# GLOBAL MULTIVIEW REGISTRATION USING NON-CONVEX ADMM

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International Conference on Image Processing 2017

## Multiview Registration

- 3D reconstruction of object from multiple images taken using range scanner.
- Different point clouds with partial overlap information merged together in a common reference frame.

**Figure:** Range scanner

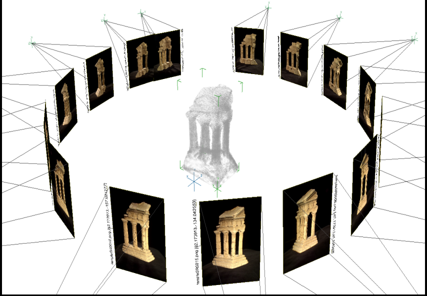
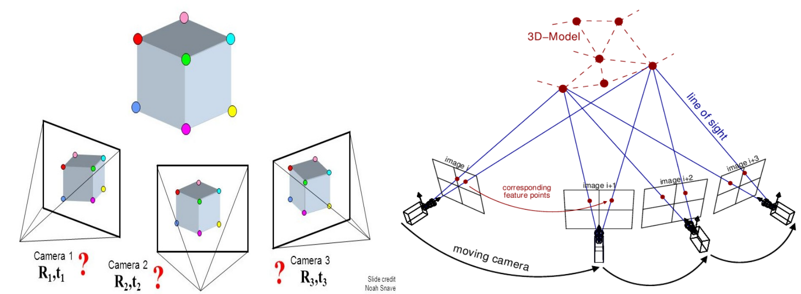


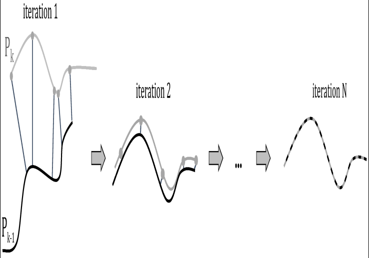
**Figure:** Multiple views

- The data collected contain vertex,face and normal informations.
- The scans with overlapping points aligned together by finding relative camera motions (A rototranslation matrix)
- For multiple scans need relative motions to align them in a common reference frame.



# Pairwise reistration

- State of the art methods like ICP [Besl et al. 1992] and it's variants to align two overlapping scans.
- Two steps:
  - Finding correspondence (assumption: closest points)
  - Registration of corresponding point sets



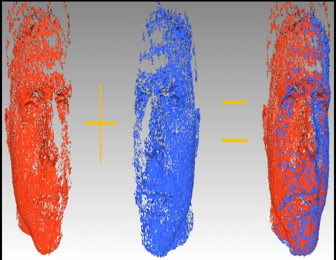
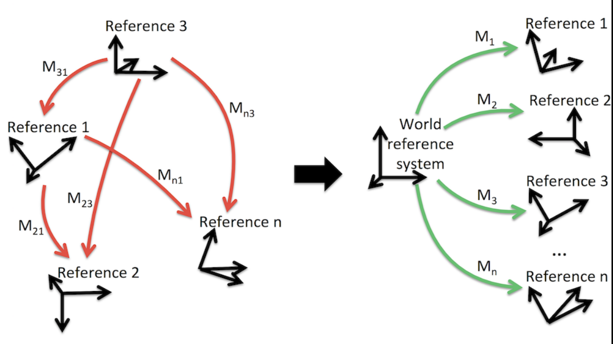


**Figure:** Aligning two surfaces using ICP

# Error propagation sequential registration

- For noisy dataset accumulation of error.
- Bad quality of reconstruction.
- Joint registration: Distribution of error



## To Counter error propagation

- Various variants of ICP [Benjemaa et al. 1996] proposed in to reduce pairwise view registration error.
  - Picky ICP [Zinsser et al. 2003]
  - Go-ICP [Yang et al. 2016]
- Ensure less error after aligning all the views sequentially.
- Relative motion averaging using Lie algebra (state of the art) [Govindu et al. 2014]

## Our approach

Objective function incorporates all the scans at a time.

M: no of point clouds

$\mathcal{P}_1, \ldots, \mathcal{P}_M$: Point clouds

Local coordinates of i-th point cloud $\mathcal{P}_i$ :

$$\{\boldsymbol{x}_{ij}^k : 1 \leq k \leq n_{ij}\}$$

Local coordinates of j-th point cloud $\mathcal{P}_j$ :

$$\{\boldsymbol{x}_{ji}^k : 1 \leq k \leq n_{ij}\}$$

Ideally, when $i \sim j$, we have for $1 \leq k \leq n_{ij}$,

$$\mathsf{R}_i \boldsymbol{x}_{ij}^k + \boldsymbol{t}_i = \mathsf{R}_j \boldsymbol{x}_{ji}^k + \boldsymbol{t}_j. \tag{1}$$

To encounter error objective function:

$$\min \quad \sum_{i \sim j} \sum_{k=1}^{n_{ij}} \| \mathsf{R}_i \boldsymbol{x}_{ij}^k + \boldsymbol{t}_i - \mathsf{R}_j \boldsymbol{x}_{ji}^k - \boldsymbol{t}_j \|^2 \qquad \text{[krishnan et al. 2005]}$$

Variables $\mathsf{R} = [\mathsf{R}_1 \cdots \mathsf{R}_M] \in \mathbb{R}^{3 \times 3M}$ and $\mathsf{T} = [\boldsymbol{t}_1 \cdots \boldsymbol{t}_M] \in \mathbb{R}^{3 \times M}$, we can write the objective in (2) as

$$\sum_{i \sim j} \sum_{k=1}^{n_{ij}} \| \mathsf{R} \boldsymbol{d}_{ij}^k + \mathsf{T} \boldsymbol{e}_{ij} \|^2, \tag{2}$$

$\boldsymbol{d}_{ij}^k = (\boldsymbol{e}_i \otimes \mathsf{I}) \boldsymbol{x}_{ij}^k - (\boldsymbol{e}_j \otimes \mathsf{I}) \boldsymbol{x}_{ji}^k, \boldsymbol{e}_{ij} = \boldsymbol{e}_i - \boldsymbol{e}_j$, and $\boldsymbol{e}_i \in \mathbb{R}^M$

- First diffrentiate the objective w.r.t $\mathsf{T}$(free variable).
- Put $\mathsf{T}^*$ in the objective.
- Objective becomes after some algebraic manipulation

$$\text{Trace}(\mathsf{C}\mathsf{R}^\top \mathsf{R})$$

- The optimization problem

$$\min_{\mathsf{R}_1,\ldots,\mathsf{R}_M} \quad \text{Trace}(\mathsf{C}\mathsf{R}^\top \mathsf{R}) \quad \text{s.t.} \quad \mathsf{R}_1,\ldots,\mathsf{R}_M \in \mathbb{SO}(3). \tag{3}$$

- Can be converted into a rank-constrained SDP

$$\min_{\mathsf{G}} \quad \text{Trace}(\mathsf{C}\mathsf{G}) \;\; \text{s.t.} \;\; \mathsf{G} \succeq 0, \mathsf{G}_{ii} = \mathsf{I}, \text{and } \text{rank}(\mathsf{G}) = 3. \tag{4}$$

# Convex Relaxation and problems

- Convex relaxation

$$\min_{G} \quad \mathrm{Trace}(CG) \quad \text{s.t.} \quad G \succeq 0 \ \text{ and } \ G_{ii} = I. \tag{5}$$

- Can be solved by ADMM [Sanyal et al. SPL 2017]
- Need to project on rank 3 matrix domain: Suboptimal Results [Chaudhury et al. 2015]
- Slow rate of convergence

# Non-convex ADMM

- So we solve

$$\min_{G} \quad \text{Trace}(CG) \quad \text{s.t.} \quad G \succeq 0, G_{ii} = I, \text{and } \text{rank}(G) \leq 3. \qquad (6)$$

- (4) and (6) are equivalent (see ICIP paper)
- we can write (6) as

$$\min_{G,H} \quad \text{Trace}(CG) \quad \text{s.t.} \quad G \in \Omega, H \in \Theta, \quad \text{and } G - H = 0. \qquad (7)$$

$\Omega$ : Set of symmetric positive semidefinite matrices , size $3M \times 3M$ , rank at most 3

$\Theta$ : Set of symmetric matrices , size $3M \times 3M$ whose $3 \times 3$ block diagonals are I.

- No need to project $G^{\star}$ in Rank 3 domain

## ADMM steps

- Augmented lagrangian

$$\mathcal{L}_\rho(\mathsf{G}, \mathsf{H}, \boldsymbol{\Lambda}) = \mathrm{Trace}(\mathsf{CG}) + \mathrm{Trace}(\boldsymbol{\Lambda}(\mathsf{G} - \mathsf{H})) + \frac{\rho}{2}\|\mathsf{G} - \mathsf{H}\|_{\mathrm{F}}^2,$$

- G update

$$\mathsf{G}^{k+1} = \underset{\mathsf{G}\in\Omega}{\mathrm{argmin}}\ \mathcal{L}_\rho(\mathsf{G}, \mathsf{H}^k, \boldsymbol{\Lambda}^k), \quad (8)$$

- H update

$$\mathsf{H}^{k+1} = \underset{\mathsf{H}\in\Theta}{\mathrm{argmin}}\ \mathcal{L}_\rho(\mathsf{G}^{k+1}, \mathsf{H}, \boldsymbol{\Lambda}^k), \quad (9)$$

- $\boldsymbol{\Lambda}$ update

$$\boldsymbol{\Lambda}^{k+1} = \boldsymbol{\Lambda}^k + \rho(\mathsf{G}^{k+1} - \mathsf{H}^{k+1}). \quad (10)$$

- Projection of G onto set $\Omega$ :

  Objective in (8) can be equivalently written as :

$$(\rho/2)\|\mathsf{G} - \mathsf{A}\|_{\mathrm{F}}^2 + c. \tag{11}$$

  Where $\mathsf{A} = (\mathsf{H}^k - \rho^{-1}(\mathsf{C} + \boldsymbol{\Lambda}^k))$

  Then $\mathsf{G}^{k+1} = \Pi_\Omega(\mathsf{A})$

  Let $\mathsf{A} = \lambda_1 \boldsymbol{u}_1 \boldsymbol{u}_1^\top + \cdots + \lambda_{3M} \boldsymbol{u}_{3M} \boldsymbol{u}_{3M}^\top$

  where $\lambda_1 \geq \cdots \geq \lambda_{3M}$

$$\Pi_\Omega(\mathsf{A}) = \sum_{i=1}^{3} \max(\lambda_i, 0) \boldsymbol{u}_i \boldsymbol{u}_i^\top.$$

- Projection of $\mathsf{H}$ onto set $\Theta$ :

Objective in (9) can be equivalently written as :

$$(\rho/2)\|\mathsf{H}-\mathsf{A}\|_{\mathrm{F}}^{2}+d. \tag{12}$$

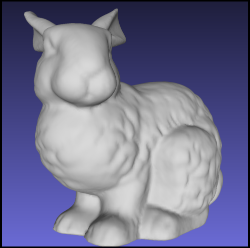
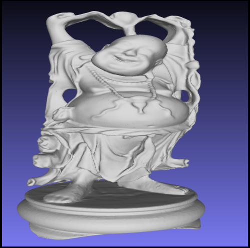
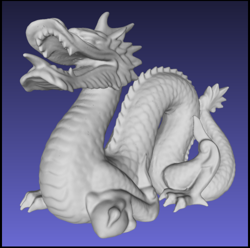
where $\mathsf{A}=(\mathsf{G}^{k+1}+\rho^{-1}\boldsymbol{\Lambda}^{k})$

Then $\mathsf{H}^{k+1}=\Pi_{\Theta}(\mathsf{A})$

$\Pi_{\Theta}(\mathsf{A})$ : $(\mathsf{A})_{[ii]}=I_3$

# Results on datsets from stanford 3D repository

- Visual and quantitative experiments done on synthetic datasets from Stanford repository.
- Results are shown for sequential ICP, MAICP and our method

3D models : Bunny, Happy Buddha and Dragon

# Bunny

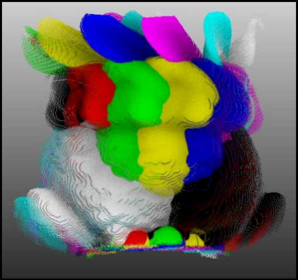


**Figure:** Before alignment

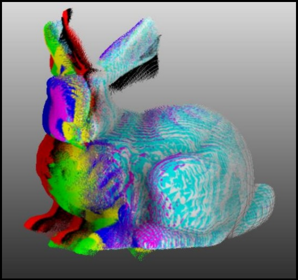


**Figure:** Sequential ICP

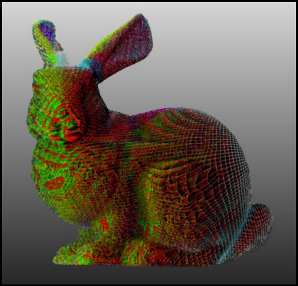


**Figure:** MAICP

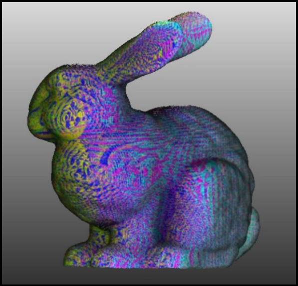


**Figure:** Our method

# Buddha

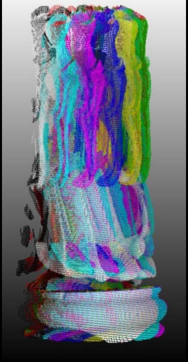


**Figure:** Before alignment

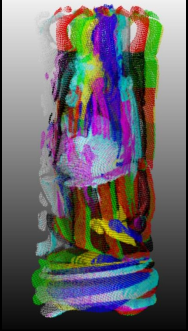


**Figure:** Sequential ICP

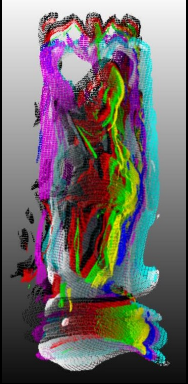


**Figure:** MAICP

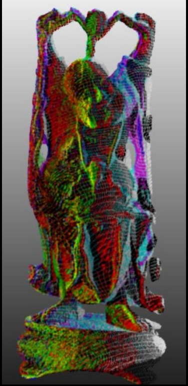


**Figure:** Our method

# Dragon

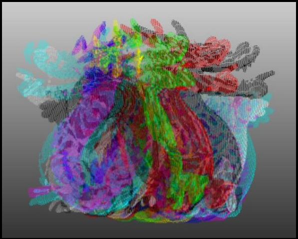


Figure: Before alignment

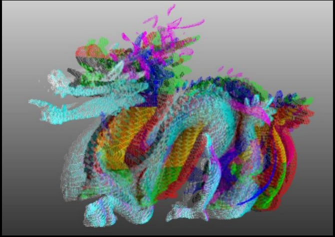


Figure: Sequential ICP

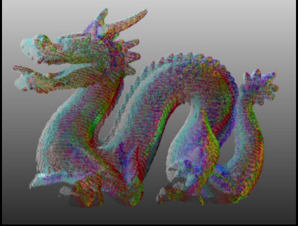


**Figure:** MAICP

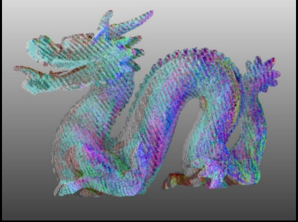


**Figure:** Our method

# Axis angle error comparison

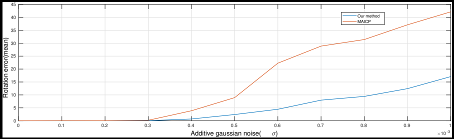



**Figure:** Bunny

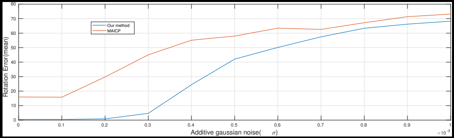



**Figure:** Buddha

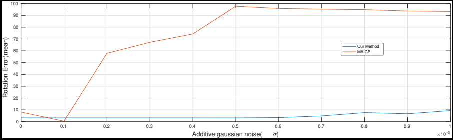



**Figure:** Dragon

# Timing Comparison

| Total time (sec) | | |
|---|---|---|
| Standard datasets | Our method | MAICP |
| Bunny | 401 | 364 |
| Buddha | 175 | 462 |
| Dragon | 165 | 771 |

# Comparison with convex formulation

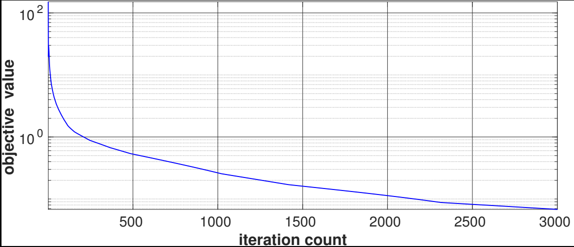



**Figure:** Convex formulation

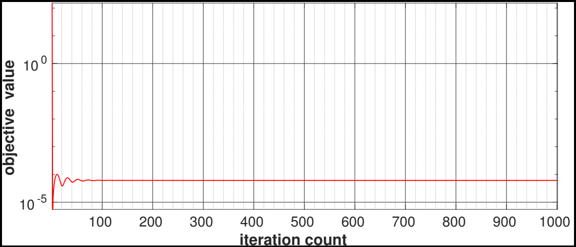



**Figure:** Non-convex formulation

Thanks for listening.