Sound field reproduction with exterior cancellation using analytical weighting of harmonic coefficients

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Abstract

Sound field reproduction for high-fidelity audio systems

- Physical reproduction of sound field using loudspeaker array

Conventional methods

- Assumption of free space (no reflection)
  - e.g., wave field synthesis and higher order ambisonics
  - Performance degradation in reverberant environment

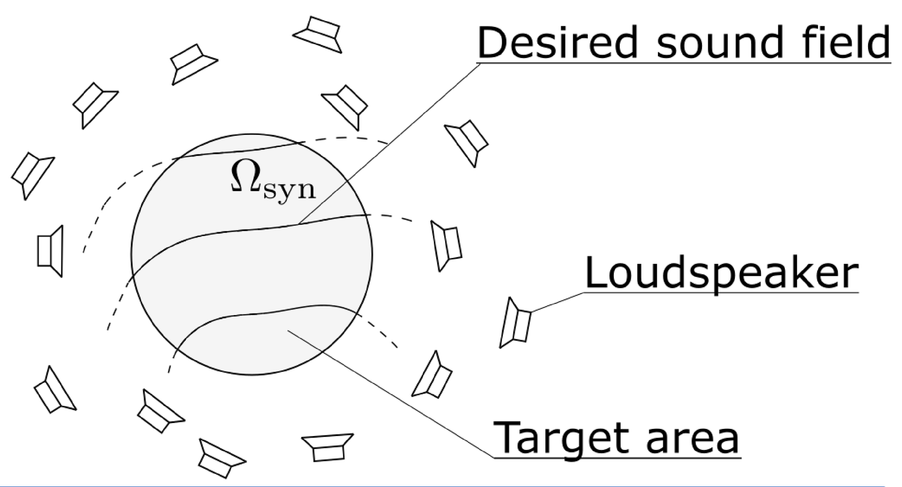
Proposed method

- Sound field reproduction with exterior cancellation
- Formulated so as to be applicable to any type of loudspeaker array (e.g., double-layer monopole array and distributed array of higher-order loudspeakers)
- Using analytical weighting of harmonic coefficients (cf. mode-matching method [1])

Problem Statement

- Two-dimensional sound field is assumed
- Our objective is jointly achieving reproduction of desired sound field within target area and suppression of exterior radiation
- lth loudspeaker's driving signal and transfer function are denoted as $d_l(\omega)$ and $g_l(\boldsymbol{r},\omega)$, respectively
- Synthesized sound field is represented as

$$u_{\mathrm{syn}}(\boldsymbol{r},\omega) = \sum_{l=1}^{L} d_l(\omega) g_l(\boldsymbol{r},\omega)$$



Optimization problem for sound field reproduction with exterior cancellation

$$\underset{\boldsymbol{d}\in\mathbb{C}^L}{\mathrm{minimize}} \quad \mathcal{J} = \mathcal{J}_{\mathrm{int}}(\boldsymbol{d}) + \gamma \mathcal{J}_{\mathrm{ext}}(\boldsymbol{d})$$

Constant parameter

- $\mathcal{J}_{\mathrm{int}}(\boldsymbol{d})$ represents interior reproduction error

$$\mathcal{J}_{\mathrm{int}}(\boldsymbol{d}) = \int_{\Omega_{\mathrm{syn}}} w(\boldsymbol{r}) |u_{\mathrm{syn}}(\boldsymbol{r}) - u_{\mathrm{des}}(\boldsymbol{r})|^2 \,\mathrm{d}\boldsymbol{r}$$

Target area: Assumed as circle

Spatial weighting: Assumed as uniform distribution

Desired sound field

- $\mathcal{J}_{\mathrm{ext}}(\boldsymbol{d})$ represents exterior radiation power

$$\mathcal{J}_{\mathrm{ext}}(\boldsymbol{d}) = \int_{\partial\Omega_{\mathrm{S}}} I_{\mathrm{ext}}(\boldsymbol{r}) \,\mathrm{d}\boldsymbol{r}$$

Circle enclosing all loudspeakers

Acoustic intensity in outward normal direction

Derivation of Optimal Driving Signals

- Based on harmonic analysis of sound field, optimal driving signals are derived

Interior sound field expansion

$$u(\boldsymbol{r}) = \sum_{\mu=-\infty}^{\infty} \mathring{u}_\mu^{\mathrm{int}}(\boldsymbol{r}_0)\varphi_\mu(\boldsymbol{r}-\boldsymbol{r}_0)$$

Interior basis function $\varphi_\mu(\boldsymbol{r}) = J_\mu(kr)\exp(\mathrm{j}\mu\phi)$

Exterior sound field expansion

$$u(\boldsymbol{r}) = \sum_{\mu=-\infty}^{\infty} \mathring{u}_\mu^{\mathrm{ext}}(\boldsymbol{r}_0)\psi_\mu(\boldsymbol{r}-\boldsymbol{r}_0)$$

Exterior basis function $\psi_\mu(\boldsymbol{r}) = H_\mu(kr)\exp(\mathrm{j}\mu\phi)$

- Objective functions are rewritten as

$$\mathcal{J}_{\mathrm{int}}(\boldsymbol{d}) = \boldsymbol{d}^{\mathsf{H}}\mathbf{A}_{\mathrm{int}}\boldsymbol{d} - \boldsymbol{d}^{\mathsf{H}}\mathbf{b}_{\mathrm{int}} - \mathbf{b}_{\mathrm{int}}^{\mathsf{H}}\boldsymbol{d} + \mathrm{const.}$$

$$(\mathbf{A}_{\mathrm{int}})_{l_1,l_2} = \sum_{\mu=-\infty}^{\infty} \mathring{w}_\mu \mathring{g}_{l_1,\mu}^{\mathrm{int}}(\boldsymbol{r}_{\mathrm{syn}})^* \mathring{g}_{l_2,\mu}^{\mathrm{int}}(\boldsymbol{r}_{\mathrm{syn}})$$

$$(\mathbf{b}_{\mathrm{int}})_{l} = \sum_{\mu=-\infty}^{\infty} \mathring{w}_\mu \mathring{g}_{l,\mu}^{\mathrm{int}}(\boldsymbol{r}_{\mathrm{syn}})^* \mathring{u}_{\mathrm{des},\mu}^{\mathrm{int}}(\boldsymbol{r}_{\mathrm{syn}})$$

$\boldsymbol{r}_{\mathrm{syn}}$: Center of Ω_{syn}

R_{syn} : Radius of Ω_{syn}

$$\mathring{w}_\mu = \int_{\Omega_{\mathrm{syn}}} w(\boldsymbol{r})|\varphi_\mu(\boldsymbol{r}-\boldsymbol{r}_{\mathrm{syn}})|^2\,\mathrm{d}\boldsymbol{r} = J_\mu(kR_{\mathrm{syn}})^2 - J_{\mu-1}(kR_{\mathrm{syn}})J_{\mu+1}(kR_{\mathrm{syn}})$$

$$\mathcal{J}_{\mathrm{ext}}(\boldsymbol{d}) = \boldsymbol{d}^{\mathsf{H}}\mathbf{A}_{\mathrm{ext}}\boldsymbol{d}$$

$$(\mathbf{A}_{\mathrm{ext}})_{l_1,l_2} = \frac{2}{\rho c k}\sum_{\mu=-\infty}^{\infty} \mathring{g}_{l_1,\mu}^{\mathrm{ext}}(\boldsymbol{r}_{\mathrm{S}})^* \mathring{g}_{l_2,\mu}^{\mathrm{ext}}(\boldsymbol{r}_{\mathrm{S}})$$

$\boldsymbol{r}_{\mathrm{S}}$: Center of Ω_{S}

- This Includes infinite sum, but can be calculated analytically using addition theorems for Bessel functions

Optimal driving signals

Regularization parameter

$$\hat{\boldsymbol{d}} = (\mathbf{A}_{\mathrm{int}} + \gamma\mathbf{A}_{\mathrm{ext}} + \lambda\boldsymbol{I}_L)^{-1}\mathbf{b}_{\mathrm{int}}$$

Comparison with Mode-Matching Method

- In mode-matching method, driving signals are obtained as

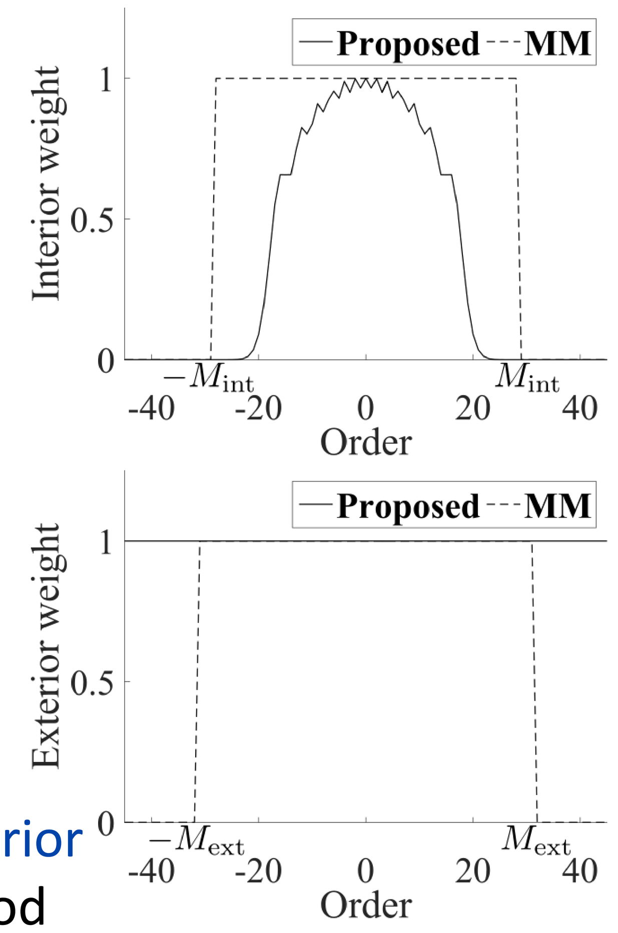
$$\hat{\boldsymbol{d}} = (\overline{\mathbf{A}}_{\mathrm{int}} + \overline{\mathbf{A}}_{\mathrm{ext}} + \lambda\boldsymbol{I}_L)^{-1}\overline{\mathbf{b}}_{\mathrm{int}}$$

$$(\overline{\mathbf{A}}_{\mathrm{int}})_{l_1,l_2} = \sum_{\mu=-M_{\mathrm{int}}}^{M_{\mathrm{int}}} \mathring{g}_{l_1,\mu}^{\mathrm{int}}(\boldsymbol{r}_{\mathrm{syn}})^* \mathring{g}_{l_2,\mu}^{\mathrm{int}}(\boldsymbol{r}_{\mathrm{syn}})$$

$$(\overline{\mathbf{b}}_{\mathrm{int}})_{l} = \sum_{\mu=-M_{\mathrm{int}}}^{M_{\mathrm{int}}} \mathring{g}_{l,\mu}^{\mathrm{int}}(\boldsymbol{r}_{\mathrm{syn}})^* \mathring{u}_{\mathrm{des},\mu}^{\mathrm{int}}(\boldsymbol{r}_{\mathrm{syn}})$$

$$(\overline{\mathbf{A}}_{\mathrm{ext}})_{l_1,l_2} = \sum_{\mu=-M_{\mathrm{ext}}}^{M_{\mathrm{ext}}} \mathring{g}_{l_1,\mu}^{\mathrm{ext}}(\boldsymbol{r}_{\mathrm{S}})^* \mathring{g}_{l_2,\mu}^{\mathrm{ext}}(\boldsymbol{r}_{\mathrm{S}})$$

- Right figures show example of interior and exterior weights in proposed and mode-matching method



Simulation Results

- Circular array (of radius 1.5m) of 12 ideal third-order loudspeakers
- Comparing proposed method with mode-matching method [1]
- Each third-order loudspeaker was assumed to be seven individual multipoles (from -3rd to 3rd)
- Evaluation criteria: signal-to-distortion ratio (SDR) and suppression-power ratio (SPR)

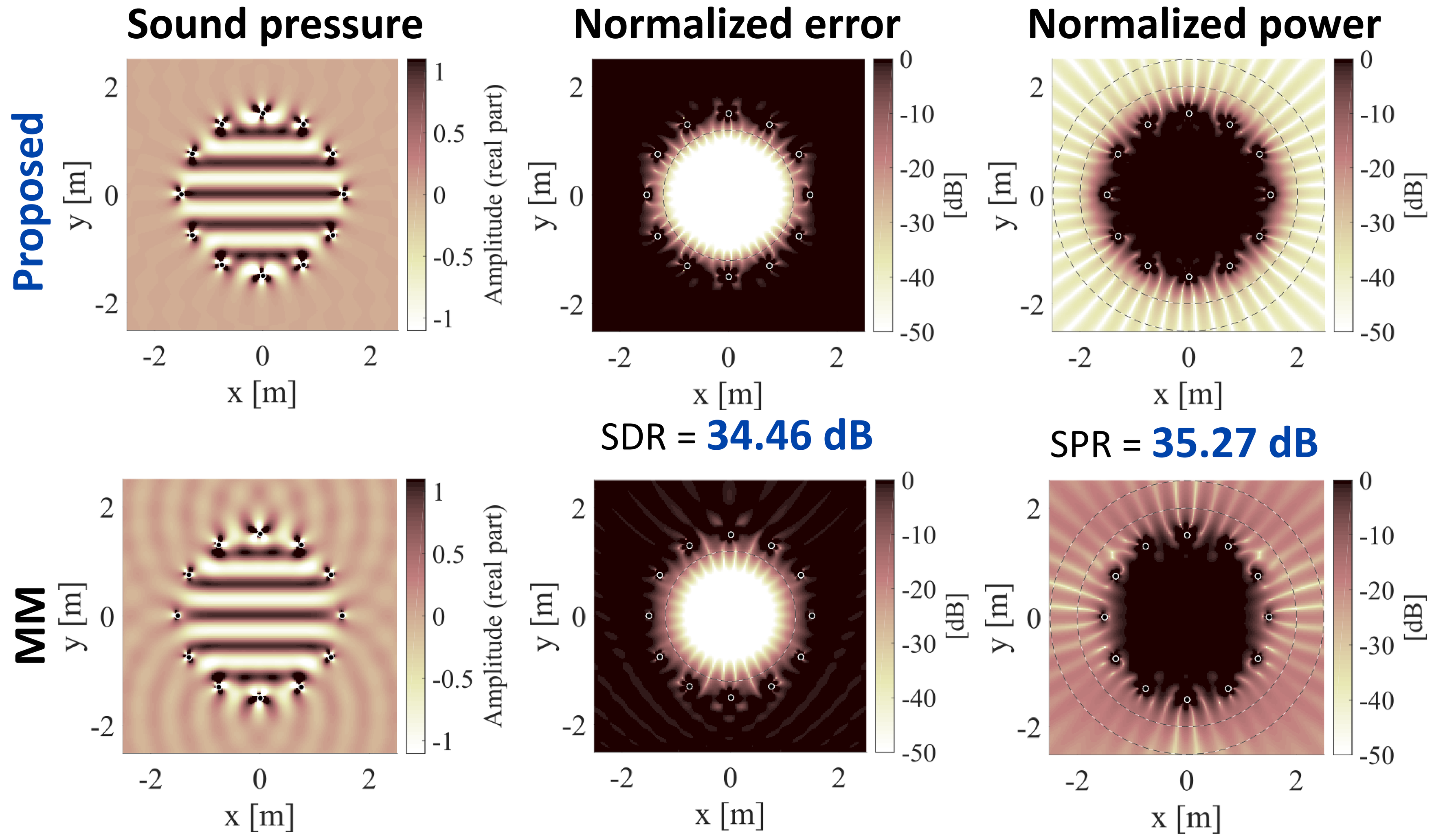
$$\mathrm{SDR}(\omega) = 10\log_{10}\frac{\int_{\Omega_{\mathrm{syn}}}|u_{\mathrm{des}}(\boldsymbol{r},\omega)|^2\,\mathrm{d}\boldsymbol{r}}{\int_{\Omega_{\mathrm{syn}}}|u_{\mathrm{syn}}(\boldsymbol{r},\omega)-u_{\mathrm{des}}(\boldsymbol{r},\omega)|^2\,\mathrm{d}\boldsymbol{r}}$$

- Ω_{syn} was circular area of radius 1.2 m

$$\mathrm{SPR}(\omega) = 10\log_{10}\frac{\int_{\Omega_{\mathrm{ext}}}|u_{\mathrm{des}}(\boldsymbol{r},\omega)|^2\,\mathrm{d}\boldsymbol{r}}{\int_{\Omega_{\mathrm{ext}}}|u_{\mathrm{syn}}(\boldsymbol{r},\omega)|^2\,\mathrm{d}\boldsymbol{r}}$$

- Ω_{ext} was area bounded by two circles of radii 2.0 and 2.5 m

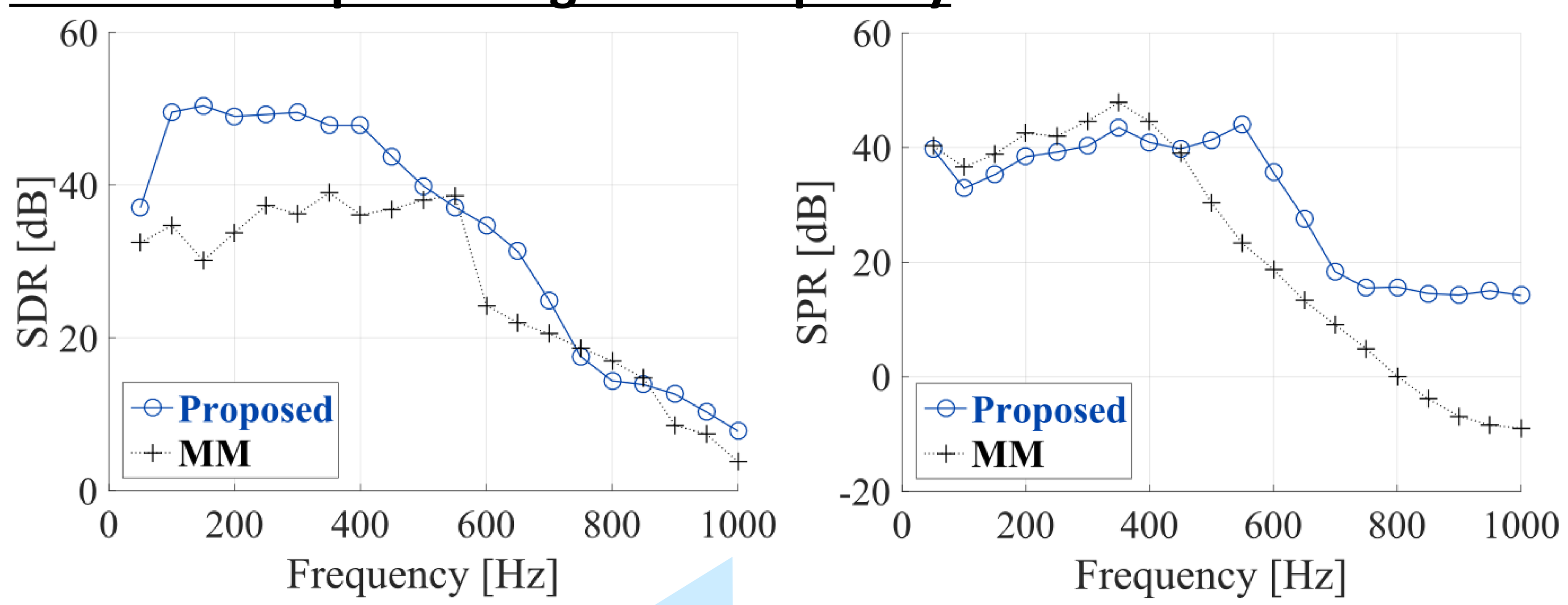
Reproduction results: plane wave, 600 Hz



Proposed: SDR = 34.46 dB, SPR = 35.27 dB

MM: SDR = 24.23 dB, SPR = 18.74 dB

SDR and SPR plotted against frequency



High reproduction accuracy and exterior power suppression are achieved by using proposed method

[1] M. A. Poletti, T. D. Abhayapala and P. Samarasinghe, "Interior and exterior sound field control using two dimensional higher-order variable-directivity sources," J. Acoust. Soc. Am., vol. 131, no. 5, pp. 3814-3823, 2012.