ILAPF: INCREMENTAL LEARNING ASSISTED PARTICLE FILTERING

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Abstract

This paper is concerned with dynamic system state estimation based on a series of noisy measurement with the presence of outliers. An incremental learning assisted particle filtering (ILAPF) method is presented, which can learn the value range of outliers incrementally during the process of particle filtering. The learned range of outliers is then used to improve subsequent filtering of the future state. Convergence of the outlier range estimation procedure is indicated by extensive empirical simulations using a set of differing outlier distribution models. The validity of the ILAPF algorithm is evaluated by illustrative simulations, and the result shows that ILAPF is more accurate and faster than a recently published state-of-the-art robust particle filter. It also shows that the incremental learning property of the ILAPF algorithm provides an efficient way to implement transfer learning among related state filtering tasks.

Problem

As we know, for conventional particle filtering (PF) methods, a major degradation in performance will happen when a significant mismatch between the leveraged model and the real mechanism that governs the systems evolution exists. A popular strategy to handle such issue of model mismatch is to employ a set of candidate models, instead of a single model, to take account of model uncertainty. To this end, a number of multiple-model strategies (MMS) based PF algorithms have been proposed in the literature. To implement MMS, human effort is required to specify a set of candidate models beforehand. The question is: is it possible to learn a single model online, instead of specifying multiple models beforehand, to handle the issue of model mismatch in PF? Here we focus on the problem of nonlinear state filtering in presence of outliers, which suffers from the aforementioned model mismatch issue.

Model

Consider a state space model as follows

$$x_k = f(x_{k-1}) + u_k \tag{1}$$
$$y_k = h(x_k) + n_k. \tag{2}$$

Now introduce a variable $o \in \{0, 1\}$ to take account of the uncertainty in the measurement model. $o_k = 1(0)$ denotes the event that y_k is (is not) an outlier. If $o_k = 0$, assume that the measurement noise n_k is Gaussian distributed by default, namely $n_k \sim \mathcal{N}(0, R)$, where R is *a priori* known. If $o_k = 1$, assume that n_k is generated from an unknown uniform distribution $\mathcal{U}(lb, ub)$, where lb and ub denote the lower and upper bounds of $\mathcal{U}$, respectively. The likelihood function can now be represented as follows

$$p(y_k|x_k) = \begin{cases} p(y_k|x_k, o_k = 1), & \text{if} \quad o_k = 1 \\ p(y_k|x_k, o_k = 0), & \text{if} \quad o_k = 0 \end{cases} \tag{3}$$

where

$$p(y_k|x_k, o_k = 1) = \begin{cases} 1/V_{lb,ub}, & \text{if} \quad e_k \in [lb, ub] \\ 0, & \text{otherwise} \end{cases} \tag{4}$$
$$p(y_k|x_k, o_k = 0) = \mathcal{N}(e_k|0, R), \tag{5}$$

where $e_k = y_k - h(x_k)$ and $V_{lb,ub}$ denotes the volume of the space bounded by lb and ub.

Outlier range estimation (ORE) procedure

Assume that the whole population of the outliers has a definite value range specified by a lower and upper bounds lb and ub. We can estimate lb and ub accurately provided that we have enough outlier data points at hand. But in practical tasks, usually, only a sparse set of outliers can be collected in a sequential way. The question under consideration here is: how to estimate lb and ub accurately using a limited number of outliers that have been found? To fit the sequential structure of the state filtering problem, we also expect that the estimation procedure can be performed in a sequential way.

The ORE method is proposed to address the above problem. It only has one free parameter I, which can be interpreted as a measure of uncertainty. Consider outliers sequentially. We make an incremental update to the estimation of lb and ub, once a new outlier arrives. Assume that the current estimations of lb and ub are $\hat{lb}$ and $\hat{ub}$, respectively, and the number of outliers that have been found is n. When the $(n+1)$th outlier, denoted as z_{n+1}, arrives, the ORE procedure updates $\hat{lb}$ and $\hat{ub}$ as follows

$$\hat{lb} = \min\{\hat{lb}, z_{n+1}\} - I/(n+1), \tag{6}$$
$$\hat{ub} = \max\{\hat{ub}, z_{n+1}\} + I/(n+1). \tag{7}$$

Convergence of the ORE procedure is demonstrated empirically by simulations. See Figure 1 as follows.

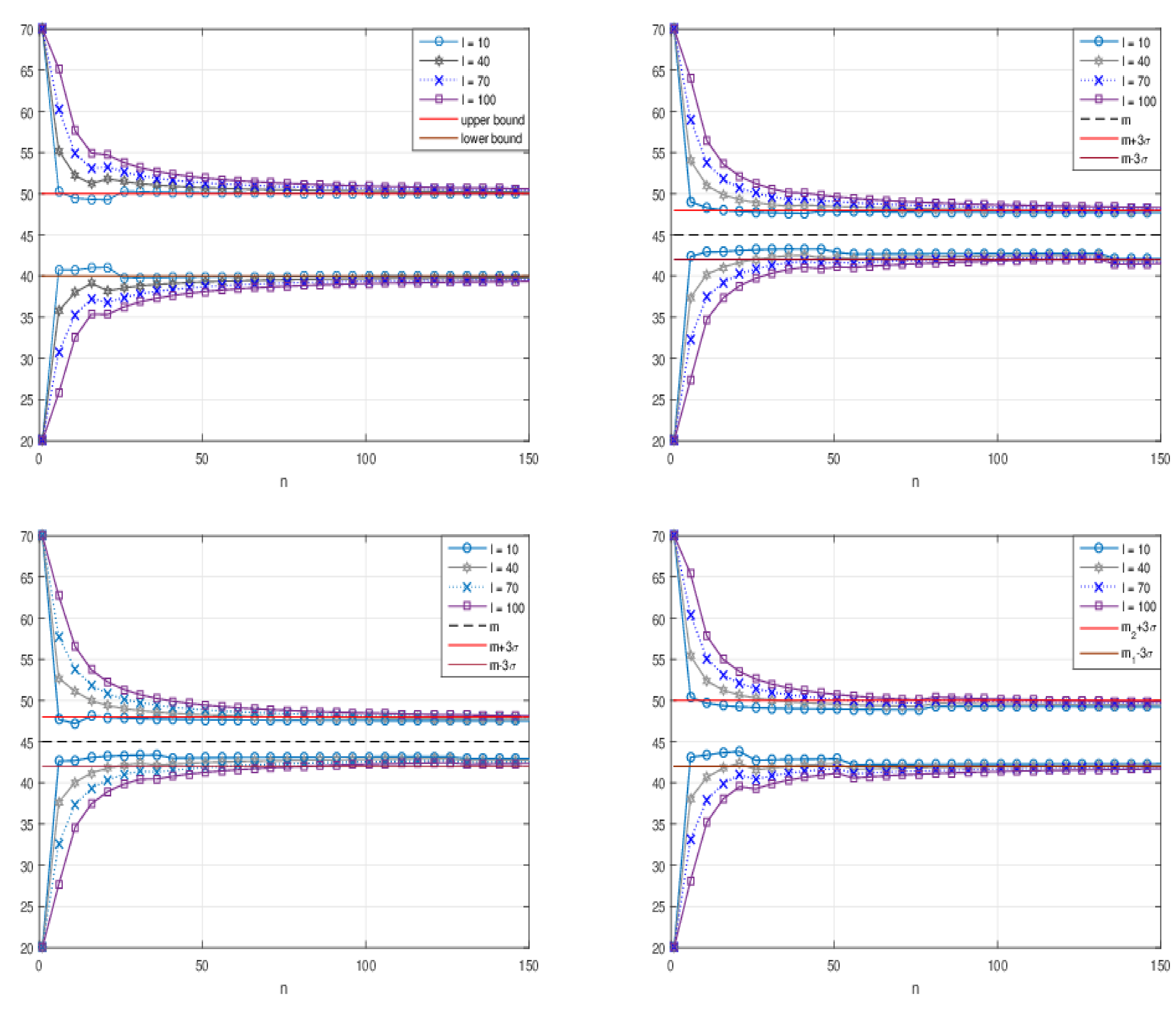

Figure 1: The simulation result of using the ORE procedure to sequentially estimate the range of the outliers.

The ILAPF Algorithm

Starting from $\{x^i_{k-1}, \omega^i_{k-1}\}^N_{i=1}$, $\hat{lb}$ and $\hat{ub}$ and the number of outliers that have been found n, the operations in one iteration of the ILAPF algorithm corresponding to time step k are shown as follows.

- Sampling step. Sample $\hat{x}^i_k$ from the state transition prior by setting $\hat{x}^i_k = f(x^i_{k-1})$, $i = 1, \ldots, N$;
- Weighting step. The same as in traditional PFs;
- ORE step. If $\pi(o_k = 1) > 0.5$, declare y_k to be an outlier, let $n = n + 1$ and update $\hat{lb}$ and $\hat{ub}$ using Eqns. (6) and (7), respectively. Note that $\pi(o_k = 1)$ is an output of the above weighting step.
- Resampling step. Sample $x^i_k \sim \sum_{j=1}^N \omega^j_k \delta_{\hat{x}^j_k}$, set $\omega^i_k = 1/N$, $i = 1, \ldots, N$. δ_x denotes the Dirac-delta function located at x.

Performance Evaluation

See the simulation setting in the paper. The filtering result is shown as follows

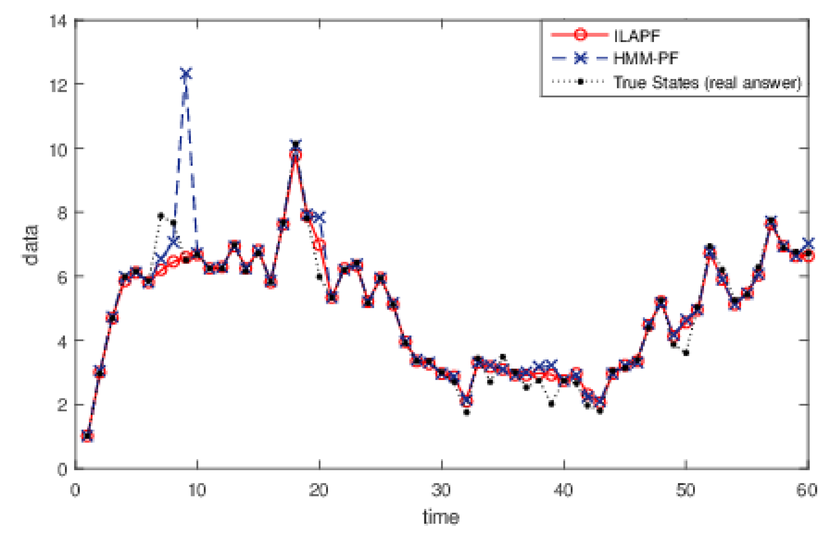

Figure 2: Filtering result of ILAPF and HMM-RPF.

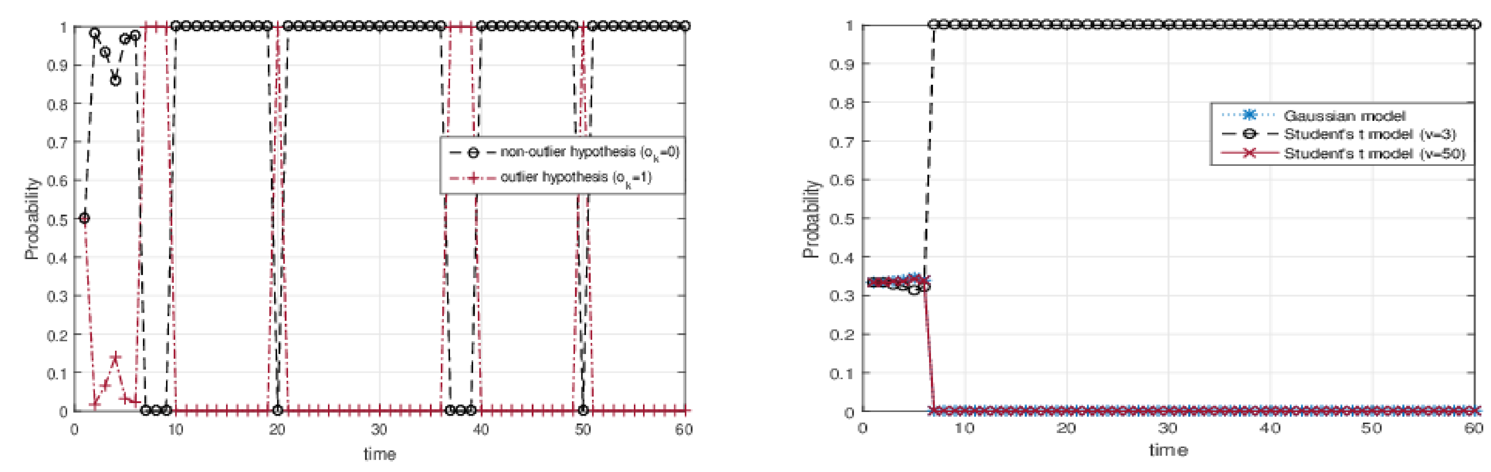

Figure 3: The left panel shows the ILAPF yielded posterior probabilities of the outlier hypothesis (corresponding to $o_k = 1$) and the non-outlier hypothesis (corresponding to $o_k = 0$). The right panel shows the HMM-RPF yielded posterior probabilities of the candidate models it employs and v denotes the degree of freedom of a Student's t model.

Table 1: Execution time (in seconds), Mean and variance of the MSE calculated over 30 independent runs of each algorithm.

Algorithm	Time	MSE	
		mean	var
ILAPF	3.998	0.365	0.007
HMM-RPF	5.509	0.582	0.109

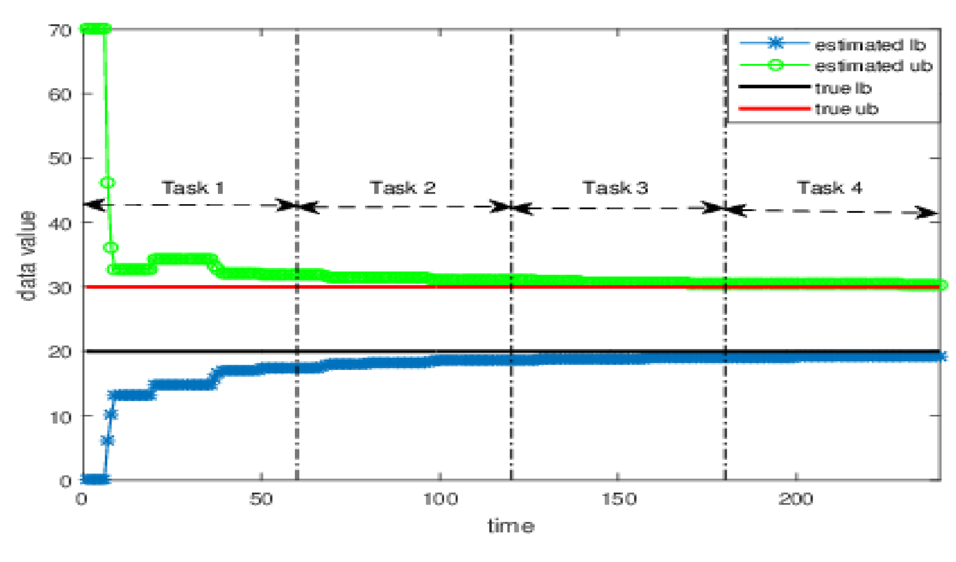

Figure 4: The estimated lower and upper bounds of the outliers' value range over 4 consecutive state filtering tasks

Table 2: Mean and variance of the MSE calculated over 30 independent runs of ILAPF for 4 consecutive tasks.

	Task 1	Task 2	Task 3	Task 4
Mean of MSE	0.365	0.360	0.333	0.272
Variance of MSE	0.007	0.005	0.004	0.003

Conclusions

MMS is a powerful solution to address nonlinear state filtering problems in presence of model uncertainty. The common practice to implement MMS is to specify a set of candidate models beforehand. In this paper, we proposed a novel way to implement MMS in the context of nonlinear state filtering in presence of outliers. Instead of specifying a set of candidate models beforehand, we select to learn a model to approximate the distribution of the outliers in a sequential way. The resulting algorithm, ILAPF, is shown to be more accurate and faster than its competitor algorithm HMM-RPF. Through simulations, we also show that the ILAPF algorithm makes transfer learning among related state filtering tasks possible.