An algorithm for multi subject fMRI analysis based on the SVD and penalized rank-1 matrix approximation

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Introduction

- Data-driven methods have been successfully applied to multi-subject fMRI data analysis using temporally or spatially concatenated datasets.
- Temporal concatenation allows for the extraction of group-level spatial activation maps.
- Spatial concatenation leads to the extraction of group-level temporal dynamics.
- Aim of this paper: separation of the joint information (group-level spatial maps) from the subject-specific information.

Background

Consider p fMRI datasets denoted by $\mathbf{Y}_i \in \mathbb{R}^{n\times N}$, $i\in[1,p]$, data driven methods with sparsity constraints aim to decompose $\mathbf{Y}_i$ as:

$$\mathbf{Y}_i = \mathbf{D}_i\mathbf{X}+\mathbf{E};\ \text{with } \mathbf{D}_i\in\mathbb{R}^{n\times k}, \mathbf{X}\in\mathbb{R}^{k\times N} \quad (1)$$

with l_2 normalized columns of $\mathbf{D}_i$ and a sparse coefficient matrix $\mathbf{X}$. With temporally concatenated datasets $\mathbf{Y}\in\mathbb{R}^{np\times N}$, a factor model decomposition is performed by solving:

$$\min_{\mathbf{D},\mathbf{X}} ||\mathbf{Y}-\mathbf{D}\mathbf{X}||_F^2+\lambda\sum_{q=1}^{N}\|\mathbf{x}_q\|_1 \text{ s.t. } \forall i,l,\ \|\mathbf{d}_{il}\|_2\le 1 \quad (2)$$

where $\mathbf{D}\in\mathbb{R}^{np\times k}$, $\mathbf{X}\in\mathbb{R}^{k\times N}$ and $\mathbf{x}_q$ is the q^{th} column of $\mathbf{X}$. The resulting $\mathbf{D}$ and $\mathbf{X}$ matrices contain k dense temporal dynamics and k sparse group level spatial maps (SM) respectively.
Learning SMs using this formulation lacks the ability to distinguish group-level (joint) SMs from subspecific ones.

The Proposed Algorithm

Let $\mathbf{Y}\in\mathbb{R}^{np\times N}$ be the temporally concatenated dataset, the proposed algorithm decomposes it in two steps:

1. Using SVD, decompose $\mathbf{Y}$ into sum of three low-rank matrices; $\mathbf{Y}=\mathbf{J}+\mathbf{I}+\mathbf{E}$.
2. Refine $\mathbf{J}$ and $\mathbf{I}$ into k dense temporal dynamics and sparse spatial maps.

The Proposed Algorithm contd.

To carryout the second stage, we aim to minimize;

$$\min \frac{1}{2}\|\mathbf{G}_0-\mathbf{A}\mathbf{B}\|_F^2+\sum_{m=1}^{k}\left(\alpha_1\|\mathbf{b}^m\|_1+\alpha_2\,\mathbf{a}_m^\top\,\boldsymbol{\Omega}\,\mathbf{a}_m\right) \quad (3)$$

where $\mathbf{a}_m$ and $\mathbf{b}^m$ are the columns and rows from matrices $\mathbf{A}\in\mathbb{R}^{n\times k}$ and $\mathbf{B}\in\mathbb{R}^{k\times N}$ respectively, $\boldsymbol{\Omega}\in\mathbb{R}^{n\times n}$ [2].
(3) can be approximately minimized via k penalized rank-1 matrix approximations via matrix deflation, i.e, by replacing $\mathbf{G}_0$ in (3) by the residual matrix $\mathbf{G}_m=\mathbf{G}_{m-1}-\mathbf{a}_m\mathbf{b}^m$ with $m=[1,\cdots,k]$. Thus, minimizing (3) is equivalent to minimizing

$$\min \frac{1}{2}\|\mathbf{G}_{m-1}-\mathbf{a}_m\mathbf{b}^m\|_F^2+\alpha_1\|\mathbf{b}^m\|_1 +\alpha_2\,\mathbf{a}_m^\top\,\boldsymbol{\Omega}\,\mathbf{a}_m;\ \text{ s.t. } \|\mathbf{a}_m\|_2=1 \quad (4)$$

Using alternating optimization framework, the solution to (4) is found by alternating between

$$\hat{\mathbf{a}}_m = (\mathbf{I}\,\|\mathbf{b}^m\|_2^2+\alpha_2\,\boldsymbol{\Omega})^{-1}\,\mathbf{G}_{m-1}\mathbf{b}^{m\top} \qquad \hat{\mathbf{a}}_m=\hat{\mathbf{a}}_m/\|\hat{\mathbf{a}}_m\|_2 \quad (5)$$

$$\hat{\mathbf{b}}^m=\text{sgn}(\mathbf{a}_k^\top\mathbf{G}_{m-1})\odot\max(0,|\mathbf{a}_k^\top\mathbf{G}_{m-1}|-\alpha_1\mathbf{1}_N) \quad (6)$$

Algorithm Overview

Algorithm 1: Proposed Data Driven Method

Input: $\mathbf{Y}$, p, r_J, r_I, k_J, k_I, α_1, α_2, $noIt$
Stage 1: $\mathbf{J}\leftarrow\mathbf{0}$, $\mathbf{I}\leftarrow\mathbf{0}$
for $it=1:noIt$ **do**
  Compute $\mathbf{X}=\mathbf{Y}-\mathbf{I}$,
  Find $\mathbf{J}$ as best r_J-rank approx. of $\mathbf{X}$ using SVD.
  for $i=1:p$ **do**
    Compute $\mathbf{Z}_i=\mathbf{Y}_i-\mathbf{J}_i$,
    Find $\mathbf{I}_i$ as best r_I-rank approx. of $\mathbf{Z}_i$ using SVD.
Stage 2:
Use Algorithm 2 to refine $\mathbf{J}$ into k_J-rank matrix pairs as $\mathbf{J}=\mathbf{A}^J\mathbf{B}^J$,
for $i=1:p$ **do**
  Use Algorithm 2 to refine $\mathbf{I}_i$ into k_I-rank matrix pairs as $\mathbf{I}_i=\mathbf{A}_i^I\mathbf{B}_i^I$,
Output: $\mathbf{J}$, $\mathbf{I}$, $\mathbf{A}^J$, $\mathbf{B}^J$, $\mathbf{A}_i^I$, $\mathbf{B}_i^I$

Algorithm Overview

Algorithm 2: Refinement Algorithm

Input: $\mathbf{G}_0$, k, $noIt$, α_1, α_2
Initialize $\mathbf{A}$ and $\mathbf{B}$ from $\mathbf{G}_0$,
for $m=1:k$ **do**
  for $it=1:noIt$ **do**
    use (5) to get $\mathbf{a}_m$,
    use (6) to get $\mathbf{b}^m$,
  Compute $\mathbf{G}_m=\mathbf{G}_{m-1}-\mathbf{a}_m\,\mathbf{b}^m$,
  Store the pair as $\mathbf{A}(:,m)=\mathbf{a}_m$ and $\mathbf{B}(m,:)=\mathbf{b}^m$,
Output: $\mathbf{A}$, $\mathbf{B}$

Simulation Results

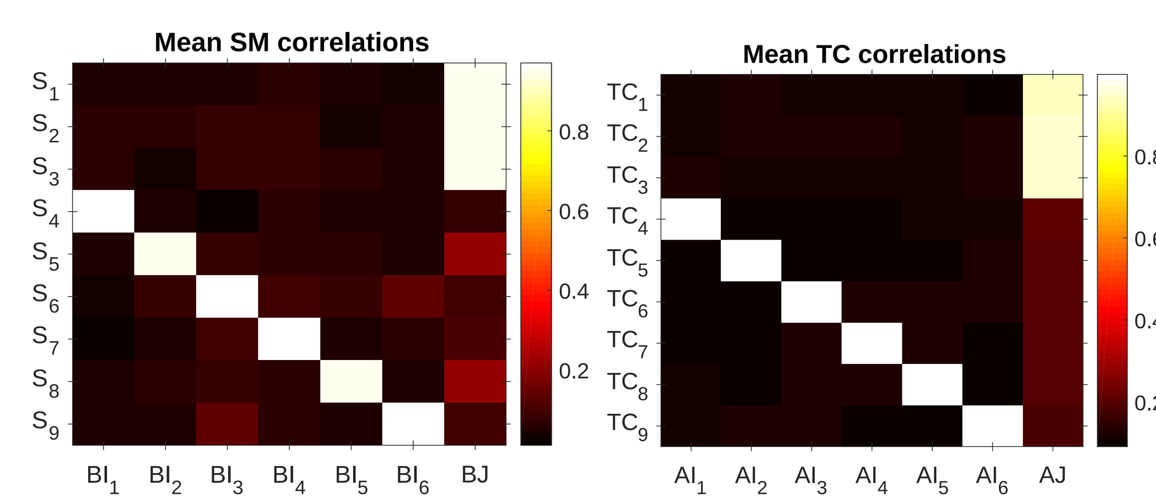
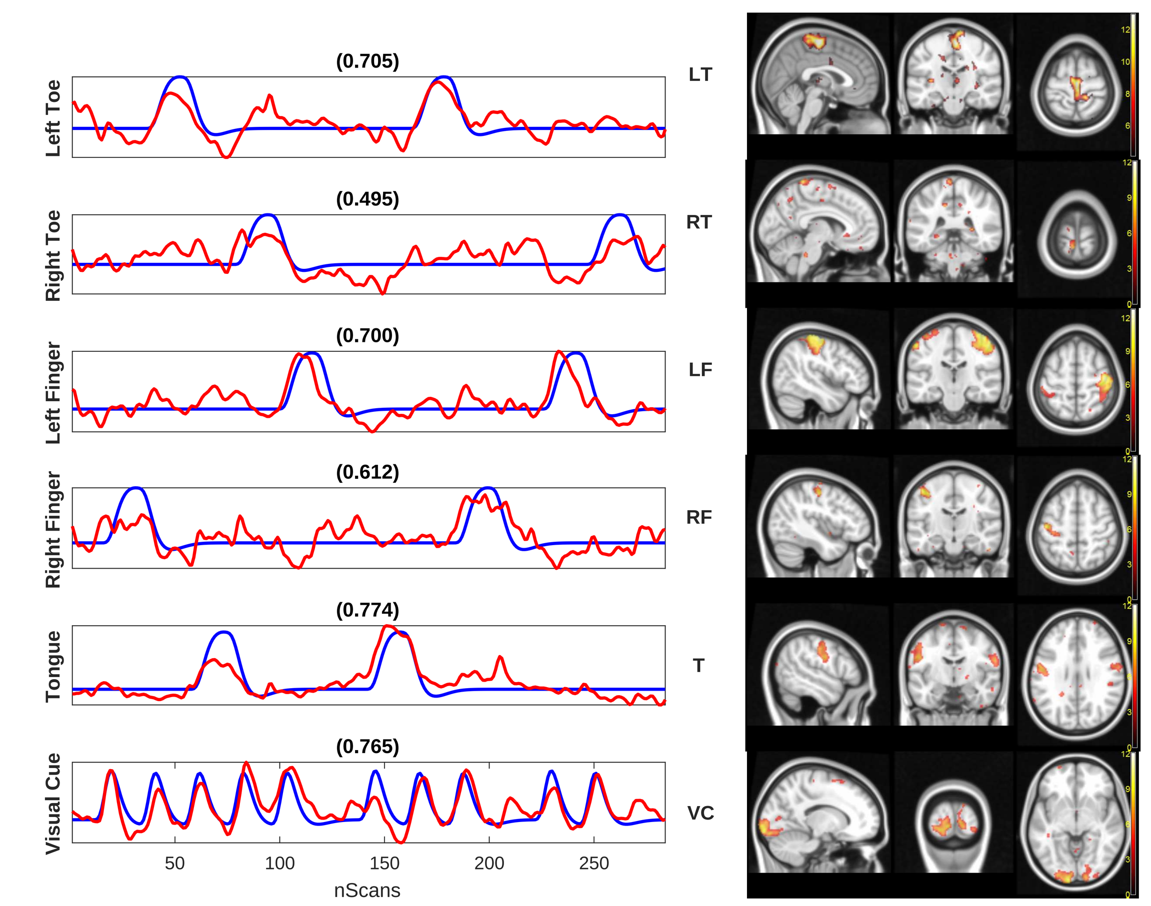


Figure 1: The mean Ground Truth (GT) SM and TC correlation coefficients over 100 trials with respect to all recovered $\mathbf{A}^J$, $\mathbf{B}^J$, $\mathbf{A}_i^I$, $\mathbf{B}_i^I$ matrices. SNR = 0 dB.

Table 1: Mean and std dev (Pearson correlation) of most correlated TCs and SMs w.r.t. ground truth as recovered by the proposed algorithm and CODL [1]. over 100 trials.

SNR dB	Algorithm	TCs		SMs	
		Mean	STD	Mean	STD
-10	Proposed	**0.99**	**0.01**	**0.89**	**0.05**
	CODL	0.95	0.03	0.79	0.05
-15	Proposed	**0.98**	**0.01**	**0.85**	**0.06**
	CODL	0.87	0.04	0.58	0.21

Real fMRI Results



(a) (b)

Figure 2: a) Most correlated average TCs from $\mathbf{A}^J$ (*red*) with their respective PTCs (*blue*) recovered by the proposed algorithm. The corresponding correlation coefficients are given above each TC plot. b) Respective activation maps from $\mathbf{B}^J$.

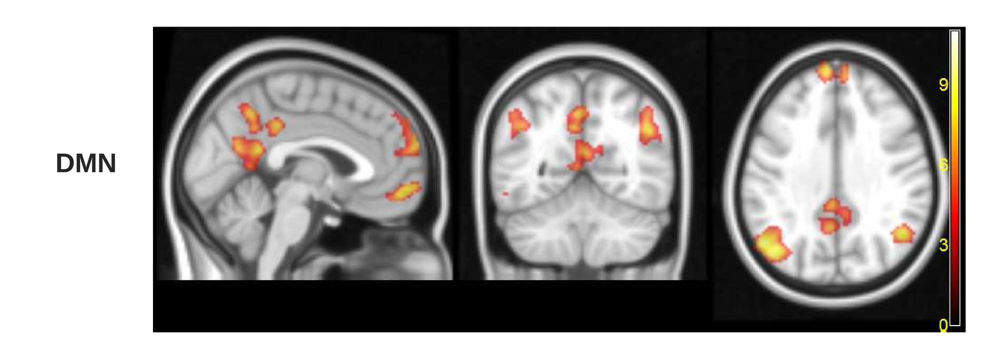


Figure 3: Default mode network from joint info matrix $\mathbf{B}^J$.

References

[1] A. Mensch, G. Varoquaux, and B. Thirion. Compressed online dictionary learning for fast resting-state fMRI decomposition. *In 2016 IEEE 13th International Symposium on Biomedical Imaging (ISBI)*, pages 1282–1285, 2016.

[2] A. K. Seghouane and A. Iqbal. Basis expansions approaches for regularized sequential dictionary learning algorithms with enforced sparsity for fMRI data analysis. *IEEE Transactions on Medical Imaging*, 36(9):1796–1807, 2017.