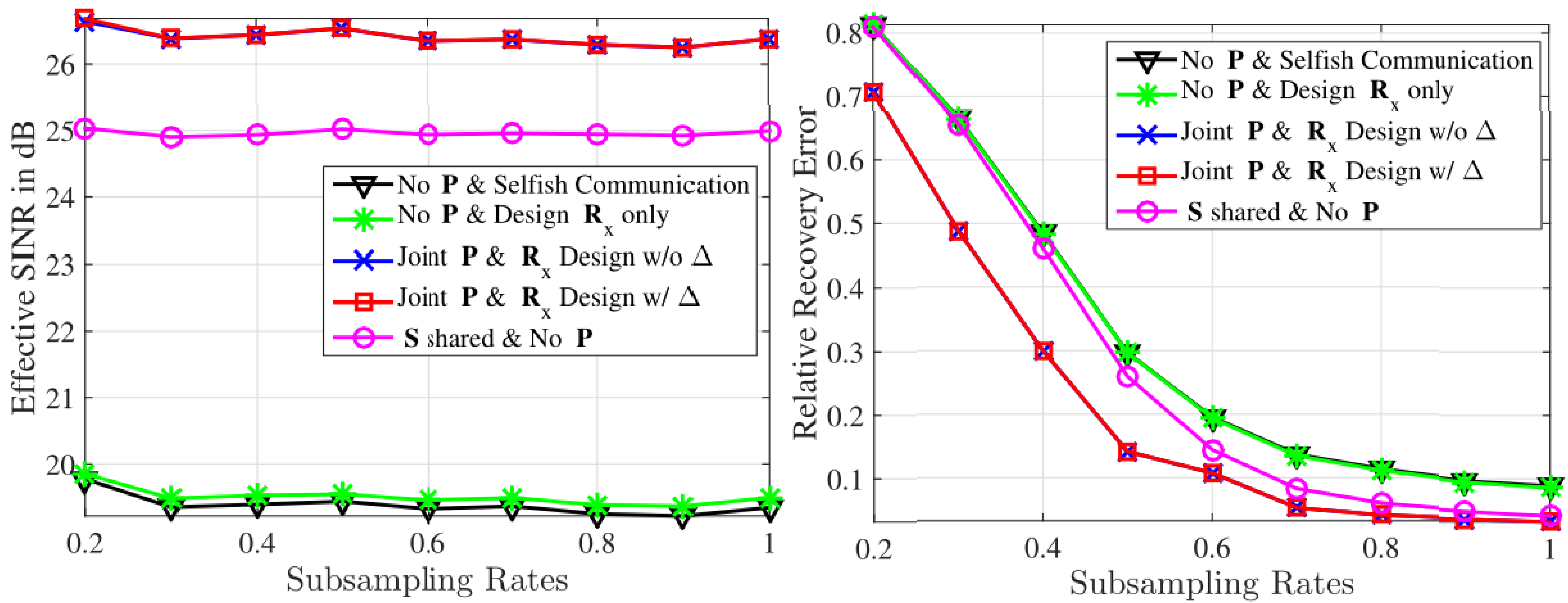


Figure 1. Spectrum sharing under different sub-sampling rates.
$M_{t,R} = 4, M_{r,R} = 8, M_{t,C} = 8, M_{r,C} = 4,$
$P_R = 10LM_{t,R}, C = 20$bits/symbol.

Simulations

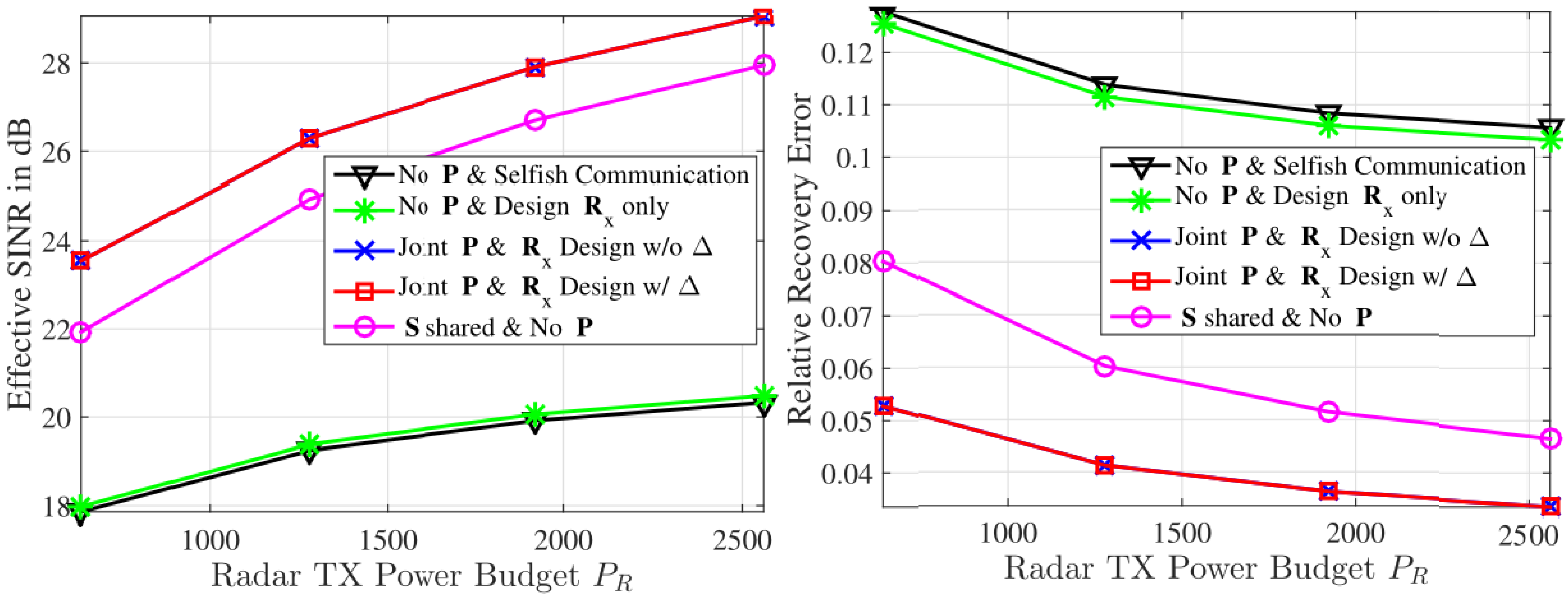


Figure 2. Spectrum sharing under different radar transmit power budget.
$M_{t,R} = 4, M_{r,R} = 8, M_{t,C} = 8, M_{r,C} = 4,$
$p = 0.8, C = 20$bits/symbol.

Observations

- The radar precoder plays an important role in the proposed spectrum sharing method.
  - The green and black curves in figures illustrate that the design of $\mathbf{R}_x$ could not benefit the MIMO-MC radar when no radar precoding is considered.
- The proposed method performs almost the same whether Ω or Δ is shared with the communication system or not.
  - Δ resulted from the uniformly random subsampling matrix Ω is a diagonal matrix with almost identical entries, which has almost no affect on the performance of the proposed method.
- Compared with our previous approach with $\mathbf{S}$ shared, the proposed method could achieve even higher ESINR and lower MC relative recovery error.
  - The reason is that the target prior information in $\mathbf{D}$ facilitates the design of $\mathbf{P}$ so that the radar power is focused on the targets while nulling the interference to the communication receiver.

Conclusions

- We have investigate a new framework for spectrum sharing between a MIMO communication system and a MIMO-MC radar system.
- Spectrum sharing is achieved by joint design of the radar precoding matrix and the communication codeword covariance matrix.
- Simulation results show that, the radar precoder plays a key role in improving the radar SINR and matrix completion recovery accuracy over previous approaches.
- Potential future directions include
  - spectrum sharing problem with targets distributed across different range bins;
  - spectrum sharing in an environment with clutter.

Thank you!