AASP-P6.10

JOINT SOURCE AND SENSOR PLACEMENT FOR SOUND FIELD CONTROL BASED ON EMPIRICAL INTERPOLATION METHOD

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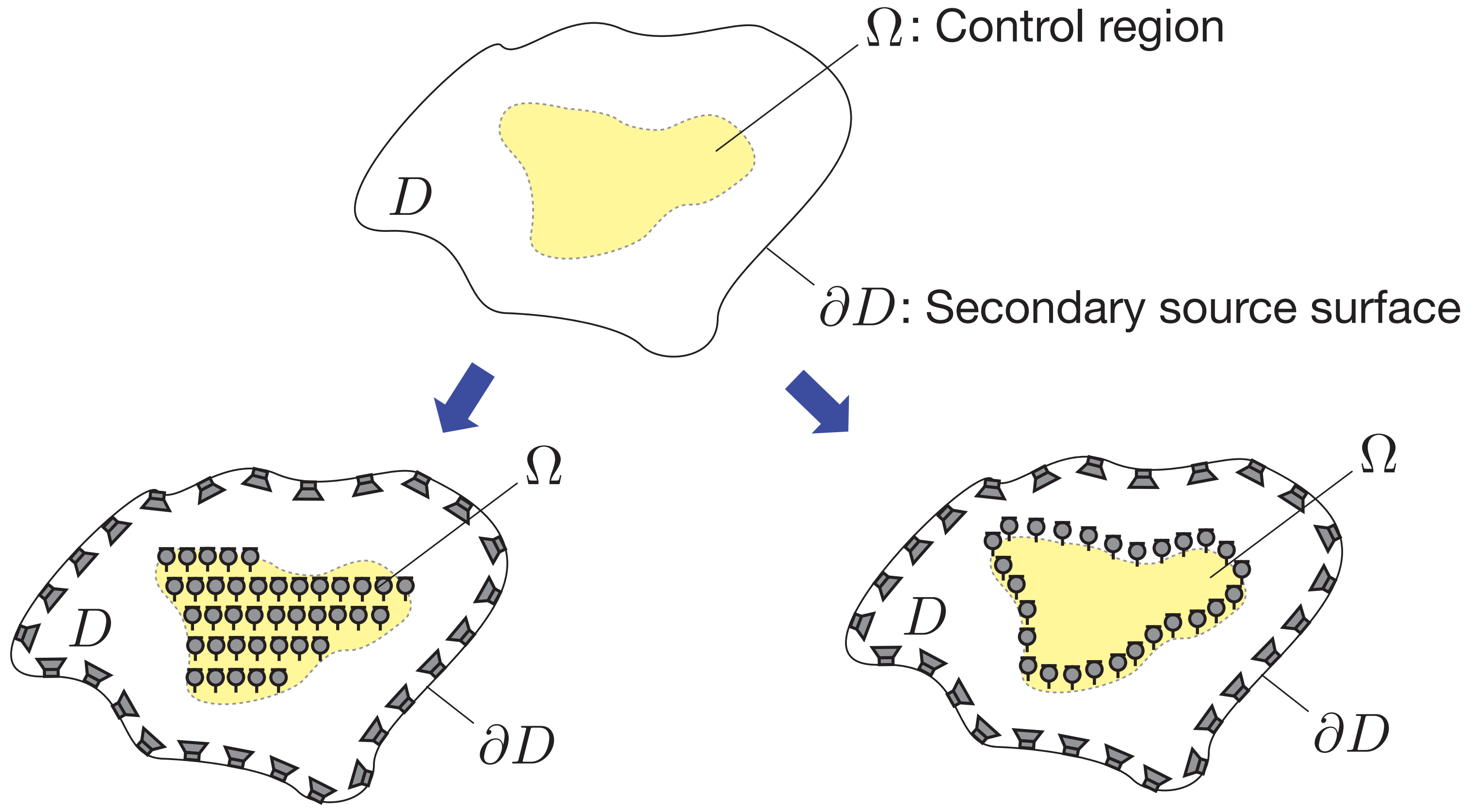
Abstract

▶ Loudspeakers and control points (microphones) placement for sound field control

- Sound field control: Synthesizing desired sound field inside region of interest with inverse filters between multiple secondary sources (loudspeakers) and sensors (control points/microphones)

➡ *Spatial audio and noise cancellation systems*

- Positions of loudspeakers and control points/microphones have significant effect on control accuracy and filter stability



Dense sampling inside Ω
- Too many loudspeakers and control points
- Unstable inverse filter due to excessively high correlation of transfer functions

Sampling only on boundary of Ω
- Significant degradation of control accuracy at several frequencies (forbidden frequency problem)

▶ What is the best placement of loudspeakers and control points?

- Current methods for source placement
  - Method based on Gram-Schmidt orthogonalization [Asano+ 1999]
  - Sparse-approximation-based method [Khalilian+ 2016]

➡ *Most algorithms depend on desired sound field*

- Current methods for sensor placement
  - Avoid forbidden frequency problem by introducing rigid baffle / directional microphones / double layer array of microphones [Poletti 2005, Betlehem+ 2005, Koyama+ 2016]

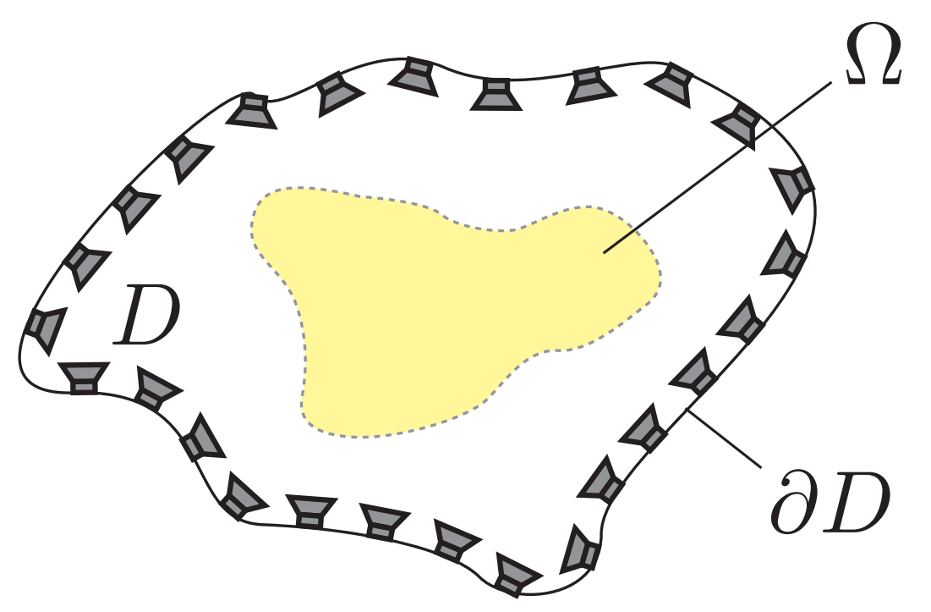
➡ *Most methods can be basically applied to simple array geometry*

➡ *Source and sensor placements are independently determined*

Proposed method: Source and sensor placements are jointly determined for arbitrary geometries of control region and secondary source surface

Problem Statement

- Synthesized sound field by L loudspeakers



$$u_{\text{syn}}(\mathbf{r},\omega)=\sum_{l=1}^{L}d_l(\omega)g_l(\mathbf{r},\omega)$$
Transfer function
Driving signal

- Minimize square error between synthesized and desired sound fields

$$\underset{d_l(\omega)}{\text{minimize}}\,\mathcal{J}=\int_{\mathbf{r}\in\Omega}\left|\sum_{l=1}^{L}d_l(\omega)g_l(\mathbf{r})-u_{\text{des}}(\mathbf{r},\omega)\right|^2 d\mathbf{r}$$
Desired sound field

➡ *Difficult to directly solve due to domain integral*

- Linear equation by discretizing Ω

$$\mathbf{u}^{\text{des}}=\mathbf{G}\mathbf{d}$$ ➡ Derivation of driving signals: $$\mathbf{d}=\mathbf{G}^{\dagger}\mathbf{u}^{\text{des}}$$
Transfer function matrix
(Pseudo) inverse

Choose the best loudspeaker and control-point locations with respect to control accuracy and filter stability from candidate locations

Source and Sensor Placement Based on EIM

▶ Idea

- Empirical Interpolation Method (EIM):
  - *Proposed in the context of numerical analysis of PDE* [Barrault+ 2004]
  - *Given functional space $\mathcal{V}$ defined on Ω, choose the best interpolation function and sampling points on Ω to approximate any function $v\in\mathcal{V}$ by greedy algorithm*
- Apply EIM to source and sensor placement problem
  - *Regard transfer function of each loudspeaker as interpolation function and control points as sampling points, and choose the best source and sensor locations to approximate any transfer function between candidate locations*
  - *Greedy algorithm for source / sensor locations using transfer functions between candidate locations*

Input: Candiate locations of loudspeakers $\mathbf{r}_l$ $(l\in\{1,\dots,L\})$ and control points $\mathbf{r}_m$ $(m\in\{1,\dots,M\})$, transfer function matrix $\mathbf{G}\in\mathbb{C}^{M\times L}$, target error tolerance ϵ_{tol}
Output: Set of indexes of loudspeakers and control points

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1: Set $Q=1$
2: while $\epsilon>\epsilon_{\text{tol}}$ do
3:     Choose loudspeaker index
           $l_Q=\underset{l=1,\dots,L}{\arg\max}\left\|\mathbf{G}_{\cdot,l}-I_{Q-1}(\mathbf{G}_{\mathbf{m}_{Q-1},l})\right\|_{\infty}$
4:     Choose control-point index
           $m_Q=\underset{m=1,\dots,M}{\arg\max}\left|\mathbf{G}_{m,l_Q}-\left(I_{Q-1}(\mathbf{G}_{\mathbf{m}_{Q-1},l_Q})\right)_m\right|$
5:     Calculate error
           $\epsilon=\max_{l=1,\dots,L}\left\|\mathbf{G}_{\cdot,l}-I_{Q-1}(\mathbf{G}_{\mathbf{m}_{Q-1},l})\right\|_2$
6:     Set $Q=Q+1$
7: end while
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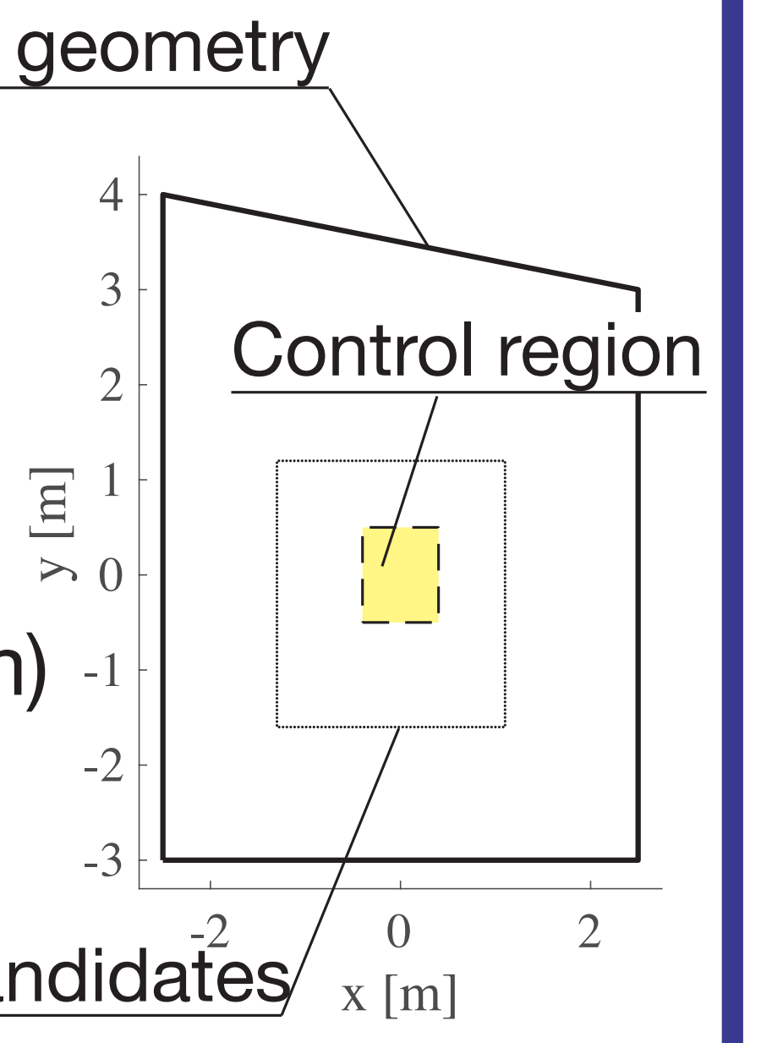
Interpolation of $\mathbf{G}$

Approximation of $\mathbf{G}$ below ϵ_{tol} and inverse filter stability are guaranteed

Experiments

▶ Simulation experiments in 2D sound field

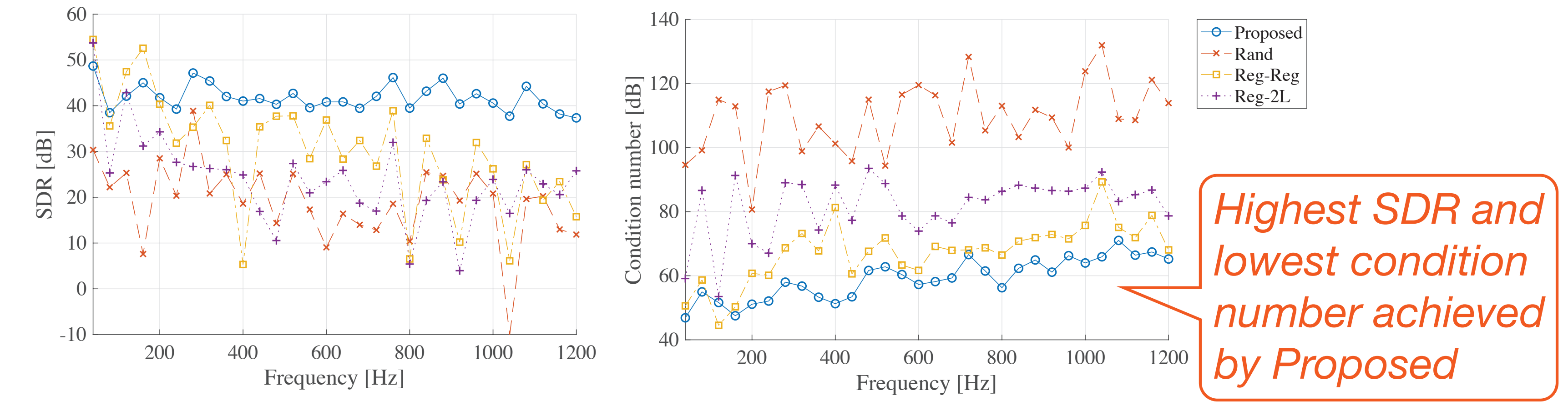


- Numerical simulation of transfer functions by using finite element method (absorption ratio: 0.10)
- Loudspeaker candidates: Boundary of rectangular region 2.4x2.8m^2 (256 points)
- Control-point candidates: Rectangular region 0.8x1.0m^2 (discretized every 0.04 m)
- Methods to be compared
  - Proposed method (**Proposed**)
  - Random placement (**Random**)
  - Regular placement (**Reg-Reg**)
  - Regular and two-layer placement (**Reg-2L**)
- Evaluation:

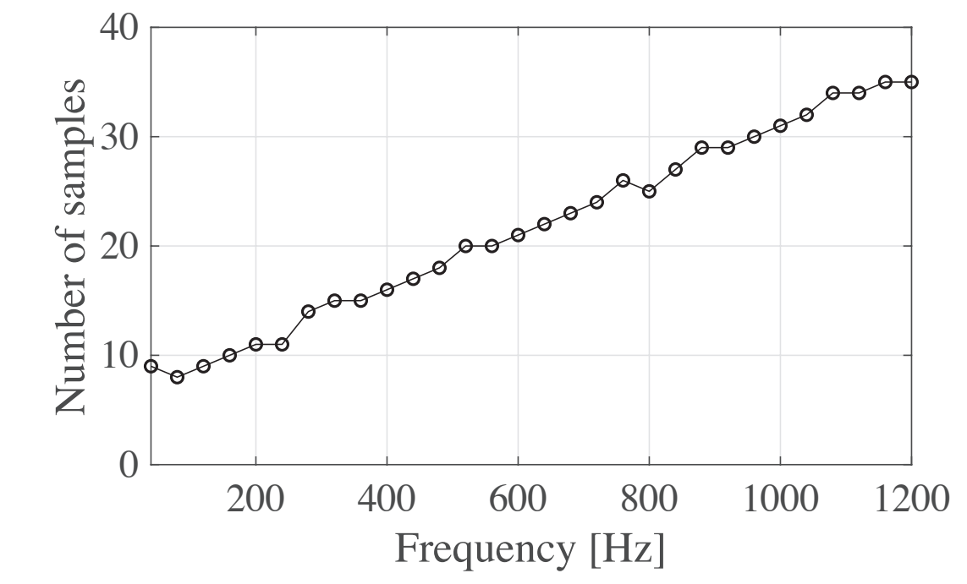
Signal-to-distortion ratio (SDR):
$$\mathrm{SDR}(\omega)=10\log_{10}\frac{\int_{\Omega}|u_{\text{des}}(\mathbf{r},\omega)|^2 d\mathbf{r}}{\int_{\Omega}|u_{\text{syn}}(\mathbf{r},\omega)-u_{\text{des}}(\mathbf{r},\omega)|^2 d\mathbf{r}}$$

Condition number in dB:
$$\kappa(\mathbf{G})=10\log_{10}\frac{\sigma^2_{\text{max}}(\mathbf{G})}{\sigma^2_{\text{min}}(\mathbf{G})}$$

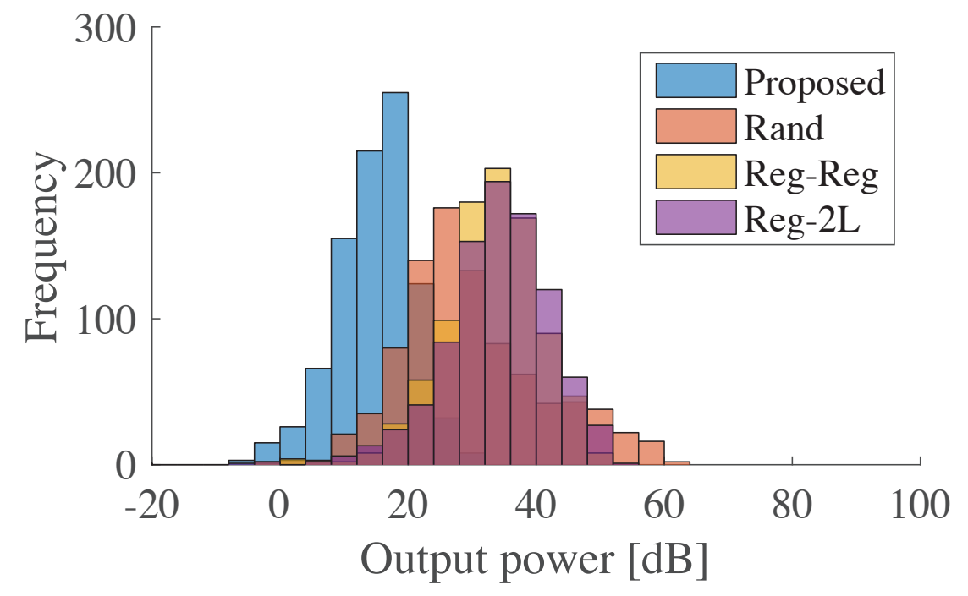
▶ Single-frequency case



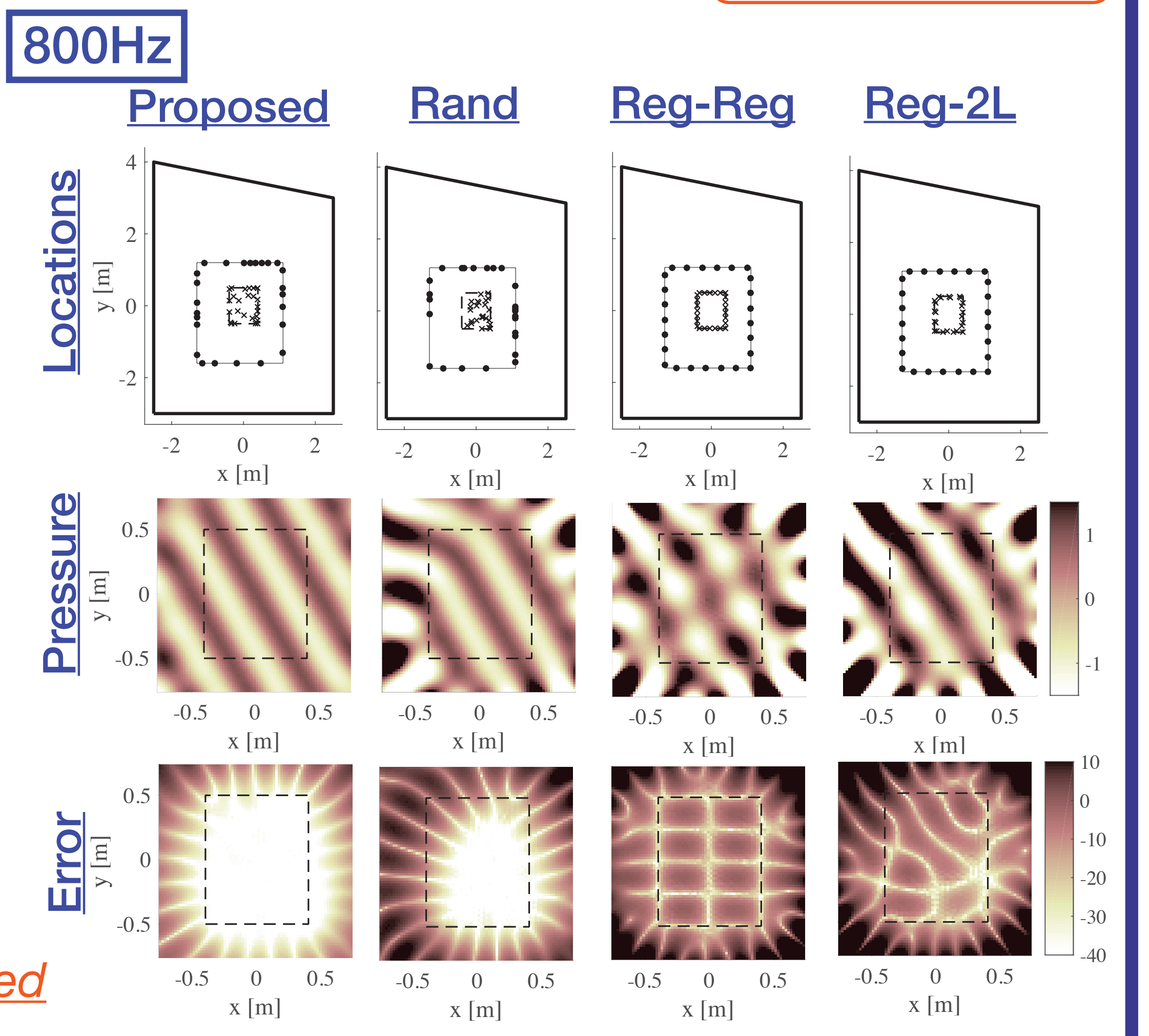
Highest SDR and lowest condition number achieved by Proposed



Number of sources/ sensors increases with frequency



Lower loudspeaker output power is achieved by Proposed



▶ Broadband case

- Locations are chosen by using transfer functions from 40 to 1200 Hz
- Gaussian noise is added to transfer function matrix
- Tikhonov regularization with optimal regularization parameter