Feature LMS Algorithms

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Presentation Outline

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Outline

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Motivations

- When we have some *a priori* knowledge about the unknown system $\mathbf{w}_o$
  - We can exploit it for accelerating the convergence rate
- When we want to obtain an estimate of the unkown system $\mathbf{w}_o$ such that a determined characteristic for the estimate is desirable
  - Lowpass, highpass, linear phase

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Proposal

- Feature LMS (F-LMS) algorithms ⇒ impose some structure on the adaptive filter's coefficients ⇒ exploit hidden sparsity in system, such as sparsity in linear combination of coefficients
- In this paper, we present the F-LMS algorithm for:
  - Unknown systems with lowpass narrowband spectrum
  - Unknown systems with highpass narrowband spectrum

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F-LMS Algorithm: Problem and Solution

- Problem:

$$\xi_{\text{F-LMS}}(k) = \underbrace{\frac{1}{2}|e(k)|^2}_{\text{standard LMS term}} + \underbrace{\alpha\mathcal{P}\left(\mathbf{F}(k)\mathbf{w}(k)\right)}_{\text{feature-inducing term}},$$

where $\mathcal{P}(\cdot)$ is the sparsity promoting penalty function and $\mathbf{F}(k)$ is the feature matrix that takes the unknown system to a sparse vector.

- For example, choose function $\mathcal{P}$ to be the l_1 norm and the feature matrix $\mathbf{F}(k)$ to be time-invariant $\mathbf{F}$

$$\xi_{\text{F-LMS}}(k) = \frac{1}{2}|e(k)|^2 + \alpha\|\mathbf{F}\mathbf{w}(k)\|_1$$

- Solution:

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu e(k)\mathbf{x}(k) - \mu\alpha\mathbf{p}(k),$$

where $\mathbf{p}(k) \in \mathbb{R}^{N+1}$ is the gradient of function $\|\mathbf{F}\mathbf{w}(k)\|_1$.

Example I: F-LMS Algorithm for Lowpass Systems

- Unknown system has lowpass narrowband spectrum $\Rightarrow$ its impulse response is smooth $\Rightarrow$ the difference between adjacent coefficients is small
- Choose $\mathbf{F} \in \mathbb{R}^{N \times (N+1)}$ as

$$\mathbf{F} = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \\ 0 & 1 & -1 & \cdots & 0 \\ \vdots & & \ddots & \ddots & \\ 0 & 0 & \cdots & 1 & -1 \end{bmatrix} \Rightarrow \mathbf{Fw}(k) \text{ is a sparse vector}$$

- Therefore, $\mathbf{p}(k) = [p_0(k) \;\cdots\; p_N(k)]^T$ is given by

$$\begin{cases} p_i(k) = \text{sgn}(w_0(k) - w_1(k)) & \text{if } i = 0, \\ p_i(k) = -\text{sgn}(w_{i-1}(k) - w_i(k)) + \text{sgn}(w_i(k) - w_{i+1}(k)) & \text{if } i = 1, \cdots, N-1, \\ p_i(k) = -\text{sgn}(w_{N-1}(k) - w_N(k)) & \text{if } i = N. \end{cases}$$

Example II: F-LMS Algorithm for Highpass Systems

- Unknown system has highpass narrowband spectrum $\Rightarrow$ adjacent coefficients have similar absolute values, but with opposite signs
- Choose $\mathbf{F} \in \mathbb{R}^{N\times(N+1)}$ as

$$\mathbf{F} = \begin{bmatrix} 1 & 1 & 0 & \cdots & 0 \\ 0 & 1 & 1 & \cdots & 0 \\ \vdots & & \ddots & \ddots & \\ 0 & 0 & \cdots & 1 & 1 \end{bmatrix} \Rightarrow \mathbf{Fw}(k) \text{ is a sparse vector}$$

- Therefore, $\mathbf{p}(k) = [p_0(k) \ \cdots \ p_N(k)]^T$ is given by

$$\begin{cases} p_i(k) = \text{sgn}(w_0(k) + w_1(k)) & \text{if } i = 0, \\ p_i(k) = \text{sgn}(w_{i-1}(k) + w_i(k)) + \text{sgn}(w_i(k) + w_{i+1}(k)) & \text{if } i = 1, \cdots, N-1, \\ p_i(k) = \text{sgn}(w_{N-1}(k) + w_N(k)) & \text{if } i = N. \end{cases}$$

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More Examples for Feature Matrix

- When unknown system is the result of upsampling a lowpass system by a factor of L (e.g., $L = 2$)

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & -1 & 0 & \cdots & 0 \\ 0 & 1 & 0 & -1 & \cdots & 0 \\ \vdots & & \ddots & \ddots & \ddots & \\ 0 & 0 & \cdots & 1 & 0 & -1 \end{bmatrix}.$$

- When unknown system is the result of interpolating a highpass system by a factor L (e.g., $L = 2$)

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 1 & 0 & 1 & \cdots & 0 \\ \vdots & & \ddots & \ddots & \ddots & \\ 0 & 0 & \cdots & 1 & 0 & 1 \end{bmatrix}.$$

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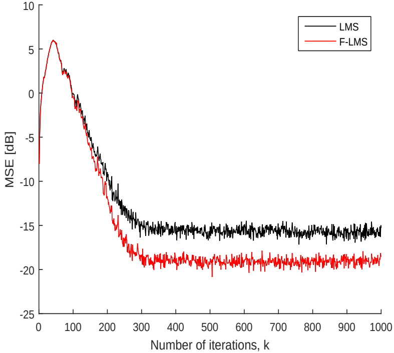
Scenario: System Identification

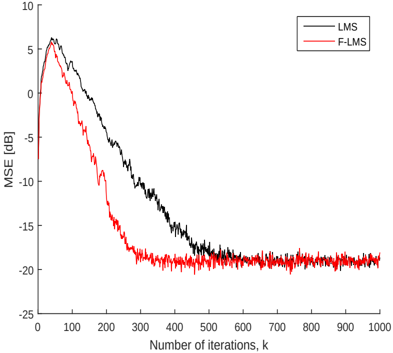
- Algorithms tested: LMS and F-LMS algorithms
- Input signal: $\mathbf{x} \sim \mathcal{N}(0, 1)$
- Filter order: $N = 39$, i.e., 40 coefficients
- $\mathbf{w}(0) = [0, \cdots, 0]^T$
- $\alpha = 0.05$
- SNR: 20 dB
- Unknown lowpass system: $\mathbf{w}_{o,l} = [0.4, 0.4, \cdots, 0.4]^T$
- Unknown highpass system: $\mathbf{w}_{o,h} = [0.4, -0.4, 0.4 \cdots, -0.4]^T$

F-LMS Algorithm Identifying Unknown System with Lowpass Spectrum

- Unknown lowpass system: $\mathbf{w}_{o,l} = [0.4, 0.4, \cdots, 0.4]^T$



(a) $\mu_{\text{LMS}} = \mu_{\text{F-LMS}} = 0.03$



(b) $\mu_{\text{LMS}} = 0.01, \ \mu_{\text{F-LMS}} = 0.03$

Figure: Learning (MSE) curves

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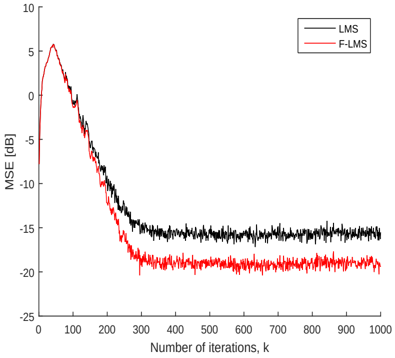
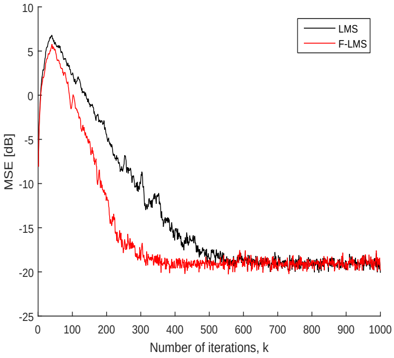
F-LMS Algorithm Identifying Unknown System with Highpass Spectrum

- Unknown highpass system: $\mathbf{w}_{o,h} = [0.4, -0.4, 0.4 \cdots, -0.4]^T$



(a) $\mu_{\text{LMS}} = \mu_{\text{F-LMS}} = 0.03$



(b) $\mu_{\text{LMS}} = 0.01, \ \mu_{\text{F-LMS}} = 0.03$

Figure: Learning (MSE) curves

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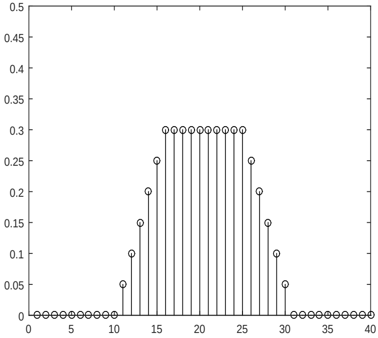
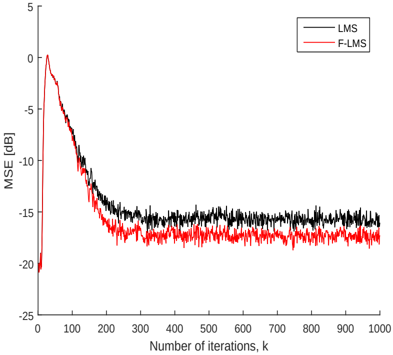
F-LMS Algorithm Identifying Unknown Block Sparse System

- Block sparse system with lowpass narrowband spectrum



(a) Unknown system

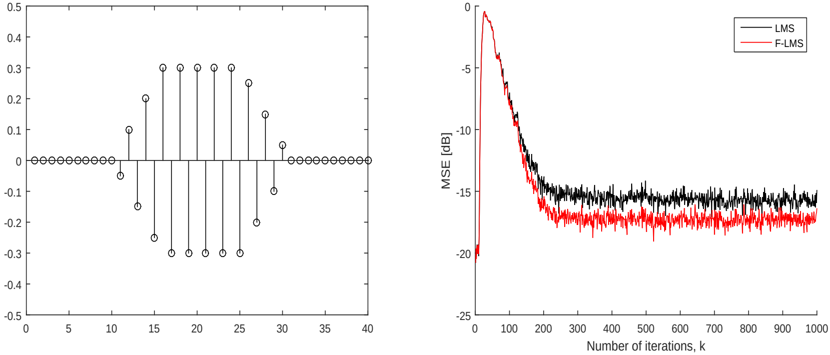


(b) $\mu_{\text{LMS}} = \mu_{\text{F-LMS}} = 0.03$

Figure: Learning (MSE) curves

F-LMS Algorithm Identifying Unknown Block Sparse System

- Block sparse system with highpass narrowband spectrum



(a) Unknown system

(b) $\mu_{\text{LMS}} = \mu_{\text{F-LMS}} = 0.03$

Figure: Learning (MSE) curves

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Conclusions

- In this presentation:
  - We have proposed a family of algorithms called feature LMS algorithm
    - We have presented some examples of the F-LMS algorithms for exploiting the lowpass and highpass characteristics of unknown systems
    - Some other characteristics can be exploited (e.g., linear phase)
  - The F-LMS algorithms have some advantages such as higher convergence rate or lower steady-state MSE
  - The computational complexity of the proposed F-LMS algorithms are close to that of the LMS algorithm

Thank You!