



Deep Residual Learning for Model-Based Iterative CT Reconstruction using Plug-and-Play Framework

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April 18, 2018

Model-Based Iterative Reconstruction (MBIR)

- Computed Tomography (CT) Reconstruction
 - Diagnostic Radiology
 - > Additive Manufacturing Inspection
- MBIR Flowchart



Advantage of MBIR over Filtered-Back Projection (FBP)

Superior Image Quality: Low Noise and High Resolution



Prior Model in MBIR

- Accurate prior modeling is critical to the image quality of MBIR.
- Typical Prior Model: MRF
 - Penalize intensity fluctuation in the neighborhood
 - Challenge: Noise-induced fluctuation vs. underlying object
- Solution: Prior Model from an Image Database



MBIR Optimization

MAP Estimation



 $x \in \mathbb{R}^N$: reconstructed image, $y \in \mathbb{R}^M$: measured CT scan

 $\mathbf{A} \in \mathbb{R}^{M \times N}$: system matrix for CT scan, $\mathbf{W} \in \mathbb{R}^{M \times M}$: measurement noise variance

 $\sigma \in \mathbb{R}$: regularization parameter, $\Phi: \mathbb{R}^N \to \mathbb{R}$: prior model

- First-order iterative optimization
 - Iterative Coordinate Descent (ICD) / Ordered Subset (OS)
 - Prior model should be first-order differentiable.

Not flexible for Data-driven Prior

Plug-and-Play (PnP) Framework

• Variable Splitting

Separate variable

$$(\hat{x}, \hat{v}) = \arg\min_{x, v} \left[\frac{1}{2} \| \mathbf{A}x - y \|_{\mathbf{W}}^{2} + \frac{1}{2\sigma^{2}} \Phi(v) \right] \text{ s.t. } x = v$$

Alternating Direction Method of Multipliers (ADMM)

$$L_{\lambda}(x,v,u) = \frac{1}{2} \|\mathbf{A}x - y\|_{\mathbf{W}}^{2} + \frac{1}{2\sigma^{2}} \Phi(v) + \frac{1}{2\lambda^{2}} \|x - v + u\|_{2}^{2} \longleftarrow \frac{\text{Augmented}}{\text{Lagrangian}}$$

Step 1: Reconstruction Module

 $u \leftarrow u + (\hat{x} - \hat{v})$

$$\hat{x} = \arg\min_{x} L_{\lambda}(x, \hat{v}, u) = \arg\min_{x} \left[\frac{1}{2} \|\mathbf{A}x - y\|_{\mathbf{W}}^{2} + \frac{1}{2\lambda^{2}} \|x - \hat{v} + u\|_{2}^{2} \right]$$
Step 2: De-noising Module
$$\hat{x} = \operatorname{I}_{\lambda}(\hat{x}, \hat{v}, u) = \operatorname{I}_{\lambda} \left[\frac{1}{2\lambda^{2}} \|\mathbf{A}x - y\|_{\mathbf{W}}^{2} + \frac{1}{2\lambda^{2}} \|x - \hat{v} + u\|_{2}^{2} \right]$$

$$\hat{v} = \arg\min_{v} L_{\lambda}(\hat{x}, v, u) = \arg\min_{v} \left[\frac{1}{2\sigma^2} \Phi(v) + \frac{1}{2\lambda^2} \|\hat{x} + u - v\|_2^2 \right]$$

Step 3: Update Dual Variable

Input noisy image

Independent Module for De-noising

Deep Residual Learning for De-noising

- Deep Neural Network
 - Powerful performance for vision tasks such as de-noising
 - Weights of a neural network learned on large training dataset
 - Challenge: Long training time
- Deep Residual Learning for Efficient Training
 - Bypassing low-freq. image



Deep Residual Learning: Training

• Cost
$$l(\Theta) = \frac{1}{2K} \sum_{k=1}^{K} \left\| R(\tilde{v}_k^{tr}; \Theta) - (\hat{v}_k^{tr} - \tilde{v}_k^{tr}) \right\|_2^2$$

 $\{\tilde{v}_k^{tr}, \hat{v}_k^{tr}\}$: noisy and clean image pairs, $R(\bullet; \Theta)$: Residual neural network

- Training Database
 - 40x40 patches for all slices, Data augmentation (flip, rotation)
 - Randomly selected 256000 patches, mini-batch size: 128



Deep Residual Learning: Testing



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MBIR Result: Qualitative @ 1 iter.



MBIR Result: RMSD



Significant Speed-up with Faster Convergence

Computational Time

- Deep Learning Training Time
 - 4 NVIDIA Titan X GPU (12GB memory)
 - Google Tensor Flow
 - > 65 minutes / 50 epochs
- Deep Learning Testing Time
 - ~10ms/slice
- Standard MBIR and PnP MBIR require similar amount of recon. time per iteration.

Conclusion

- Summary
 - 1. Image prior modeling from FBP/MBIR database
 - Deep Residual Learning for Image De-noising
 - 2. Incorporating the prior model from a database into MBIR
 - Plug-and-Play Optimization Framework
- Deep Residual Learning is effective in reducing the noise and enhancing the resolution in FBP.
- PnP MBIR with deep learning prior significantly improves the image quality compared with standard MBIR.