# A Novel Thresholding Technique for the Denoising of Multicomponent Signals

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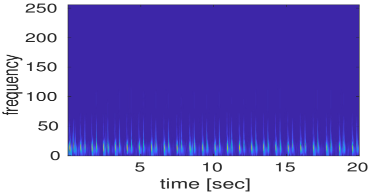
# Plan

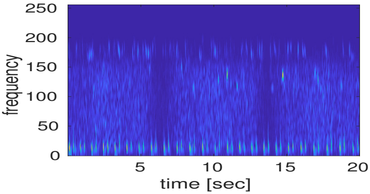
1. Introduction
2. Definitions
3. Signal Denoising from STFT
4. New Method for the Denoising of the STFT of MCSs
5. Conclusions and Perspectives

# Context

- Denoising oscillatory signals or superposition of AM/FM modes (called *multicomponent signals* (MCSs)) from their Short-Time Fourier Transform (STFT).
- Illustration of the STFTs of oscillatory signals: real PCG (phonocardiogram) signals (SiSEC 2016 database):




(a)




(b)

Figure: (a): STFT of noise-free PCG; (b): STFT of noisy PCG

# Objectives

- Hard-Thresholding (HT) probably the most popular thresholding technique: **keep only STFT coefficients above a certain level in terms of their magnitude** before signal reconstruction [1].
- Our goal here is to improve HT in the context of MCS: **one wants to recover the modes while denoising them**.

[1] D. Donoho and I. Johnstone, *Ideal spatial adaptation via wavelet shrinkage*, Biometrika, vol. 81, pp. 425-455, 1994.

# STFT and Reconstruction Formula

- The time-frequency representation our denoising algorithm is based on is STFT computed as:

$$V_f^g(m, \frac{k}{N}) = \sum_{n \in \mathbb{Z}} f[n] g[n-m] e^{-i2\pi \frac{k(n-m)}{N}}.$$

  in which $N$ is the number of frequency bins.

- STFT is invertible through (assuming $\|g\|_2 = 1$):

$$f[n] = \sum_{m \in \mathbb{Z}} \sum_{k=0}^{N-1} V_f^g(m, \frac{k}{N}) g[n-m] \frac{e^{i2\pi \frac{k(n-m)}{N}}}{N},$$

# Multicomponent Signals (MCSs)

- *Multicomponent signals* are defined as:

$$f[n] = \sum_{l=1}^{K} f_l[n], \text{ with } f_l[n] = A_l[n]e^{i2\pi\phi_l[n]}, \ n = 0, \cdots, L-1$$

  for some $K$, where $A_l[n] > 0$, $\phi_l'[n] > 0$ for any $n$ and $l$, $(m, \phi_l'(m))$ is called *ridge* associated with the $l$th mode.

- $f_l$s, called *modes or components*, are separated with resolution $\Delta$, if for all $l \in \{1, \cdots, K-1\}$:

$$\phi_{l+1}'[n] - \phi_l'[n] > 2\Delta \ \forall n.$$

# HT Denoising Technique

- Assume $f$ complex signal contaminated by complex white Gaussian noise $\Phi$ with variance $\sigma^2$: $\tilde{f}[n] = f[n] + \Phi[n]$
- $V_\Phi^g(m, \frac{k}{N})$ also Gaussian with zero mean and satisfies:

$$\mathrm{Var}\left(\Re\{V_\Phi^g(m, \frac{k}{N})\}\right) = \mathrm{Var}\left(\Im\{V_\Phi^g(m, \frac{k}{N})\}\right) = \sigma^2\|g\|_2^2,$$

- HT in that context can be viewed as thresholding the coefficients:

$$\overline{V}_{\tilde{f}}^g(m, \frac{k}{N}) = \begin{cases} V_{\tilde{f}}^g(m, \frac{k}{N}), \text{ if } |V_{\tilde{f}}^g(m, \frac{k}{N})| \geq 3\sigma\|g\|_2 \\ 0 \quad \text{otherwise,} \end{cases}$$

$\frac{\Re\{V_\Phi^g(m, \frac{k}{N})\}^2 + \Im\{V_\Phi^g(m, \frac{k}{N})\}^2}{\sigma^2\|g\|_2^2}$ is $\chi_2$ distributed with two degrees of freedom, a threshold of 9 is associated with a level of significance 0.01

- Reconstruction: replace $V_f^g(m, \frac{k}{N})$ by $\overline{V}_{\tilde{f}}^g(m, \frac{k}{N})$ in reconstruction formula.

# HT in the Context of MCS

- **Ridges extraction**: $\phi_l'$ approximated by some $\psi_l$ following an existing approach [2].
- **STFT thresholding**: Select the coefficients above the threshold provided by HT in the vicinity of the detected ridge.

$$\eta_{l,m}^{[1]} = \operatorname*{argmax}_k \left\{ \frac{k}{N} < \psi_l(m), |V_{\tilde{f}}^g(m, \frac{k}{N})| < 3\sqrt{2}\sigma \|g\|_{2,m} \right\}$$
$$\eta_{l,m}^{[2]} = \operatorname*{argmin}_k \left\{ \frac{k}{N} > \psi_l(m), |V_{\tilde{f}}^g(m, \frac{k}{N})| < 3\sqrt{2}\sigma \|g\|_{2,m} \right\},$$

and then define: $J_{l,m} := [N\psi_l(m) - \eta_{l,m}^{[1]}, N\psi_l(m) + \eta_{l,m}^{[2]}]$.

- **Mode reconstruction**:

$$f_l[n] = \sum_{m \in \mathbb{Z}} \sum_{k \in J_{l,m}} V_{\tilde{f}}^g(m, \frac{k}{N}) g[n-m] \frac{e^{i2\pi \frac{k(n-m)}{N}}}{N}.$$

[2] R. Carmona, W. Hwang, and B. Torrésani, *Characterization of signals by the ridges of their wavelet transforms*, IEEE Transactions on Signal Processing, vol. 45, no. 10, pp. 2586-2590, Oct 1997.

# Illustration of HT on a Mono Component Signal

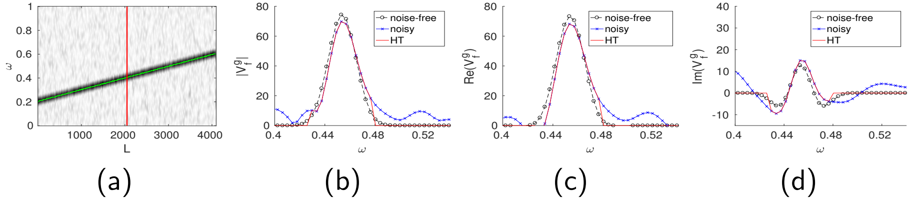



Figure: (a): modulus of the STFT of a noisy linear chirp (SNR = 0 dB) along with its ridge detection (green line); (b): modulus of STFTs (noise-free, noisy and hard-thresholded) at $m = L/2$ (red line); (c): the real part of STFT displayed in (b); (d): same as (c) but for the imaginary part.

Several drawbacks of HT:

- Potential shift of the local maxima compared with noise-free case
- Lack of symmetry and regularity (the noise-free transform has the regularity of $g$)

# Local Linear Chirp Mode Approximation

- Second-order phase approximation close to the $l$th ridge:

$$V_{f_l}^g(m, \frac{k}{N}) \approx \left| V_{f_l}^g(m, \frac{k}{N}) \right| e^{i2\pi\Psi_l(m, \frac{k}{N})},$$

when $g$ is the Gaussian window $\sigma_s^{-\frac{1}{2}} e^{-\pi \frac{x^2}{\sigma_s^2}}$, with

$$\Psi_l(m, \frac{k}{N}) = -\frac{\phi_l''(m)}{2r(m)^2} \left( \frac{k}{N} - \phi_l'(m) \right)^2 - \frac{\theta(m)}{4\pi} + \phi_l(m)$$

$$\left| V_{f_l}^g(m, \frac{k}{N}) \right| = A_l(m) \sigma_s^{-\frac{1}{2}} r(m)^{-\frac{1}{2}} e^{-\frac{\pi\left(\frac{k}{N} - \phi_l'(m)\right)^2}{\sigma_s^2 r(m)^2}}$$

with $\theta(m) = \arctan(-\phi''(m)\sigma_s^2)$ and $r(m) = \sigma_s^{-2} \left(1 + \phi''(m)^2 \sigma_s^4\right)^{\frac{1}{2}}$

- $\left| V_{f_l}^g(m, \frac{k}{N}) \right|_k$, $\Re\left( V_{f_l}^g(m, \frac{k}{N}) \right)_k$ and $\Im\left( V_{f_l}^g(m, \frac{k}{N}) \right)_k$ even functions in the vicinity of the $l$th ridge, and all reach an extremum at $\frac{k}{N} = \phi_l'(m)$.

# Novel Algorithm for MCS Denoising

- **Compensate for potential shift:** shift the imaginary and real parts of STFT so that their extrema correspond to $\psi_l(m)$.
- **Compensate for lack of symmetry:** enforce the symmetry of the real and imaginary part of STFT with respect to $\psi_l(m)$, by defining:

$$\eta = \max(\eta_{l,m}^{[1]}, \eta_{l,m}^{[2]}) \text{ and then}$$

$$\forall \nu \in [0,\eta] : \frac{\left(V_{\tilde{f}}^{g}(m,\psi_l(m)+\frac{\nu}{N}) + V_{\tilde{f}}^{g}(m,\psi_l(m)-\frac{\nu}{N})\right)}{2} \to X$$

$$X \to V_{\tilde{f}}^{g}(m,\psi_l(m) \pm \frac{\nu}{N})$$

- **Compensate for lack of regularity**: smooth the just obtained real and imaginary parts using piecewise cubic Hermite interpolation.

# Illustration on a Simple Example

Process called *SSR-HT*:
*Shifted-Symmetrized-Regularized Hard-Thresholding*.

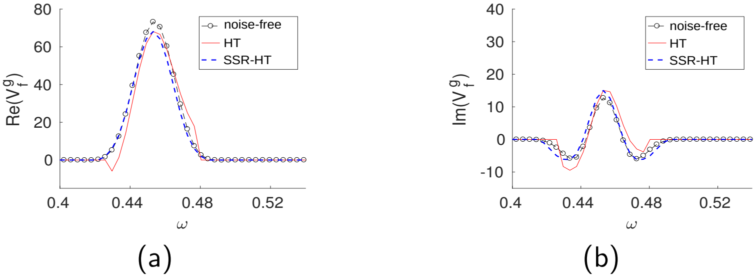



Figure: Illustration of SSR-HT on the real and imaginary parts of the STFT displayed in the previous Figure

# Numerical Experiments: a Simulated MCS

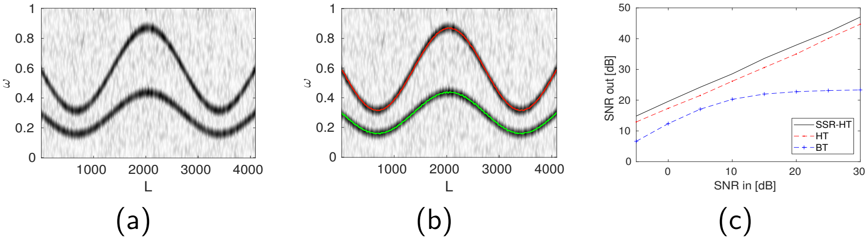



Figure: (a): STFT of the simulated MCS; (b): detected ridges of the signal displayed in (a); (c): denoising performance comparison of the studied methods (BT (Block-Thresholding [3]), HT (Hard-Thresholding), SSR-HT (our new method))

[3] G. Yu, S. Mallat, and E. Bacry, *Audio denoising by time-frequency block thresholding*, IEEE Transactions on Signal Processing, vol. 56, no. 5, pp. 1830-1839, May 2008.

# Numerical Experiments: Reals Noisy PCGs

- 16 PCGs were recorded with a sampling rate 1KHz by a cardiac microphone MLT210 on three healthy volunteers (SiSEC 2016 database [4]).
- Artificially contaminated by different real interference (radio, cough, pseudo-periodic breathing noise, etc.)

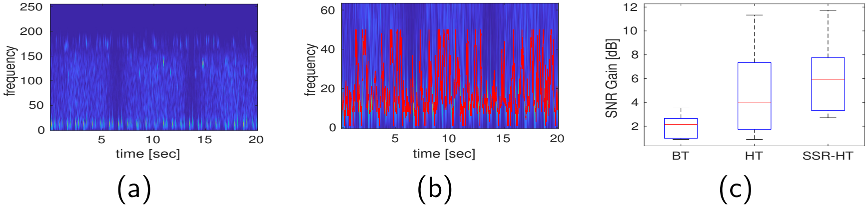



Figure: (a): STFT of a noisy PCG; (b): ridge detection on a low-pass filtered version with a cutoff frequency 64Hz of signal in (a) (assuming number of modes $K = 1$); (c): denoising performance comparison (using boxplot) .

[4] A. Liutkus, et *al.*, *The 2016 signal separation evaluation campaign*, in Int. Conf. on Latent Variable Anal. and Sig. Separation, 2017, pp. 323-332.

# Conclusions and Perspectives

- We have detailed a new method for the denoising of multicomponent signals based on an improvement of hard-thresholding.
- This approach can be extended to the denoising of STFT computed using any symmetric and regular window.
- Further works should be carried out to adapt the proposed new technique to noise other than Gaussian.

# Thank you for your attention!