Scheduling of Multistatic Sonobuoy Fields using Multi-Objective Optimization

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Department of Defence
Science and Technology

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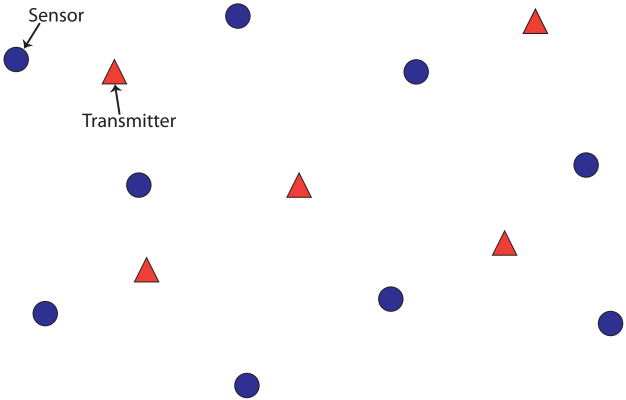
Outline

1. Multistatic Sonobuoy Fields
   - Two Tasks $\Longrightarrow$ Search for and track underwater targets
   - Performance dependent on scheduling sonobuoys
2. Recap on Tracking in Sonobuoy Fields
   - Geometric Modelling and Measurements
   - Tracking algorithm used to track targets
3. Multi-Objective Scheduling Framework
   - Optimization Problem $\Longrightarrow$ Two reward functions
   - Tracking Reward Function
   - Search Reward Function
4. Simulation Results
5. Conclusions

Multistatic Sonobuoy Fields

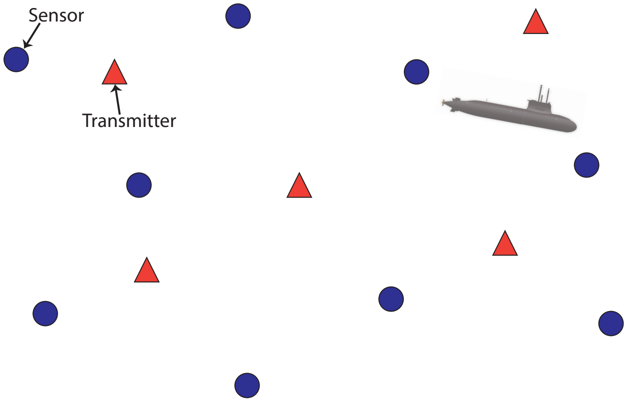
A network of transmitters and sensors distributed across a large search region



Multistatic Sonobuoy Fields

Two tasks of the system:

- Detect targets that are unknown to the system



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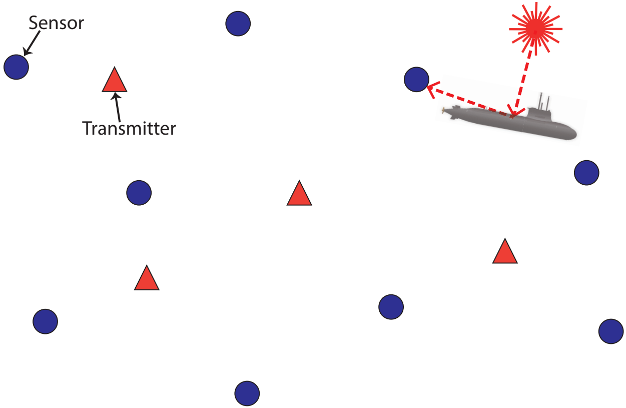

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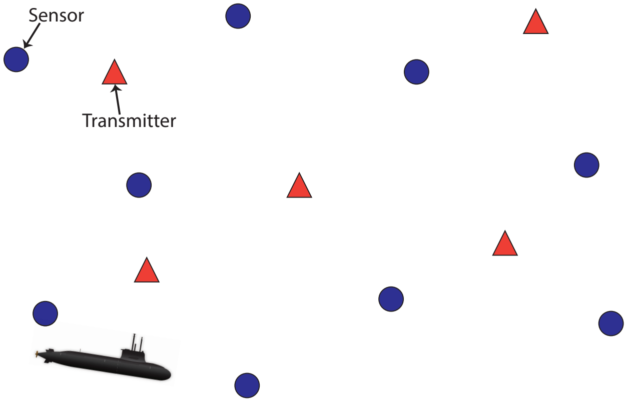
- Accurately track targets known to the system



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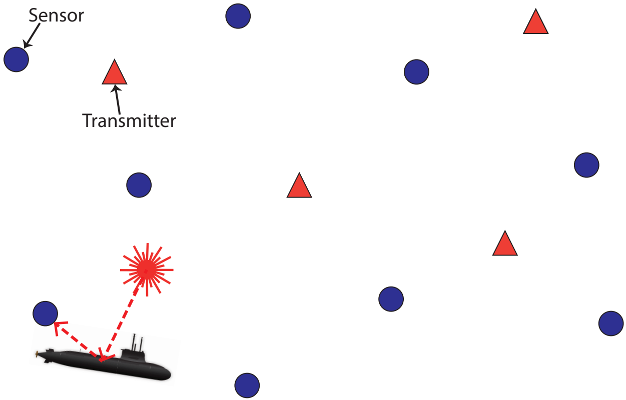

Scheduling Problem

- Choose sequence of transmitters and waveforms to satisfy tasks

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At one transmission time:

$$\text{Choose a Transmitter:} \quad \mathcal{T} = \{j_1, j_2, \ldots, j_{N_T}\}$$

where N_T is the number of transmitters in the field

$$\text{Choose a Waveform:} \quad \mathcal{W} = \{w_1, w_2, \ldots, w_{N_d}\}$$

where N_d is the number of possible waveforms

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Possible waveforms:

- Continuous Wave (CW) or Frequency Modulated (FM) waveform
- 1kHz or 2kHz frequency
- 2 second or 8 second duration

Scheduling Problem

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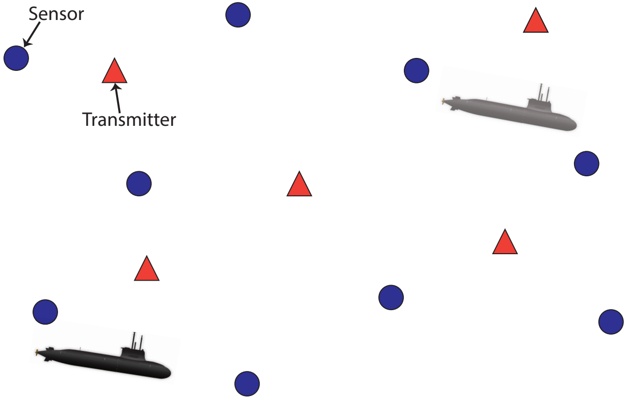
Action space:

Choose an action: $a \in \mathcal{A}, \quad \mathcal{A} = \mathcal{T} \times \mathcal{W}$

Conflicting Objectives

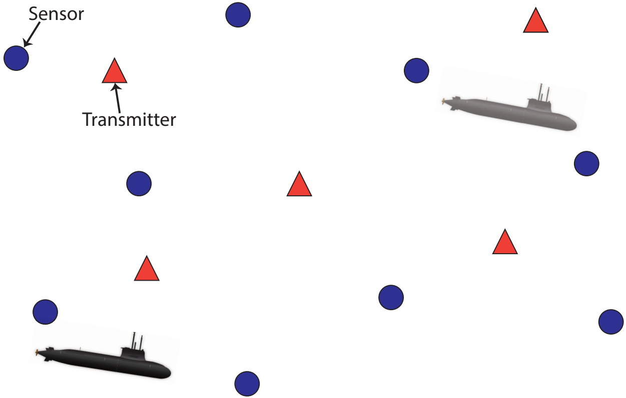
Track vs Search $\Longrightarrow$ Which transmitter to choose...



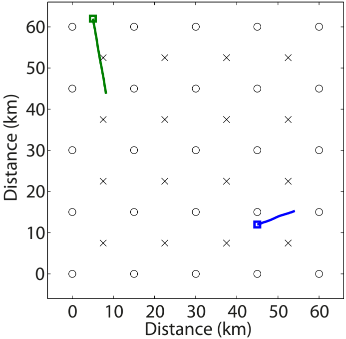
Conflicting Objectives

Our Approach:

Combine both tasks in multi-objective framework and use multi-objective optimization to decide scheduling



Modelling, Measurements & Tracking Algorithm



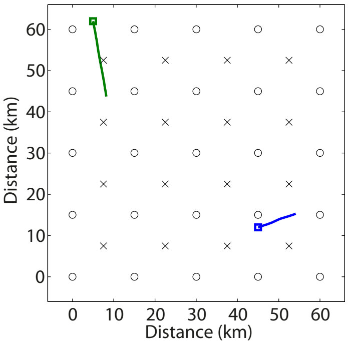
'x' = Transmitters, 'o' = Receivers

Sonobuoy Field Description:

- Transmitter positions
$$\mathbf{s}_j = \left[x_s^j, y_s^j\right]^{\mathrm{T}}$$
- Receiver positions
$$\mathbf{r}_i = \left[x_r^i, y_r^i\right]^{\mathrm{T}}$$
- Assume positions are known at all times*

*Each buoy contains RF communications and may contain GPS equipment

Modelling, Measurements & Tracking Algorithm



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Target Description:

- Target Position at time t_k:
$$\mathbf{p} = [x_k, y_k]^{\mathrm{T}}$$
- Target Velocity at time t_k:
$$\mathbf{v} = [\dot{x}_k, \dot{y}_k]^{\mathrm{T}}$$
- Time-varying state
$$\mathbf{x}_k = [\mathbf{p}_k^{\mathrm{T}}, \mathbf{v}_k^{\mathrm{T}}]^{\mathrm{T}}$$

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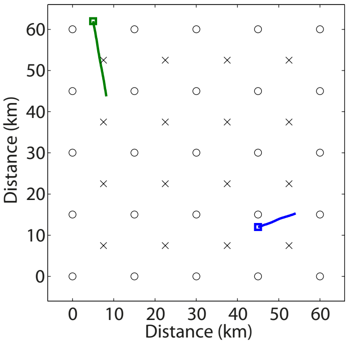

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Target Motion:

- Noisy linear constant-velocity model

$$\mathbf{x}_k = \underbrace{\left(\begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \otimes \mathbf{I}_2 \right) \mathbf{x}_{k-1}}_{f(\mathbf{x}_{k-1})} + \mathbf{e}_k$$

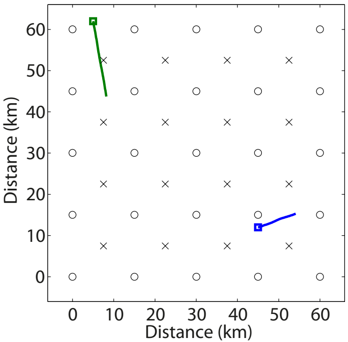
- Process noise $\mathbf{e}_k$ is Gaussian with variance

$$\mathbf{Q} = \omega \begin{bmatrix} T^3/3 & T^2/2 \\ T^2/2 & T \end{bmatrix} \otimes \mathbf{I}_2$$

where $T = t_k - t_{k-1}$ is the sampling in time

$\otimes$ is the Kronecker product and $\mathbf{I}_2$ is 2×2 identity matrix

Modelling, Measurements & Tracking Algorithm



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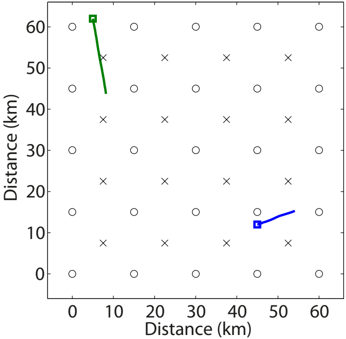
Measurements:

- Signal amplitude β and Kinematic measurement $\mathbf{z}$
$$\mathbf{z} = \mathbf{h}_j^{(i)}(\mathbf{x}_k) + \mathbf{w}_j^{(i)}$$
- Measurements collected from a subset of receivers
- Buoys have two waveform modalities
  - Frequency Modulated (FM)
  - Continuous Wave (CW)

Modelling, Measurements & Tracking Algorithm

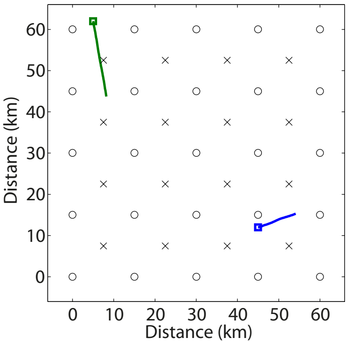


'x' = Transmitters, 'o' = Receivers

Using FM waveforms:

- Bistatic Range:
  $$|\mathbf{p}_k - \mathbf{r}_i| + |\mathbf{p}_k - \mathbf{s}_j|$$
- Angle from Receiver:
  $$\arctan\left(\frac{y_k - y_r^i}{x_k - x_r^i}\right)$$
- Good positional information

Modelling, Measurements & Tracking Algorithm



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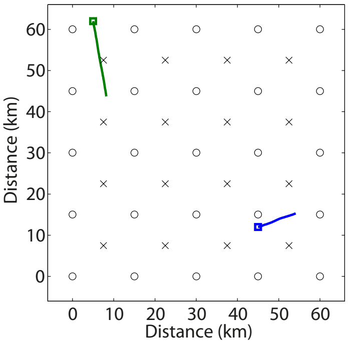
Using CW waveforms:

- Bistatic Range:
$$|\mathbf{p}_k - \mathbf{r}_i| + |\mathbf{p}_k - \mathbf{s}_j|$$
- Angle from Receiver:
$$\arctan\left(\frac{y_k - y_r^i}{x_k - x_r^i}\right)$$
- Bistatic Range-Rate:
$$\mathbf{v}^{\mathrm{T}}\left[\frac{\mathbf{p}_k - \mathbf{r}_i}{|\mathbf{p}_k - \mathbf{r}_i|} + \frac{\mathbf{p}_k - \mathbf{s}_i}{|\mathbf{p}_k - \mathbf{s}_i|}\right]$$
- Good velocity information

Modelling, Measurements & Tracking Algorithm



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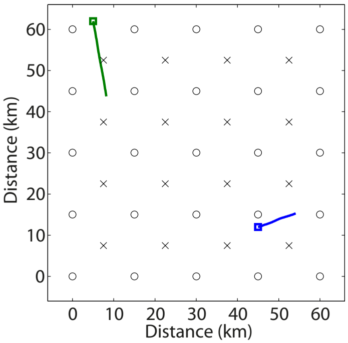
Tracking Challenges:

- High levels of clutter
- Non-linear measurements
- Low probability of detection

Many possible algorithms: ML-PDA, MHT, PMHT, JIPDA, PHD/CPHD, ... etc

Modelling, Measurements & Tracking Algorithm



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The tracker:

- Multi-Sensor Bernoulli filter[1] (optimal multi-sensor Bayesian filter for a single target)
- Linear Multi-Target (LMT) Paradigm[2]
- Gaussian mixture model implementation[3]
- Process FM & CW measurements

[1] B. Ristic *el al.*, '*A tutorial on Bernoulli filters: Theory, implementation and applications*', IEEE Trans. Signal Process., 2013.
[2] D. Mušicki and B. La Scala, '*Multi-Target Tracking in Clutter without Measurement Assignment*', IEEE Trans. Aerosp. Electron. Syst., 2008.
[3] B. Ristic *el al.*, '*Gaussian Mixture Multitarget Multisensor Bernoulli Tracker for Multistatic Sonobuoy Fields*', IET Radar, Sonar & Navig., 2017.

Multi-Objective Framework for choosing

Maximising rewards:

- $R_{\text{Search}}(a) \Rightarrow$ Reward for searching to detect unknown targets
- $R_{\text{Track}}(a) \Rightarrow$ Reward for continued tracking of known targets

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Combine rewards via convex sum:

$$\max_{a} \quad \{\alpha R_{\text{Track}}(a) + (1-\alpha) R_{\text{Search}}(a)\}$$

where $\alpha \in [0, 1]$

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Performance depends on $\alpha \Rightarrow$ Controls trade-off
$\looparrowright$ Different solutions depending on the value of α

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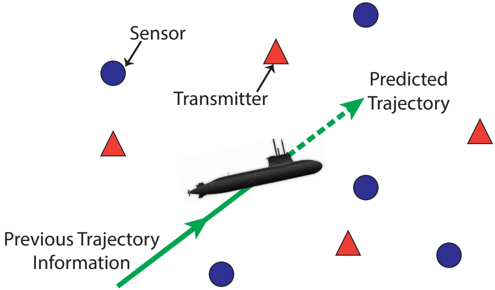
Pareto Optimality:

A point is Pareto optimal if there is no other point that can improve one objective without degrading the other.

Problem characterised $\Longrightarrow$ Set of Pareto optimal points

$\looparrowright$ Pareto Frontier

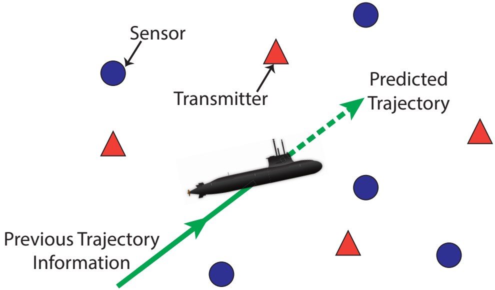
Tracking Reward



Given previous tracking:

- Measure the gain in tracking information from action a

Tracking Reward



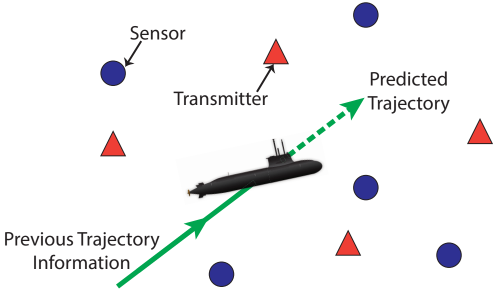
Approximate information matrix:

Single track: $$\text{trace}\left[\mathbf{J}_{\text{Predict}} + \sum_{i\in\mathcal{R}} P_d^i(a)\mathbf{J}^i_{\text{Measure}}(a)\right]$$

Trace of only the positional elements of information matrix

$P_d^i(a)$ Expected probability of detecting track

Tracking Reward

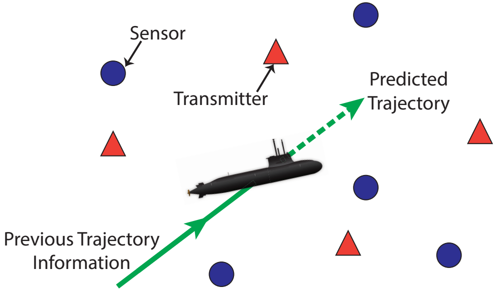


Predicted Information Matrix:

$$\underbrace{\mathbf{J}_{\text{Predict}} = \left[\mathbf{F}_{k-1}\mathbf{P}_{k-1}\left[\mathbf{F}_{k-1}\right]^{\mathrm{T}}\right]^{-1}}_{\text{Propagation of error covariance due to motion model}}$$

where $\mathbf{F}_{k-1}$ is the Jacobian of $f(\mathbf{x}_{k-1})$ and $\mathbf{P}_{k-1}$ is the error covariance from tracker

Tracking Reward

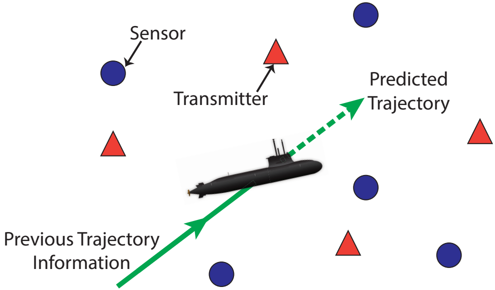


Measurement Information Matrix:

$$\underbrace{\mathbf{J}_{\text{Measure}} = \left[\mathbf{H}_k^i(a)\right]^{\mathrm{T}} \left[\mathbf{R}_k^i(a)\right]^{-1} \mathbf{H}_k^i(a)}_{\text{Gain in information from action}}$$

where $\mathbf{H}_k^i(a)$ is the Jacobian of $h_a(\mathbf{x}_{k-1})$ and $\mathbf{R}_k^i(a)$ is the measurement covariance

Tracking Reward



Multiple tracks:

$$R_{\text{Track}}(a) = \sum_{\tau=1}^{T} \omega_\tau \,\text{trace}\left[\mathbf{J}^{\tau}_{\text{Predict}} + \sum_{i \in \mathcal{R}} P_d^{i,\tau}(a) \mathbf{J}^{i,\tau}_{\text{Measure}}(a) \right]$$

$\omega_\tau \Rightarrow$ Normalised weights ($\propto 1/$existence probability)

Search Reward

Reduction of the probability of undetected targets in sonar field

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Reduction of the probability of undetected targets in sonar field

Modelling this probability[1]:

- Define Threat Map $P_{T,k}$ $\Rightarrow$ Discrete 2D grid of probabilities
- Probabilities evolve over time
  - Increases $\Rightarrow$ Drift & diffusion of undetected targets
  - Decreases $\Rightarrow$ Transmitters emits a ping

[1] D. Krout *el al.*, '*Probability of target presence for multistatic sonar ping sequencing*', IEEE J. Ocean. Eng., 2009.

Search Reward

Reduction of the probability of undetected targets in sonar field

Drift & diffusion process:

- Matrix $G \Rightarrow$ Probability of targets entering from adjacent cells
- Update to Threat Map $\Rightarrow$ Filter $P_{T,k}$ with G
- Pre-calculate G using Monte-Carlo simulations

e.g. for a 60 s interval, grid size of 1 km, uniformly distributed target speed between 0 and 10 knots

$$G = \begin{bmatrix} 0.0036 & 0.0582 & 0.0036 \\ 0.0582 & 0.7526 & 0.0582 \\ 0.0036 & 0.0582 & 0.0036 \end{bmatrix}$$

Search Reward

Reduction of the probability of undetected targets in sonar field

Transmitting a ping:

Apply Bayesian update at each cell of $P_{T,k}$

$$P_{T,k}(\mathrm{x}, a) = \frac{(1 - P_d(\mathrm{x}, a))P_{T,k-1}(\mathrm{x})}{(1 - P_d(\mathrm{x}, a))P_{T,k-1}(\mathrm{x}) + (1 - P_{\text{fa}}(1 - P_{T,k-1}(\mathrm{x}))}$$

- $P_d(\mathrm{x}, a)$ is the probability a target is detected after action a
- P_{fa} is the false alarm probability
- $\mathrm{x} = (x, y)$ is the 2D grid point

Search Reward

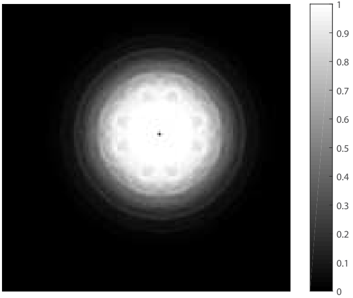
Reduction of the probability of undetected targets in sonar field

Obtaining $P_d(\mathrm{x}, a)$:

Generate probabilities using Monte-Carlo simulations and the realistic simulator (BRISE)

e.g.

- 160 × 160 km area
- 1km × 1km grid resolution
- 5 × 5 transmitter grid
- 6 × 6 receiver grid
- Buoy separation = 15km
- FM, 1 kHz waveform with 2 s duration.

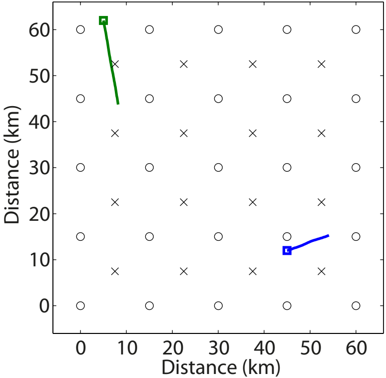


Search Reward

Reduction of the probability of undetected targets in sonar field and finally...

$$R_{\text{search}}(a) = \sum_{\mathrm{x}} P_{T,k-1}(\mathrm{x}) - P_{T,k}(\mathrm{x}, a)$$

Analysis of Scheduler - Set Up



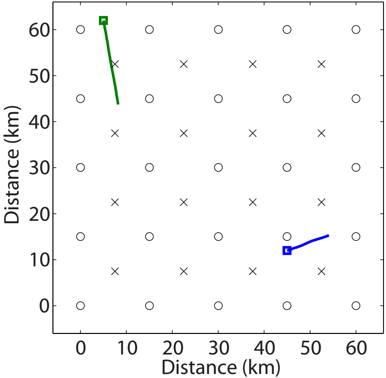
'x' = Transmitters, 'o' = Receivers

Set-up:

- 4 × 4 transmitter grid
- 5 × 5 receiver grid
- Buoy separation = 15km
- 50 Minute Scenario
- 1 transmission/minute
- Blue target present for whole duration
- Green target appears after 10 minutes

Realistic measurements $\Longrightarrow$ Bistatic Range Independent Signal Excess (BRISE) simulation environment

Analysis of Scheduler - Set Up



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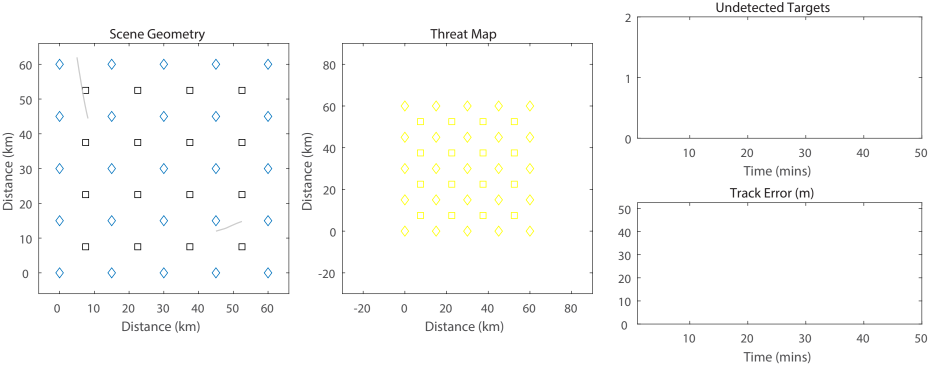
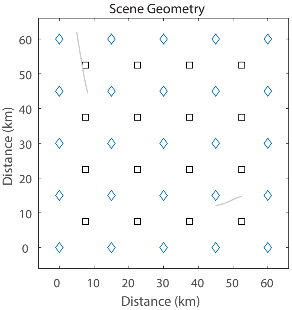
Analyse the performance of the scheduler as α varies

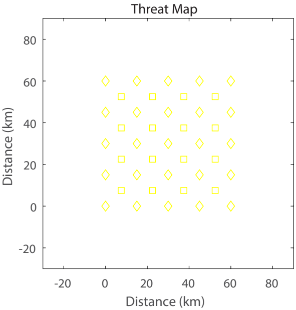
Analysis of Scheduler - Demo

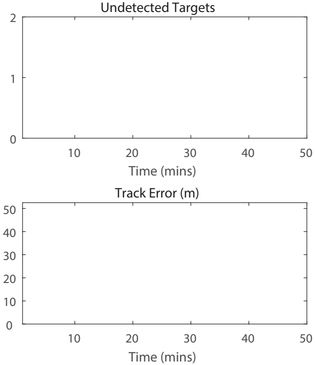


$$\alpha = 0.35$$

Analysis of Scheduler - Demo

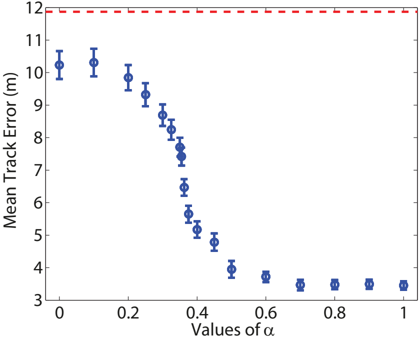




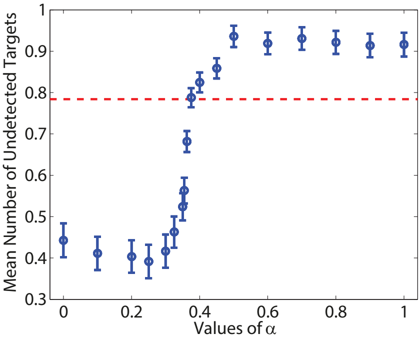


$$\alpha = 0.35$$

Analysis of Scheduler - Results



Mean Track Error (m)



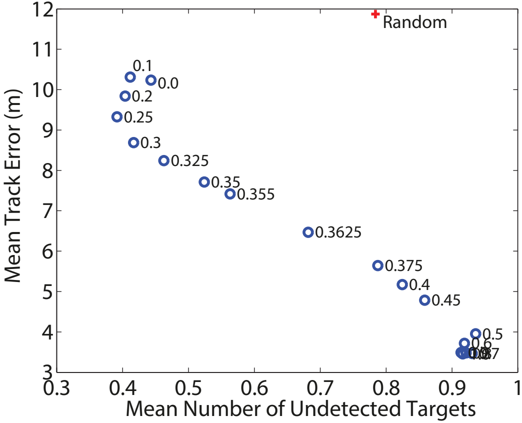
Mean Number of Undetected Targets

Error bars = 95% confidence intervals for the estimated values
Red dashed line = Performance from random scheduling

Values averaged over 300 Monte-Carlo simulations and every transmission

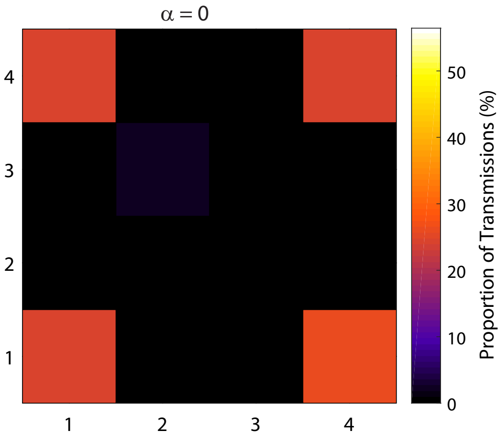
Analysis of Scheduler - Results

Pareto-esque Frontier:



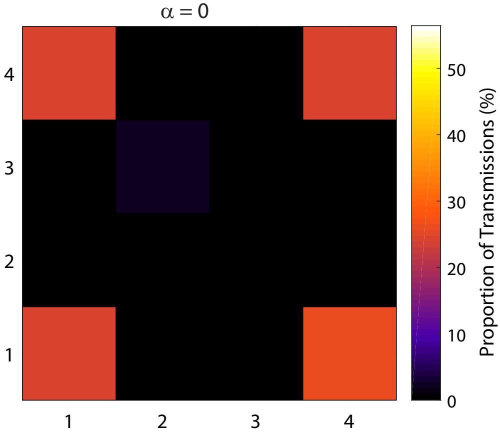
Values averaged over 300 Monte-Carlo simulations and every transmission

Analysis of Scheduler - Transmitter Choice



2D histogram showing the proportion of waveforms transmitted

Analysis of Scheduler - Transmitter Choice



2D histogram showing the proportion of waveforms transmitted

Conclusions

- Introduced scheduling of multistatic sonobuoy fields
  - Search $\implies$ Detect targets that are unknown
  - Track $\implies$ Accurately track known targets
- Presented multi-objective framework for scheduling
  - Each task is treated as a separate objective
  - Objectives combined via weighted sum
  - Weight α controls priority placed on each objective
- Analysed proposed scheduling via realistic simulations
  - Demonstrated trade-off between search and track as α varies
  - Trade-off characterised in terms of points on the Pareto front

The End

Thank you for listening