Multiple-Model and Reduced-Order Kalman Filtering for Pathological Hand Tremor Extraction

Vahid Khorasani Ghassab[†], Arash Mohammadi[†], Seyed Farokh Atashzar[‡] & Rajni V. Patel[‡]

[†] Concordia Institute for Information System Engineering, Montreal, QC, Canada

[‡] Electrical and Computer Engineering, Western University, London, ON, Canada

Motivations/Contributions

Motivation:

- Active tremor suppression is applied to reduce the need for increasing the medication dosage or conducting the surgical option for Parkinson's disease (PD) or Essential tremor (ET).
- Tremor extraction techniques are considered as the central component of several rehabilitative robotic technologies, and the accuracy of such filters can directly affect the performance of the aforementioned technologies.

Contribution:

- E-BMFLC is a Fourier linear combiner based filtering technique and is applied to extract the Pathological hand tremors and uses embedded least mean square (LMS) estimation approach[1].
- This research proposes an adaptive estimation framework, referred to as Multiple Adaptive Reduced-order Kalman filtering (KFE-BMFLC), with the goal of improving the performance and reducing the computational overhead in comparison with E-BMFLC technique.

Problem Formulation

- The complete hand motion including the involuntary components and voluntary components is modeled as

$$\eta_p(k) = \eta_{p-v}(k) + \eta_{p-i}(k) \qquad (1)$$

- By dividing the total hand movement frequency into a finite number, $\eta_p(k)$ can be modeled as follows

$$\eta_p(k)=\sum_{r=0}^{L}\left[a_r\sin(2\pi(f_{\min}+r\Delta f)k)+b_r\cos(2\pi(f_{\min}+r\Delta f)k)\right] \qquad (2)$$

- Let us define

$$x_r(k) = \begin{cases} sin(2\pi(f_{min}+r\Delta f)k), & 0\leq r\leq L \\ cos(2\pi(f_{min}+((r-L)-1)\Delta f)k), & L+1\leq r\leq 2L+1 \end{cases} \qquad (3)$$

- Considering Eq. (3), the model in Eq. (2) can be rewritten as $\eta_p(k) = \boldsymbol{w}_{\eta_p}^T(k)\boldsymbol{x}_{\eta_p}(k)$, where

$$\boldsymbol{w}_{\eta_p}(k)=[a_0(k), a_1(k), \ldots, a_L(k), b_0(k), b_1(k), \ldots, b_L(k)]^T \qquad (4)$$

$$\text{and} \quad \boldsymbol{x}_{\eta_p}(k) = [x_0(k), x_1(k)\ldots, x_{2L+1}(k)]^T \qquad (5)$$

- Considering $N_{\min} = (\omega_{\min} - f_{\min})/\Delta f$ and $N_{\max} = (\omega_{\max} - f_{\min})/\Delta f$ we have

$$\hat{\boldsymbol{w}}_{\eta_{p-i}}(k)=[a_{N_{\min}}(k),\ldots,a_{N_{\max}}(k),b_{N_{\min}}(k),\ldots,b_{N_{\max}}(k)]^T$$
$$\boldsymbol{x}_{\eta_{p-i}}(k) = [x_{N_{\min}}(k),\ldots,x_{N_{\max}}(k),x_{L+N_{\min}}(k),\ldots,x_{L+N_{\max}}(k)]^T \qquad (6)$$

- Accordingly, the involuntary hand motion is estimated as

$$\eta_{p-i}(k) = \boldsymbol{x}_{\eta_{p-i}}^T(k)\hat{\boldsymbol{w}}_{\eta_{p-i}}(k) \qquad (7)$$

The proposed KFE-BMFLC

- The complete hand motion (the observation model within the KF recursion) is modeled as follows

$$\eta_p(k) = \boldsymbol{x}_{\eta_p}^T(k)\boldsymbol{w}_{\eta_p}(k) + v(k), \qquad (8)$$

- The state space model is constructed as follows with the state vector defined a $\boldsymbol{w}_{\eta_p}(k)$

$$\boldsymbol{w}_{\eta_p}(k) = \boldsymbol{F}(k)\boldsymbol{w}_{\eta_p}(k-1) + \boldsymbol{\psi}(k), \qquad (9)$$

- $\boldsymbol{\psi}(k)$ is the state uncertainties and $\boldsymbol{F}(k) \triangleq \rho\boldsymbol{I}$, where ρ is the dynamic memory windowing pole adopted from E-BMFLC.
- Instead of using the high-dimensional KF, the proposed reduced-order tremor extraction approach runs $N_{\text{RO}} > 1$ number of reduced-order state-space models. This can be achieved by spatially decomposing the state mode given by Eq. (9) as follows

$$\begin{bmatrix} \boldsymbol{w}_{\eta_p}^{(1)}(k) \\ \vdots \\ \boldsymbol{w}_{\eta_p}^{(l)}(k) \\ \vdots \\ \boldsymbol{w}_{\eta_p}^{(N_{\text{RO}})}(k) \end{bmatrix} = \begin{bmatrix} \rho^{(1)}\boldsymbol{w}_{\eta_p}^{(1)}(k-1) \\ \vdots \\ \rho^{(l)}\boldsymbol{w}_{\eta_p}^{(l)}(k-1) \\ \vdots \\ \rho^{(N_{\text{RO}})}\boldsymbol{w}_{\eta_p}^{(N_{\text{RO}})}(k-1) \end{bmatrix} + \begin{bmatrix} \boldsymbol{\psi}^{(1)}(k) \\ \vdots \\ \boldsymbol{\psi}^{(l)}(k) \\ \vdots \\ \boldsymbol{\psi}^{(N_{\text{RO}})}(k) \end{bmatrix}. \qquad (10)$$

- In order to estimate $\hat{\boldsymbol{w}}_{\eta_p}(k|k)$, we need to combine $\hat{\boldsymbol{w}}_{\eta_p}^{(l)}(k|k)$s for $1 \leq l \leq N_{\text{RO}}$,

$$\begin{aligned}
\hat{\boldsymbol{w}}_{\eta_p}^{(l)}(k|k-1) &= \rho^{(l)} \times \hat{\boldsymbol{w}}_{\eta_p}^{(l)}(k-1|k-1) \\
\boldsymbol{P}^{(l)}(k|k-1) &= [\rho^{(l)}]^2 \times \boldsymbol{P}^{(l)}(k-1|k-1) + \boldsymbol{Q}^{(l)} \\
\boldsymbol{K}^{(l)}(k) &= \boldsymbol{P}^{(l)}(k|k-1)\boldsymbol{x}_{l\eta_p}(k)\left[\boldsymbol{x}_{l\eta_p}^T(k)\boldsymbol{P}^{(l)}(k|k-1)\boldsymbol{x}_{l\eta_p}(k)+R^{(l)}\right]^{-1} \\
\hat{\boldsymbol{w}}_{\eta_p}^{(l)}(k|k) &= \hat{\boldsymbol{w}}_{\eta_p}^{(l)}(k|k-1) + \boldsymbol{K}^{(l)}(k)(y^{(l)}(k) - \boldsymbol{x}_{l\eta_p}^T(k)\hat{\boldsymbol{w}}_{\eta_p}^{(l)}(k|k-1)) \\
\boldsymbol{P}^{(l)}(k|k) &= \left[\boldsymbol{I} - \boldsymbol{K}^{(l)}(k)\boldsymbol{x}_{l\eta_p}^T(k)\right]\boldsymbol{P}^{(l)}(k|k-1).
\end{aligned} \qquad (11)$$

- We propose to use multiple adaptive models for each of the original RKFs to not only consider the effects of localized noise statistics but also consider different localized manipulation factors. We suppose to use n adaptive models where $n > 1$.
- The innovation residual $\boldsymbol{z}_i^{(l)}(k|k-1)$ and innovation covariance $S_i^{(l)}(k|k-1)$ provide the prediction error of each filter and can be defined as $z_i^{(l)}(k|k-1)=\boldsymbol{w}_{i\eta_p}^{(l)}(k) - \hat{\boldsymbol{w}}_{i\eta_p}^{(l)}(k|k-1)$ and $S_i^{(l)}(k|k-1)=[\boldsymbol{x}_{i\eta_p}^{(l)}(k)]^T\boldsymbol{P}_i^{(l)}(k|k-1)\boldsymbol{x}_{i\eta_p}^{(l)}(k) + R_i^{(l)}$ where $\boldsymbol{P}_i^{(l)}(k|k-1)$ is the error covariance matrix and $R_i^{(l)}$ is the measurement uncertainties covariance.
- The normalized weights can be represented as $\Theta_i^{(l)} = \gamma_i^{(l)}(k|k-1)\omega_i^{(l)}(k-1|k-1)$ & $\omega_i^{(l)}(k|k) = \frac{\Theta_i^{(l)}}{\sum_{j=1}^{n}\Theta_j^{(l)}}$ where $\gamma_i^{(l)}(k|k-1)$ is defined as

$$\gamma_i^{(l)}(k|k-1) = \det(S_i^{(l)}(k|k-1))^{-\frac{1}{2}} \times \exp\left[-\frac{1}{2}[\boldsymbol{z}_i^{(l)}(k|k-1)]^T[S_i^{(l)}(k|k-1)]^{-1}\boldsymbol{z}_i^{(l)}(k|k-1)\right] \qquad (12)$$

- By fusing all the n estimates we will have the final estimates as $\hat{\boldsymbol{w}}_{\eta_p}^{(l)}(k|k) = \sum_{i=1}^{n}\omega_i^{(l)}(k|k)\hat{\boldsymbol{w}}_{i\eta_p}^{(l)}(k|k)$.

Conclusion

- The proposed KFE-BMFLC framework is capable of reducing the computational overhead of the KF-based implementation of E-BMFLC by decomposing the overall large scale estimation problem into several lower dimensional sub-systems.
- KFE-BMFLC filtering is able to deal with the inherit structural uncertainty of the constructed reduced-order state-space model by utilization of multiple-models adaptive estimation techniques.

References

[1] S.F. Atashzar, M. Shahbazi, O. Samotus, M. Tavakoli, M.S. Jog, R.V. Patel, "Characterization of Upper-Limb Pathological Tremors: Application to Design of an Augmented Haptic Rehabilitation System," *IEEE J. Sel. Topics Signal Process.*, 2016.

Experimental Results

- The *actual* tremor signal is extracted from the complete hand movement using an *offline post-processing* method.

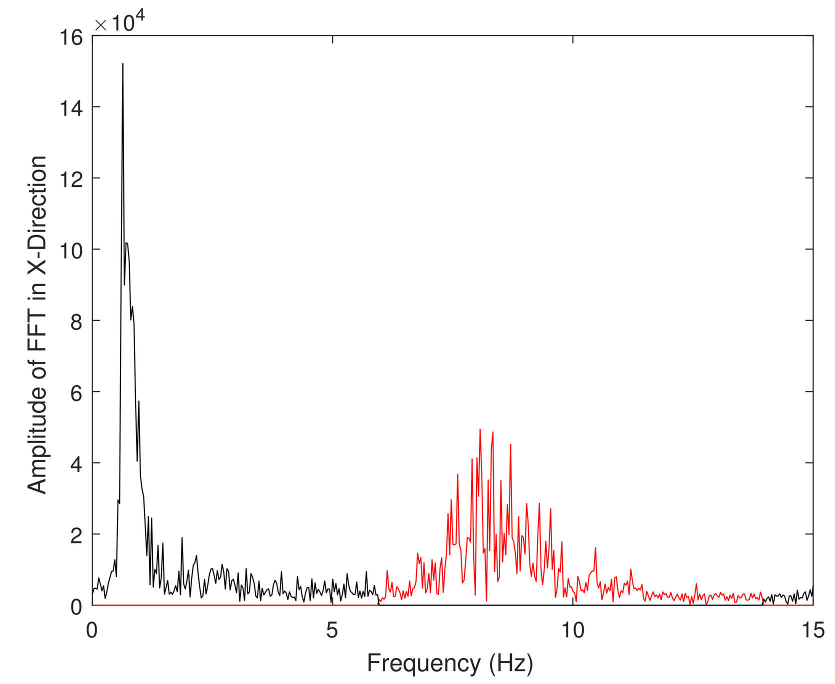

Figure 1: The tremor truncation in the frequency domain. The red part of the signal represents the involuntary hand motion regarding a PD patient.

- In this test, the KF is reduced into three subsystems.
- The first localized KF is allocated to low frequencies, the second one is for the frequencies of the considered tremor (i.e., $6-14Hz$) and the third one is allocated to high frequencies.
- The frequency difference has been considered as $\Delta f = 0.5Hz$.
- The Normalized Root Mean-Square Error (NRMSE) is considered to measure the estimation inaccuracy which is defined as $\text{NRMSE} = \text{RMSE}/(s_{\max} - s_{\min})$ when

$$\text{RMSE} = \sqrt{(\sum_{k=1}^{n}(\hat{s}(k) - s(k))^2)/n}$$

Estimation Methods	NRMSE 1	NRMSE 2	NRMSE 3	NRMSE 4
KFE-BMFLC	0.0426	0.0538	0.0568	0.0517
E-BMFLC	0.0551	0.0603	0.0654	0.0582

- Accuracy evaluation of KFE-BMFLC & E-BMFLC for the extracted tremor with four different tremor data. It is observed that KFE-BMFLC is able to increase the estimation precision in every single tremor data in comparison with E-BMFLC.

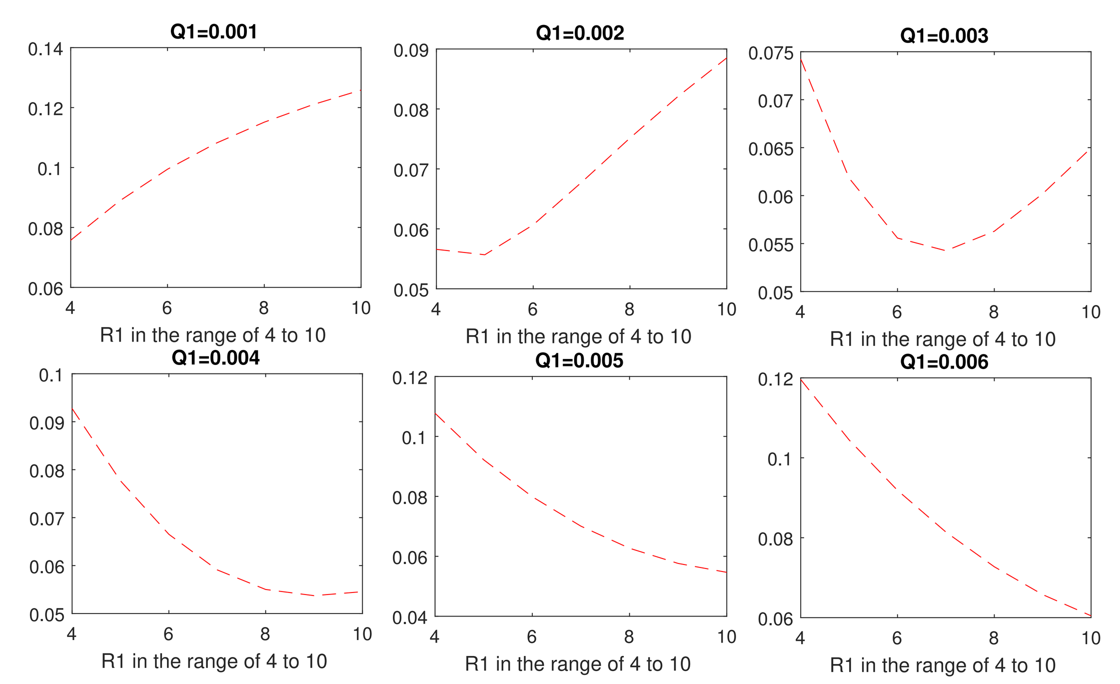

Figure 2: NRMSE sensitivity analysis regarding multiple adaptive reduced-order Kalman filtering based on the variation of $Q_1^{(l)}$ and $R_1^{(l)}$.