Performance of Interleaved Training for Single-User Hybrid Massive MIMO Downlink

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Background

Massive MIMO: high multiplexing and array gain.

↓ **Implementation difficulties for full-digital structure: too many radio frequency (RF) chains**

Hybrid beamforming structure: a combination of analog RF processing (e.g., phase shifter networks) and low-dimensional baseband processing

↓ **Difficulty of obtaining full antenna-domain channel state information (CSI)**

Codebook-based beam training: beam-domain effective channel estimation; finding the desired beams;

The remaining difficulty: finding the desired beams with affordable training overhead**.**

Existing codebook-based beam training schemes

1. **Exhaustive search: large training overhead [Liu et.al., 2017]**

2. **Hierarchical search: sensitive to training SNR and multi-path [Alkhateeb et.al., 2014]**

3. **Local search: sub-optimal, sensitive to training initialization [Gao et.al., 2016]**

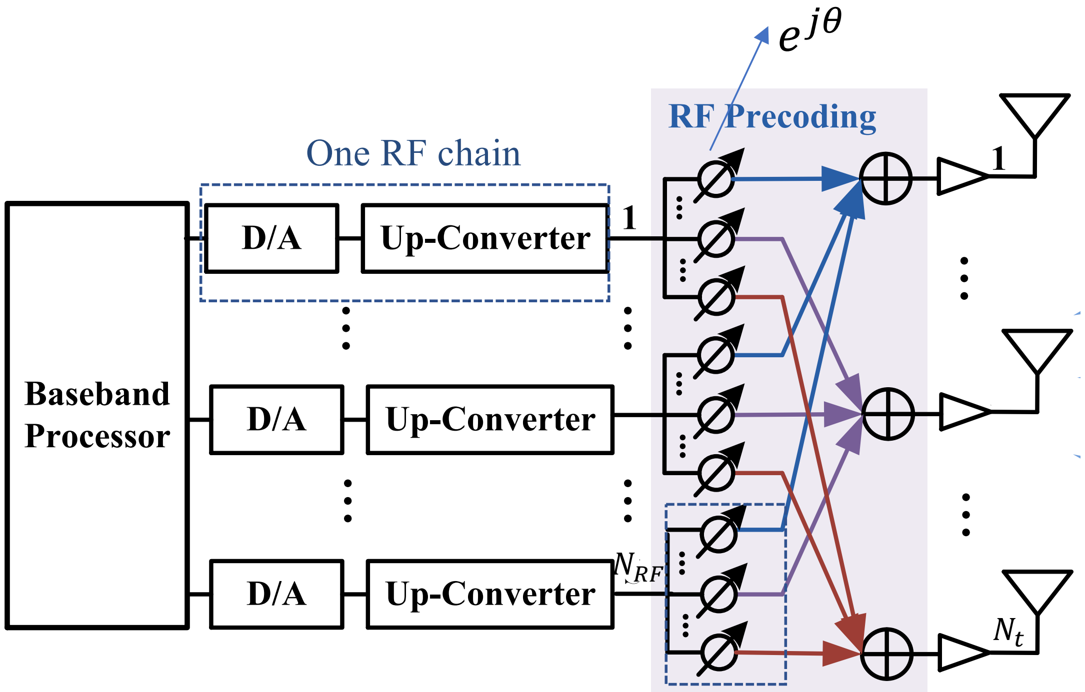
Motivations:

1. **De-coupled training and transmission design**
2. **Existing works focus on the throughput or diversity gain**

↓ **exploiting the interleaved training idea [Koyuncu et.al., 2015]**

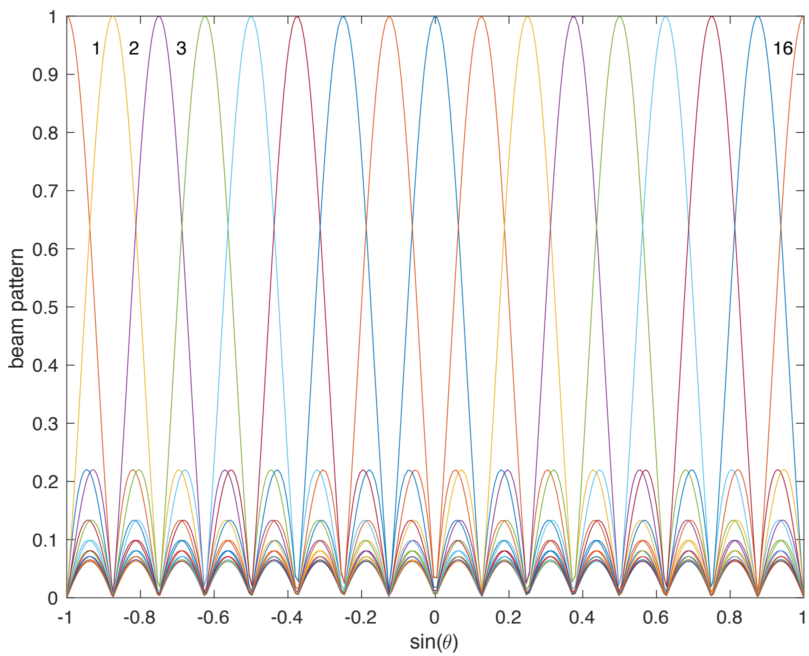
Beam-based training design jointly with the data transmission design for single-user hybrid massive antenna downlink with outage probability as the performance metric

Hybrid massive MIMO downlink



$N_{RF} < N_t$

Beam pattern of Analog Beams in the codebook



Channel model

L-path channel with DFT direction matrix

$$\mathbf{h} = \mathbf{D}\bar{\mathbf{h}} = [\mathbf{d}_1 \quad \cdots \quad \mathbf{d}_{N_t}] \begin{bmatrix} \bar{h}_1 \\ \vdots \\ \bar{h}_{N_t} \end{bmatrix}$$

The i-th direction vector

$$\mathbf{d}_i = [1, e^{-j2\pi(i-1)/N_t}, ..., e^{-j2\pi(i-1)(N_t-1)/N_t}]^T, \forall i$$

Direction indices of L paths $\mathcal{I} = \{I_1, ..., I_L\}$

discrete uniform distribution with each element on $[1, N_t]$

Channel gain for the i-th direction

$$\bar{h}_i \sim \mathcal{CN}(0, 1/L) \quad \text{for } i \in \mathcal{I} \text{ and } \bar{h}_i = 0 \text{ for } i \notin \mathcal{I}$$

Transmission model

Transmit Power — Analog beamforming — Baseband beamforming — Signal — Local noise

$$y = \sqrt{P}\mathbf{h}^T \mathbf{F}_{RF}\mathbf{f}_{BB} s + n = \sqrt{P}\bar{\mathbf{h}}^T \mathbf{D}\mathbf{F}_{RF}\mathbf{f}_{BB} s + n,$$

An outage happens when

$$\|\bar{\mathbf{h}}^T \mathbf{D}\mathbf{F}_{RF}\mathbf{f}_{BB}\|^2 \leq \alpha \triangleq \frac{2^{R_{th}} - 1}{P},$$

Target rate (bps/Hz)

Normalized signal-to-noise-ratio (SNR) threshold

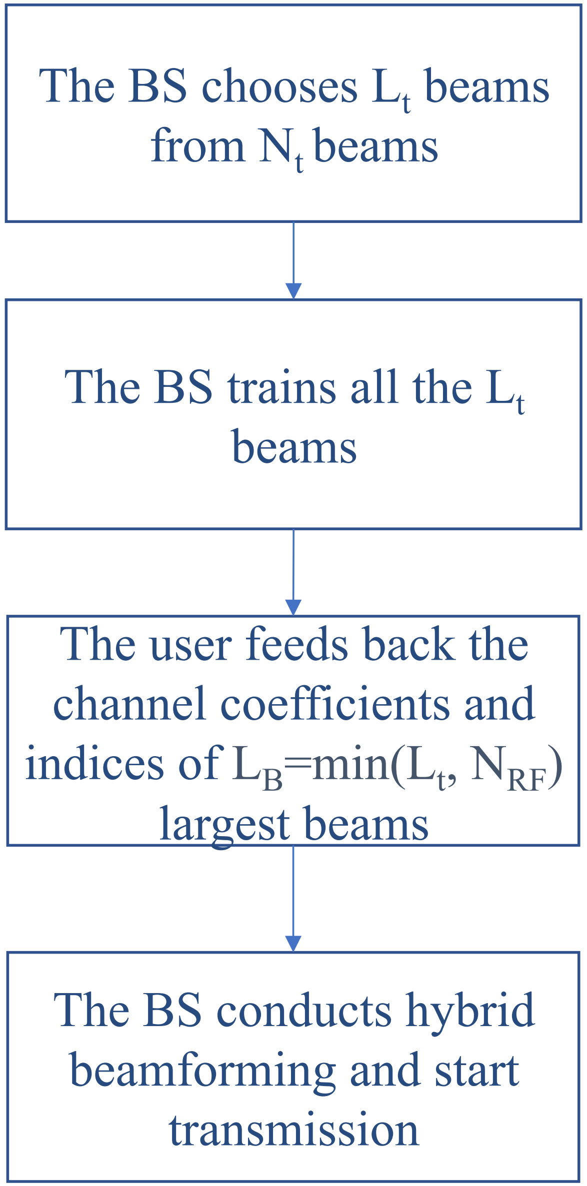
Outage Probability: $\text{out}(\mathbf{F}_{RF}, \mathbf{f}_{BB}) = \Pr(\|\bar{\mathbf{h}}^T \mathbf{D}\mathbf{F}_{RF}\mathbf{f}_{BB}\|^2 \leq \alpha).$

With beams $\mathcal{S} = \{s_1, ..., s_{L_s}\}$ **chosen**:

$$\mathbf{F}_{RF} = \left[\frac{\mathbf{d}^*_{s_1}}{\sqrt{N_t}}, ..., \frac{\mathbf{d}^*_{s_{L_s}}}{\sqrt{N_t}}\right], \mathbf{f}_{BB} = \frac{[\bar{h}_{s_1}, ..., \bar{h}_{s_{L_s}}]^H}{\sqrt{\|\bar{h}_{s_1}\|^2 + ... + \|\bar{h}_{s_{L_s}}\|^2}}$$

$$\Pr\left(\sum_{i=1}^{L_s} \|\bar{h}_{s_i}\|^2 \leq \alpha / N_t\right)$$

Existing Non-interleaved schemes (NIT-SU)

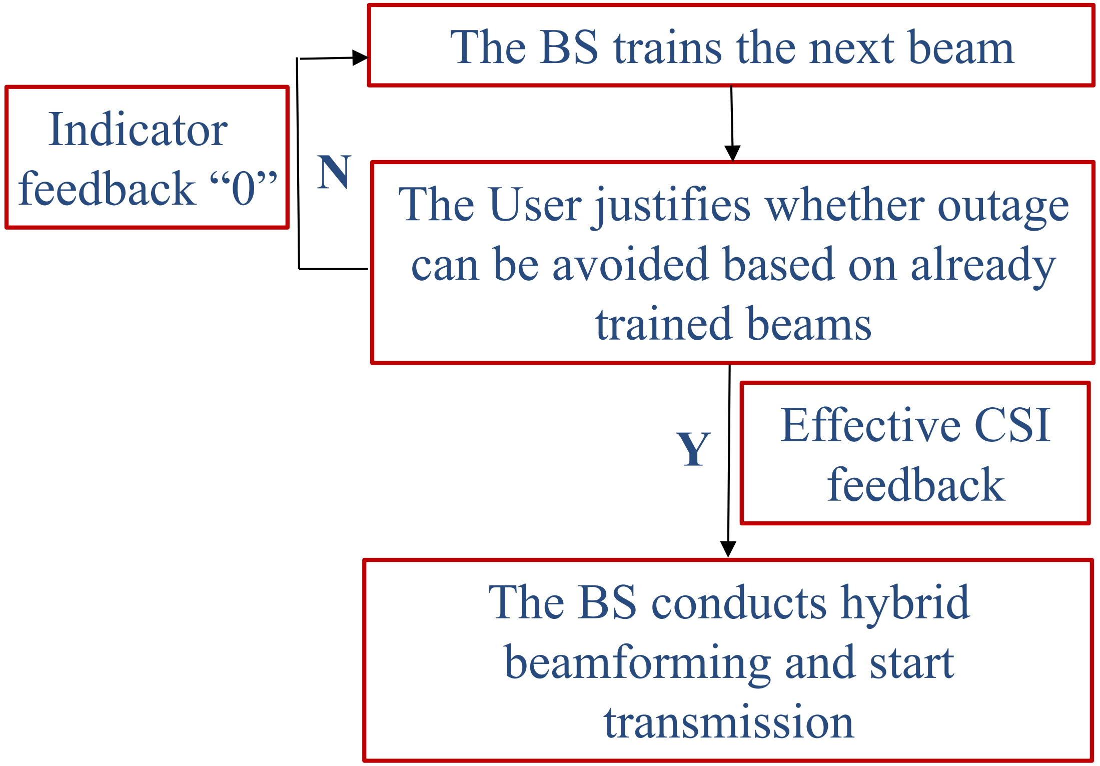


DFT codebook N_t beams

If $L_t = N_t$, full training

If $L_t < N_t$, partial training

Interleaved Training and Joint Transmission (IT-SU)



- Interleaved beam training and CSI/indicator feedback
- The user itself monitors the training procedure
- No outage performance loss
- For each channel realization: stop training when just-enough beams are trained, leading to smaller average training length

Training Length Result

For channels finite path number **L**, when **Nt** >> 1, with any given normalized SNR threshold **α** and RF chain number N_{RF}, the average training length of the IT-SU scheme is

$$T_{\text{IT-SU}} = \frac{N_t}{L+1} + \mathcal{O}(1).$$

- **Compared with non-interleaved scheme with full training, $T_{IT\text{-}SU}$ can be shortened by a scale of 1/(L+1).**
- **$T_{IT\text{-}SU}$ becomes asymptotically independent of α.**

General analytical expression of average training length is referred to Theorem 1 in the full paper [Zhang et.al., 2018]

Outage Performance Result

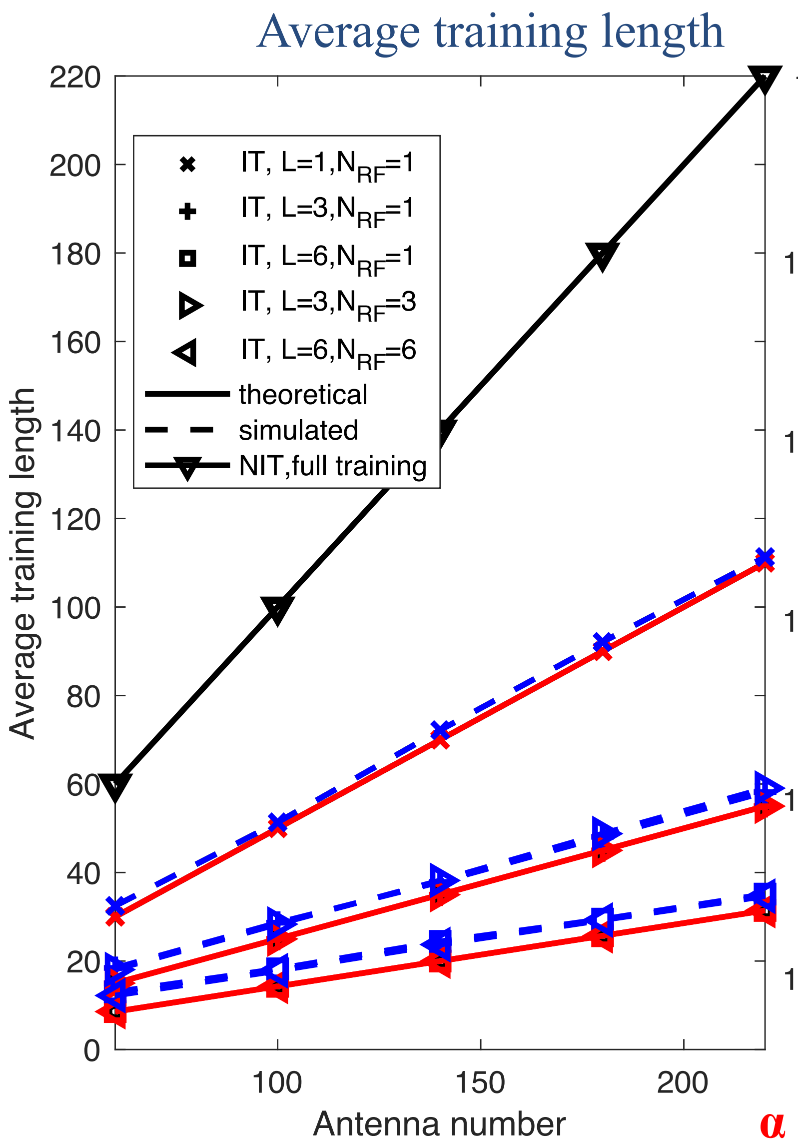
For channels with finite path number L, when Nt >> 1 and $N_{RF} = 1$, with any given normalized SNR threshold α, the outage probability of IT-SU scheme is

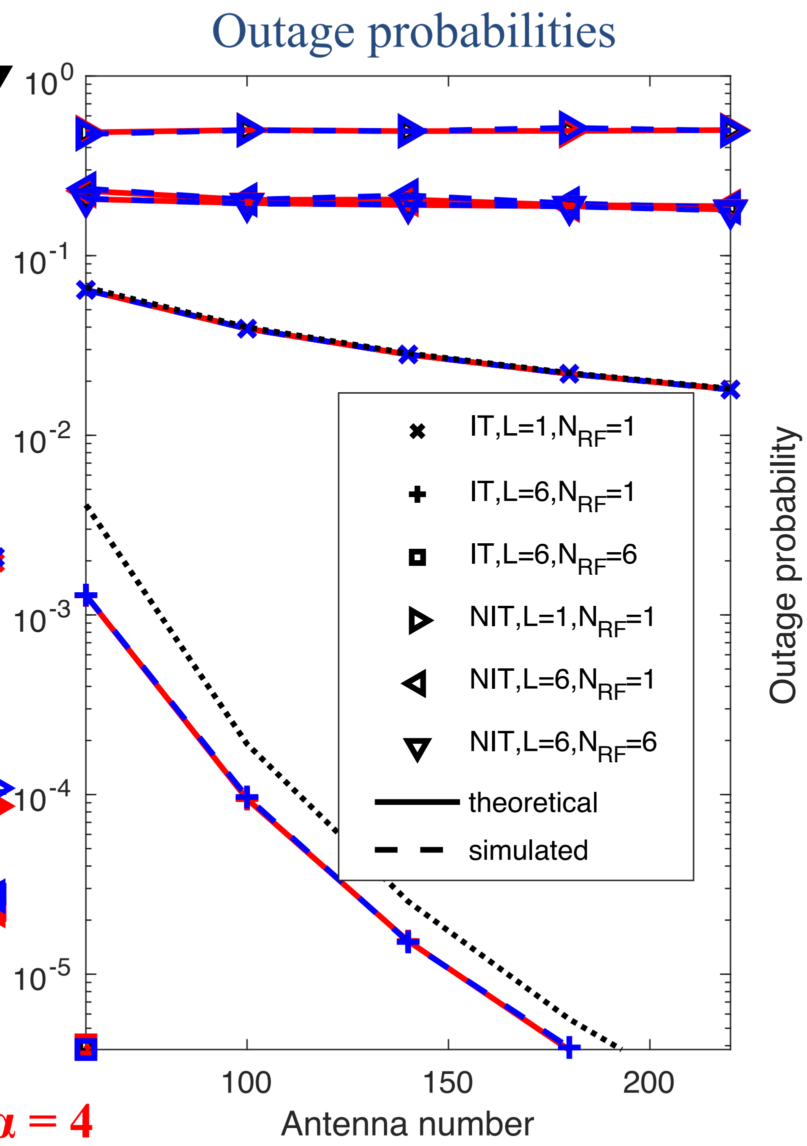
$$\mathrm{out}(IT\text{-}SU) = (\alpha L/N_t)^L + \mathcal{O}\left(N_t^{-(L+1)}\right)$$

- **More RF chains can further reduce the outage probability.**

Analytical expressions of outage probability of NIT-SU scheme with full and partial training are referred to Theorem 2 and Corollary 2.

Simulations





$\alpha = 4$

Conclusions

- ✓ Single-user hybrid massive antenna downlink
- ✓ finite L-path channels
- ✓ outage probability

Interleaved beam-based training and joint beamforming design

1. Making training length adaptive to channel realizations;
2. Explicit expressions of the average training length and outage probability;
3. The same outage performance as the traditional full-training scheme but with significantly lower training overhead.

Extended works

1. **Performance analysis for single-user case and channel with linearly increasing L with respect to N_t**
   - ✓ **The joint effect of N_{RF} and SNR threshold and the increasing ratio (channel angular spread) on the average training length**
2. **Low complexity design of interleaved training scheme for multiuser case**
   - ✓ **Low complexity beam assignment + the BS plays the decision maker**

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Other problems:

1. **Effect of beam-space channel estimation error**
2. **Feedback overhead**
3. **Multi-cell and cooperative systems**

Thank you!

Any questions?