DATA 61

Fusion of Multiple Multiband Images with Complementary Spatial and Spectral Resolutions

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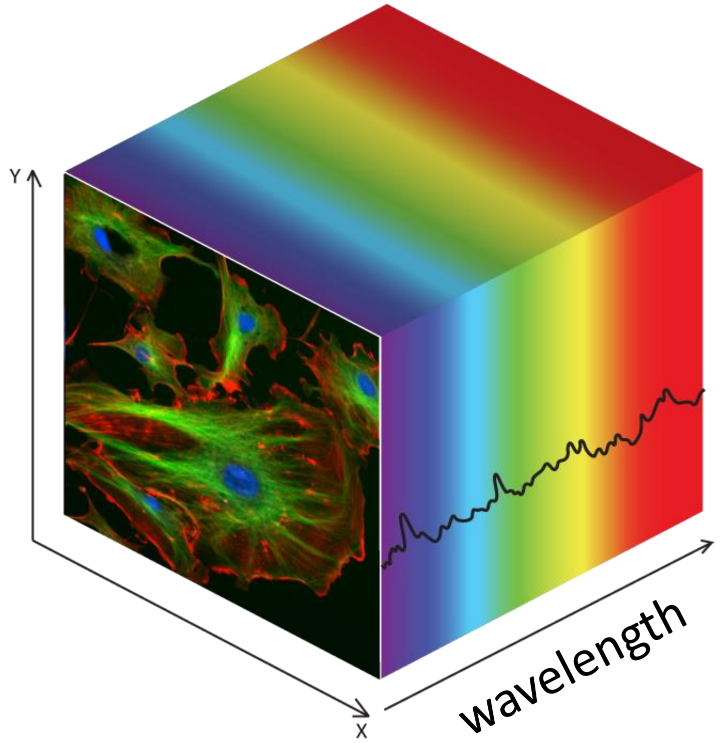
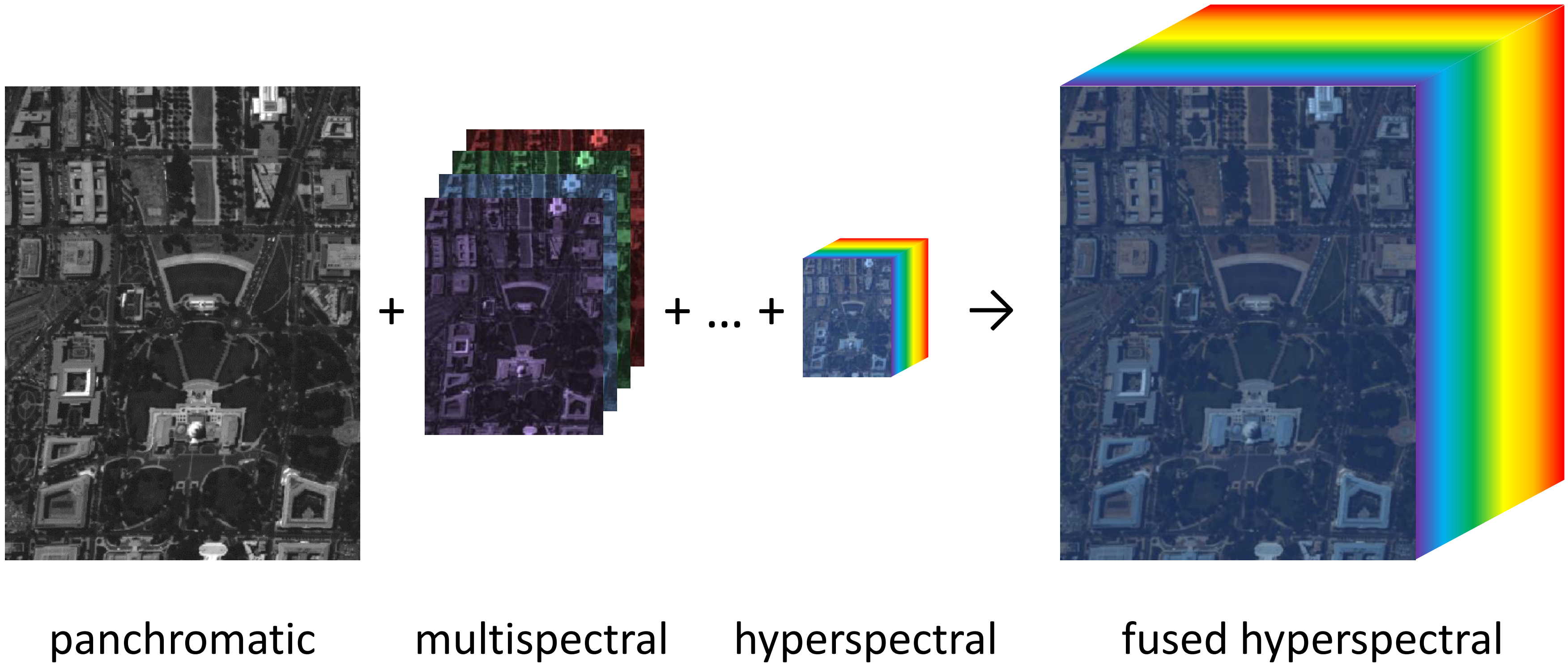
Spectral imaging

- combination of *spectroscopy* and *image forming*
- spatial and spectral information is analysed to detect, identify, or discriminate objects, patterns, or chemical composition of material
- application in computer vision, remote sensing, biomedicine, surveillance, precision agriculture, environmental studies, forensics, nanoparticle research, food science, mining, forestry, etc.



Problem


+
+ ... +
→
panchromatic
multispectral
hyperspectral
fused hyperspectral

Forward observation model

K observed multiband images with L_k bands & N_k pixels:

$$\mathbf{Y}_k = \mathbf{R}_k \mathbf{X} \mathbf{B}_k \mathbf{S}_k + \mathbf{P}_k, k = 1, \dots, K$$

$\mathbf{X} \in \mathbb{R}^{L \times N}$: target multiband image with L spectral bands & N pixels

$\mathbf{R}_k \in \mathbb{R}^{L_k \times N}$: spectral response of the kth sensor

$\mathbf{B}_k \in \mathbb{R}^{N \times N}$: band-independent spatial blurring matrix representing a 2D convolution with the point-spread function of the kth sensor

$\mathbf{S}_k \in \mathbb{R}^{N \times N_k}$: sparse matrix with N_k ones and zeros elsewhere representing a 2D uniform downsampling of ratio $D_k = \sqrt{N/N_k}$

$\mathbf{P}_k \in \mathbb{R}^{L_k \times N_k}$: additive perturbation representing the noise or error associated with the observation of $\mathbf{Y}_k$

Linear mixture model

$$\mathbf{X} = \mathbf{E}\mathbf{A} + \mathbf{P}$$

$\mathbf{E} \in \mathbb{R}^{L \times M}$: matrix of M endmembers

$\mathbf{A} \in \mathbb{R}^{M \times N}$: matrix of endmember abundances

$\mathbf{P} \in \mathbb{R}^{L \times N}$: perturbation matrix accounting for any inaccuracy or mismatch in the model

nonnegativity and sum-to-one assumptions:

$$\mathbf{A} \geq 0, \mathbf{1}_M^\top \mathbf{A} = \mathbf{1}_N^\top, i = 1, \ldots, N$$

Fusion model

$$\mathbf{Y}_k = \mathbf{R}_k \mathbf{E} \mathbf{A} \mathbf{B}_k \mathbf{S}_k + \breve{\mathbf{P}}_k$$

aggregate perturbations:

$$\breve{\mathbf{P}}_k = \mathbf{P}_k + \mathbf{R}_k \mathbf{P} \mathbf{B}_k \mathbf{S}_k$$

- Instead of estimating $\mathbf{X}$ directly, we estimate $\mathbf{A}$ from the observations $\mathbf{Y}_k$, $k = 1, \ldots K$, given the endmember matrix $\mathbf{E}$.
- This substantially reduces the dimensionality of the fusion problem and consequently its computational complexity.
- Estimating $\mathbf{A}$ then $\mathbf{X}$ gives an unmixed fused image.
- It requires the prior knowledge of the endmembers.
- The endmembers can be selected from a library of known spectral signatures or extracted from $\mathbf{Y}_k$.

Solution

assumption: $\breve{\mathbf{P}}_k$, $k = 1, \ldots, K$, are statistically independent and

$$\breve{\mathbf{P}}_k \sim \mathcal{MN}_{L \times N}(\mathbf{0}_{L \times N}, \mathbf{\Lambda}_k, \mathbf{I}_N)$$

maximum-likelihood estimate of $\mathbf{A}$ is the solution of

$$\min_{\mathbf{A}} \frac{1}{2} \sum_{k=1}^{K} \left\| \mathbf{\Lambda}_k^{-1/2} (\mathbf{Y}_k - \mathbf{R}_k \mathbf{E} \mathbf{A} \mathbf{B}_k \mathbf{S}_k) \right\|_{\mathrm{F}}^2$$

- Unregularized MLE problem is usually ill-posed and unidentifiable.

regularized maximum-likelihood:

$$\min_{\mathbf{A}} \frac{1}{2} \sum_{k=1}^{K} \left\| \mathbf{\Lambda}_k^{-1/2} (\mathbf{Y}_k - \mathbf{R}_k \mathbf{E} \mathbf{A} \mathbf{B}_k \mathbf{S}_k) \right\|_{\mathrm{F}}^2 + \alpha \| \nabla \mathbf{A} \|_{2,1} + \imath(\mathbf{A})$$

Solution

regularized maximum-likelihood:

$$\min_{\mathbf{A}} \frac{1}{2}\sum_{k=1}^{K} \left\| \mathbf{\Lambda}_k^{-1/2}(\mathbf{Y}_k - \mathbf{R}_k\mathbf{E}\mathbf{A}\mathbf{B}_k\mathbf{S}_k) \right\|_{\mathrm{F}}^2 + \alpha\|\nabla\mathbf{A}\|_{2,1} + \iota(\mathbf{A})$$

$\|\nabla\mathbf{A}\|_{2,1}$: isotropic vector total-variation penalty

$$\nabla\mathbf{A} = [(\mathbf{A}\mathbf{D}_h)^\top, (\mathbf{A}\mathbf{D}_v)^\top]^\top$$

$\mathbf{D}_h$ and $\mathbf{D}_v$: discrete differential matrix operators

$\alpha \geq 0$: regularization parameter

$$\iota(\mathbf{A}) = \begin{cases} 0 & \mathbf{A} \in \{\mathbf{A} | \mathbf{A} \geq 0, \mathbf{1}_M^\top\mathbf{A} = \mathbf{1}_N^\top\} \\ +\infty & \mathbf{A} \notin \{\mathbf{A} | \mathbf{A} \geq 0, \mathbf{1}_M^\top\mathbf{A} = \mathbf{1}_N^\top\} \end{cases}$$

Algorithm

variable splitting:

$$\min_{\mathbf{A},\{\mathbf{U}_k\},\mathbf{V},\mathbf{W}} \frac{1}{2}\sum_{k=1}^{K} \left\| \mathbf{\Lambda}_k^{-1/2}(\mathbf{Y}_k - \mathbf{R}_k\mathbf{E}\mathbf{U}_k\mathbf{S}_k) \right\|_{\mathrm{F}}^2 + \alpha\|\mathbf{V}\|_{2,1} + \iota(\mathbf{W})$$

$$\text{subject to: } \mathbf{U}_k = \mathbf{A}\mathbf{B}_k,\ \mathbf{V} = \nabla\mathbf{A},\ \mathbf{W} = \mathbf{A}$$

augmented Lagrangian:

$$\mathcal{L}(\mathbf{A}, \mathbf{U}_1, \ldots, \mathbf{U}_K, \mathbf{V}, \mathbf{W}, \mathbf{F}_1, \ldots, \mathbf{F}_K, \mathbf{G}, \mathbf{H})$$
$$= \frac{1}{2}\sum_{k=1}^{K} \left\| \mathbf{\Lambda}_k^{-1/2}(\mathbf{Y}_k - \mathbf{R}_k\mathbf{E}\mathbf{U}_k\mathbf{S}_k) \right\|_{\mathrm{F}}^2 + \alpha\|\mathbf{V}\|_{2,1} + \iota(\mathbf{W})$$
$$+ \frac{\mu}{2}\sum_{k=1}^{K} \|\mathbf{A}\mathbf{B}_k - \mathbf{U}_k - \mathbf{F}_k\|_{\mathrm{F}}^2 + \frac{\mu}{2}\|\nabla\mathbf{A} - \mathbf{V} - \mathbf{G}\|_{\mathrm{F}}^2 + \frac{\mu}{2}\|\mathbf{A} - \mathbf{W} - \mathbf{H}\|_{\mathrm{F}}^2$$

Algorithm

alternating direction method of multipliers:

$$\mathbf{A}^{(n)} = \underset{\mathbf{A}}{\operatorname{argmin}} \sum_{k=1}^{K} \left\| \mathbf{A}\mathbf{B}_k - \mathbf{U}_k^{(n-1)} - \mathbf{F}_k^{(n-1)} \right\|_{\mathrm{F}}^2 + \left\| \nabla\mathbf{A} - \mathbf{V}^{(n-1)} - \mathbf{G}^{(n-1)} \right\|_{\mathrm{F}}^2 + \left\| \mathbf{A} - \mathbf{W}^{(n-1)} - \mathbf{H}^{(n-1)} \right\|_{\mathrm{F}}^2$$

$$\mathbf{U}_k^{(n)} = \underset{\mathbf{U}_k}{\operatorname{argmin}} \frac{1}{2} \left\| \mathbf{\Lambda}_k^{-1/2} (\mathbf{Y}_k - \mathbf{R}_k \mathbf{E} \mathbf{U}_k \mathbf{S}_k) \right\|_{\mathrm{F}}^2 + \frac{\mu}{2} \left\| \mathbf{A}^{(n)} \mathbf{B}_k - \mathbf{U}_k - \mathbf{F}_k^{(n-1)} \right\|_{\mathrm{F}}^2$$

$$\mathbf{V}^{(n)} = \underset{\mathbf{V}}{\operatorname{argmin}} \ \alpha \|\mathbf{V}\|_{2,1} + \frac{\mu}{2} \left\| \nabla\mathbf{A}^{(n)} - \mathbf{V} - \mathbf{G}^{(n-1)} \right\|_{\mathrm{F}}^2$$

$$\mathbf{W}^{(n)} = \underset{\mathbf{W}}{\operatorname{argmin}} \ \iota(\mathbf{W}) + \frac{\mu}{2} \left\| \mathbf{A}^{(n)} - \mathbf{W} - \mathbf{H}^{(n-1)} \right\|_{\mathrm{F}}^2$$

$$\mathbf{F}_k^{(n)} = \mathbf{F}_k^{(n-1)} - \left(\mathbf{A}^{(n)} \mathbf{B}_k - \mathbf{U}_k^{(n)} \right), k = 1, \ldots, K$$

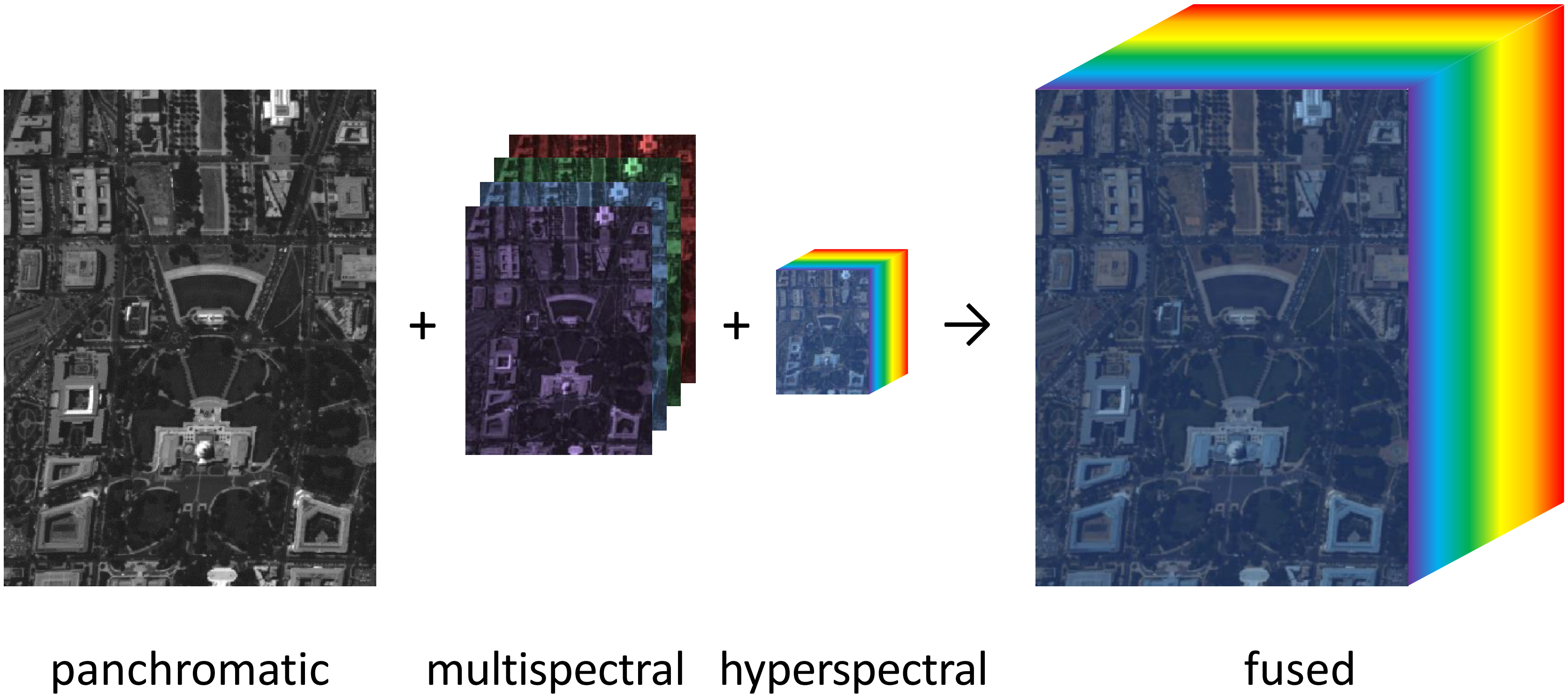
$$\mathbf{G}^{(n)} = \mathbf{G}^{(n-1)} - \left(\nabla\mathbf{A}^{(n)} - \mathbf{V}^{(n)} \right)$$

$$\mathbf{H}^{(n)} = \mathbf{H}^{(n-1)} - \left(\mathbf{A}^{(n)} - \mathbf{W}^{(n)} \right)$$

Simulations

experiment:

fusing three multiband images, viz. a panchromatic image, a multispectral image, and a hyperspectral image

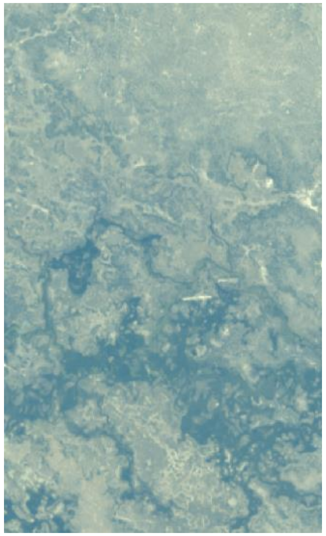


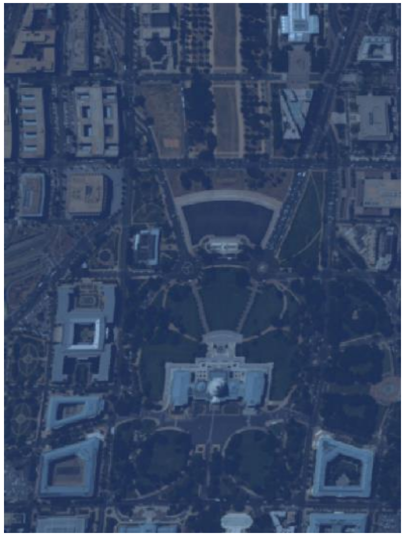
Simulations

datasets:

- **Botswana**: 400×240 pixels and 145 spectral bands, captured by the Hyperion sensor aboard the Earth Observing 1 (EO-1) satellite
- **Washington DC Mall**: 400×300 pixels and 191 bands, captured by the airborne-mounted Hyperspectral Digital Imagery Collection Experiment (HYDICE)

Both cover the visible near-infrared (VNIR) and short-wavelength infrared (SWIR) ranges with uncalibrated, noisy, and water-absorption bands removed.





Simulations

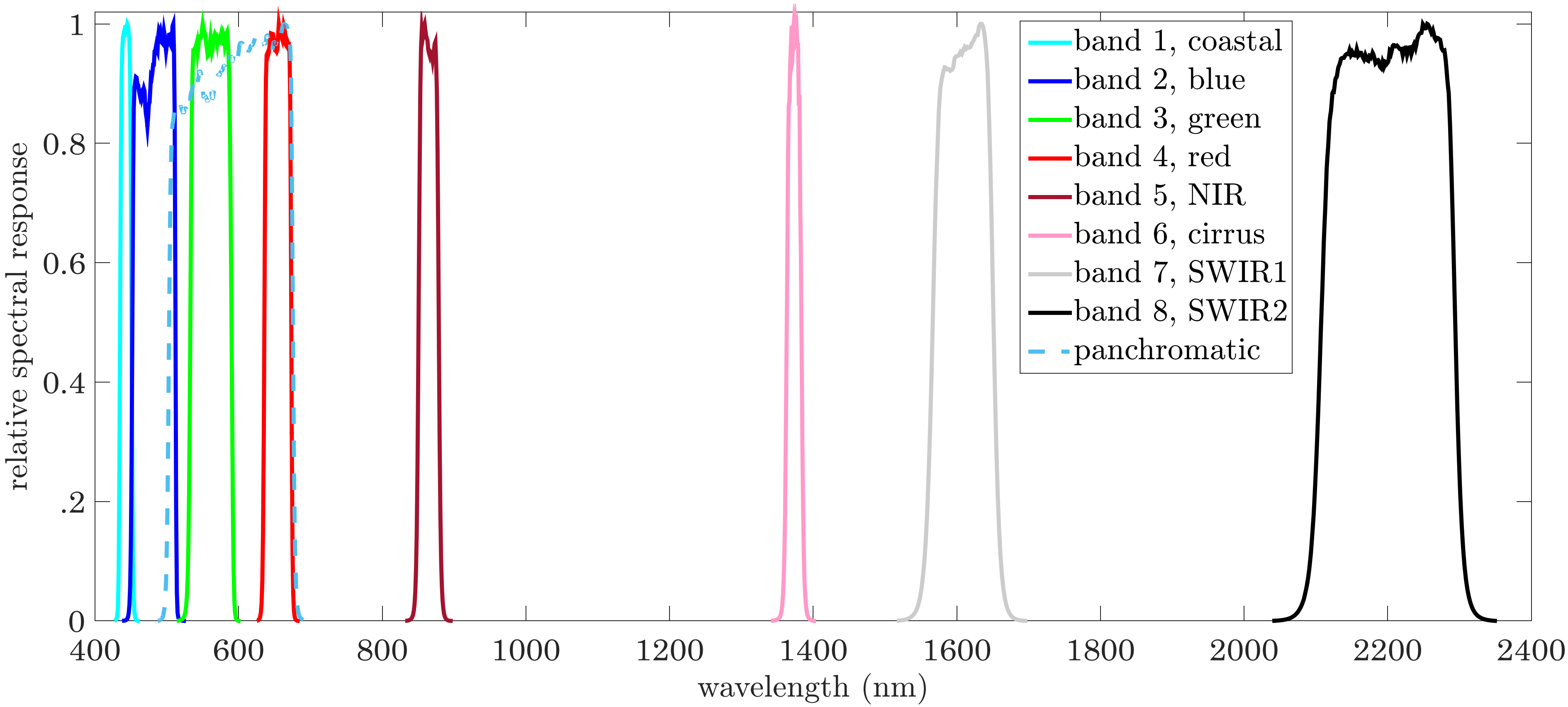
synthesis of the images from the datasets:

- **hyperspectral image**: Gaussian blur filter, kernel size 5×5, variance 1.28, downsampling ratio 4
- **multispectral image**: Gaussian blur filter, kernel size 3×3, variance 0.64, downsampling ratio 2, the spectral responses of the Landsat 8 multispectral sensor
- **panchromatic image**: the panchromatic band of the Landsat 8 sensor with no spatial blurring or downsampling

Gaussian white noise is added to each band so that the band-specific SNR is 30 dB for the multispectral and hyperspectral images and 40 dB for the panchromatic image.

Simulations

spectral responses of the Landsat 8 multispectral sensor:



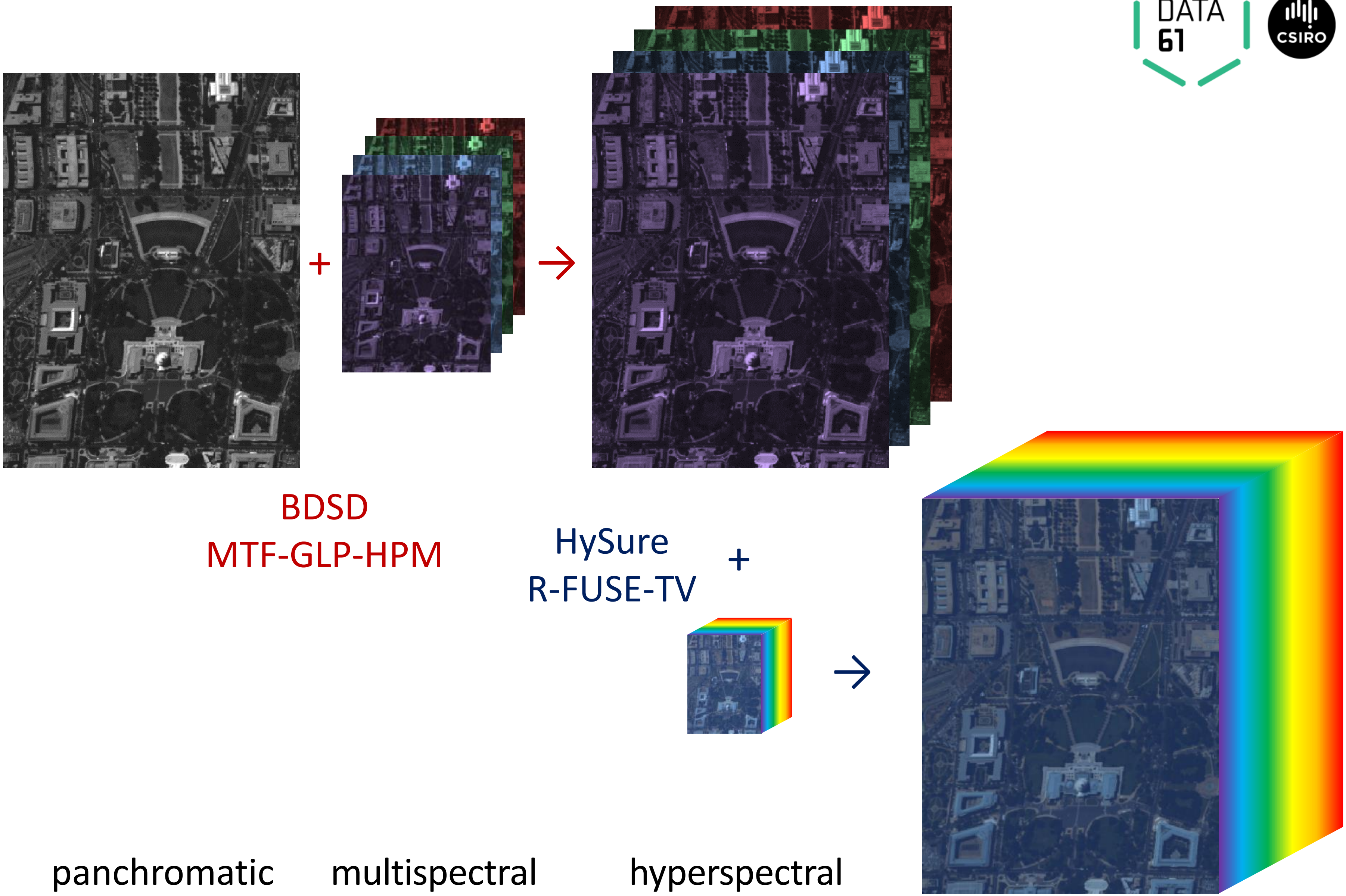
Simulations

benchmark algorithms:

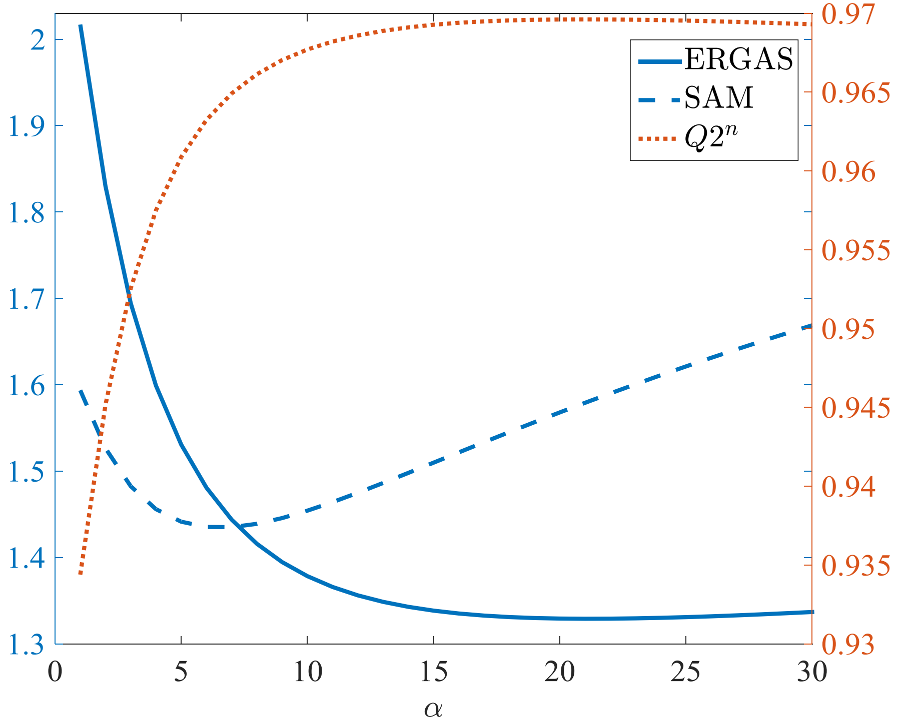
- panchromatic + multispectral:
  - band-dependent spatial detail (**BDSD**)
  - modulation-transfer-function generalized Laplacian pyramid with high-pass modulation (**MTF-GLP-HPM**)
- pansharpened multispectral + hyperspectral:
  - **HySure**
  - **R-FUSE-TV**

performance metrics:

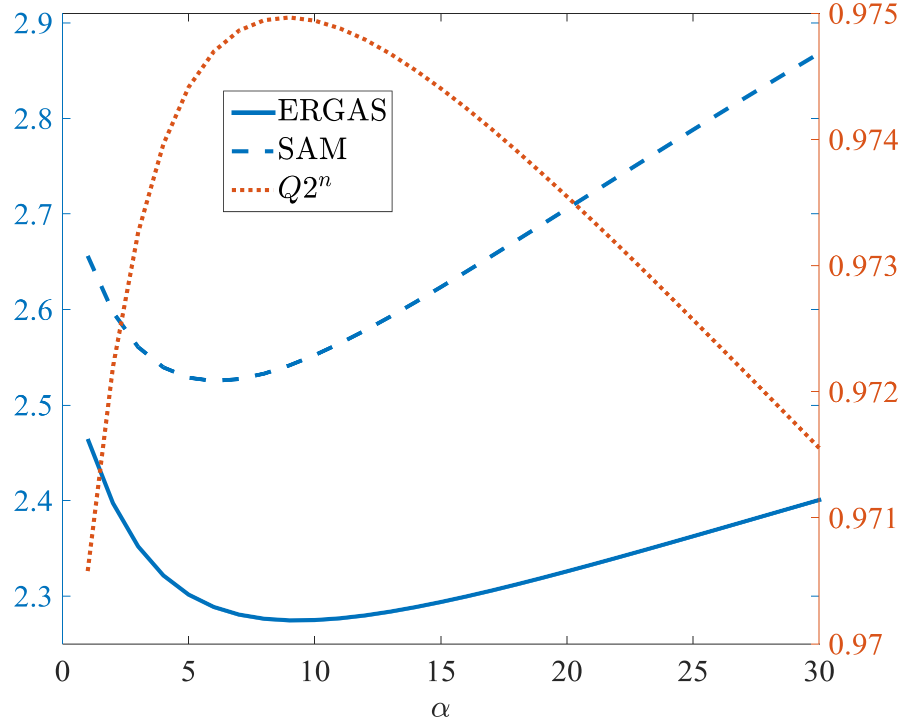
- relative dimensionless global error in synthesis (**ERGAS**)
- spectral angle mapper (**SAM**)
- $\boldsymbol{Q2^n}$


+
BDSD
MTF-GLP-HPM
HySure
R-FUSE-TV
+
panchromatic
multispectral
hyperspectral

Simulation results



Botswana



Washington DC Mall

Simulation results

Botswana				
	ERGAS	SAM (°)	$Q2^n$	time (s)
proposed	**1.378**	**1.454**	**0.969**	**47.01**
BDSD & HySure	2.268	2.228	0.923	62.58
BDSD & R-FUSE-TV	2.276	2.238	0.923	62.10
MTF-GLP-HPM & HySure	2.034	2.256	0.938	62.78
MTF-GLP-HPM & R-FUSE-TV	2.044	2.265	0.938	62.20

Simulation results

Washington DC Mall				
	ERGAS	SAM (°)	$Q2^n$	time (s)
proposed	**2.276**	**2.533**	**0.975**	**59.52**
BDSD & HySure	4.039	4.767	0.923	79.68
BDSD & R-FUSE-TV	4.141	4.787	0.921	78.41
MTF-GLP-HPM & HySure	4.240	4.809	0.916	79.28
MTF-GLP-HPM & R-FUSE-TV	4.354	4.827	0.913	78.13