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Lecture Session SPTM-L7.2: Signal Processing on Networks

Distributed Approximate Message Passing with Summation Propagation

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Outline

1. Introduction

2. Preliminaries
   1. AMP Algorithm
   2. Consensus Propagation

3. Proposed Method: Distributed AMP Algorithm

4. Simulation Result

5. Conclusion

Outline

1. **Introduction**
2. Preliminaries
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Compressed Sensing [1]

reconstruct a **sparse** vector $\boldsymbol{x} \in \mathbb{R}^N$
from its **underdetermined** linear measurement $\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{v} \in \mathbb{R}^M \, (M < N)$

$\boldsymbol{x} \in \mathbb{R}^N$: unknown sparse vector (most elements are zero)

$\boldsymbol{A} \in \mathbb{R}^{M \times N}$: measurement matrix $(M < N)$

$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} + \underbrace{\boldsymbol{v}}_{\text{noise vector}} \in \mathbb{R}^M$: measurement vector

Application

✦ magnetic resonance imaging (MRI) [2]
✦ wireless channel estimation [3]

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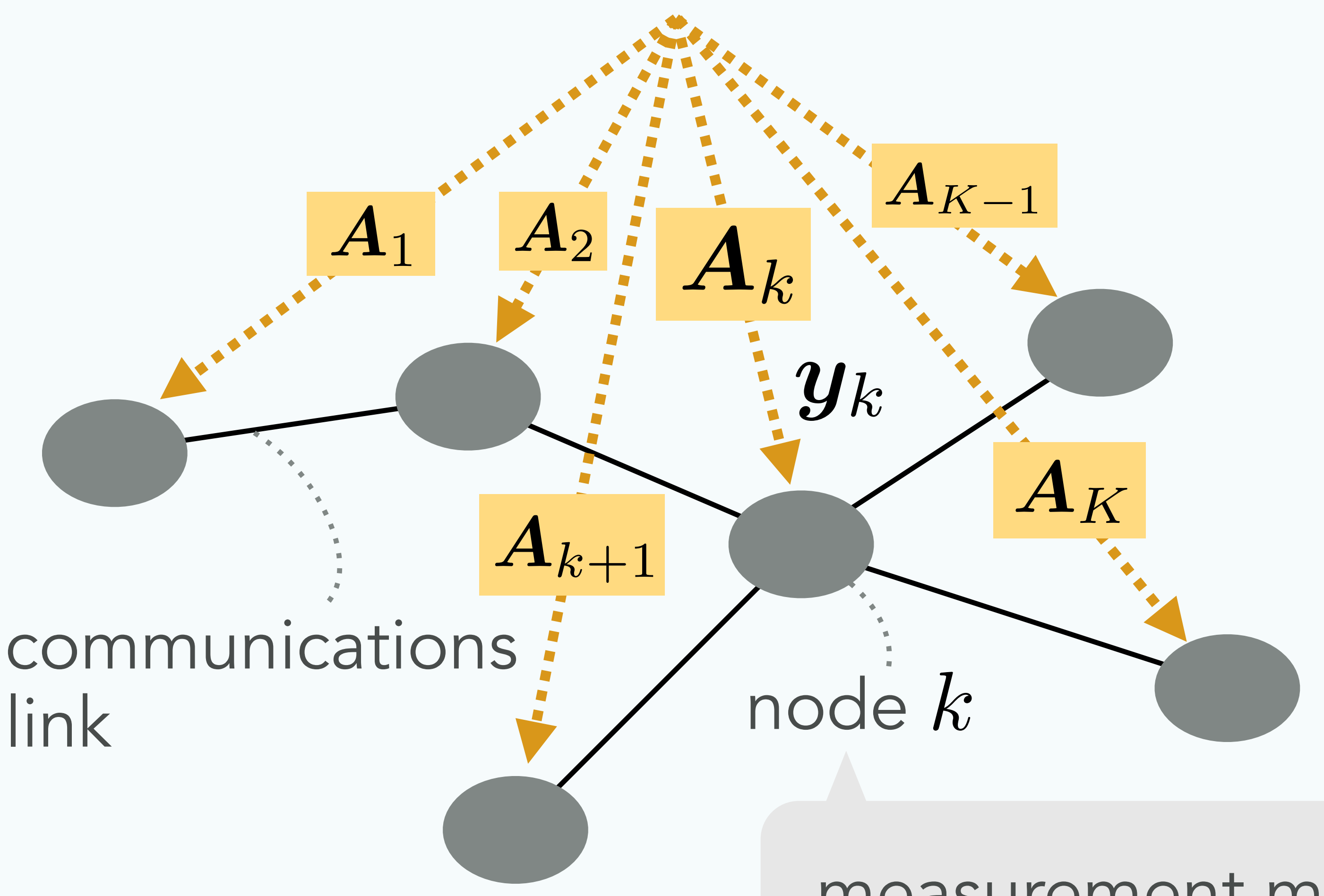
[1] D. L. Donoho, "Compressed sensing," *IEEE Trans. Inf. Theory*, vol. 52, no. 4, pp. 1289–1306, Apr. 2006.

[2] M. Lustig, D. L. Donoho, J. M. Santos, and J. M. Pauly, "Compressed sensing MRI," *IEEE Signal Process. Mag.*, vol. 25, no. 2, pp. 72–82, Mar. 2008.

[3] K. Hayashi, M. Nagahara, and T. Tanaka, "A user's guide to compressed sensing for communications systems," *IEICE Trans. Commun.*, vol. E96-B, no. 3, pp. 685–712, Mar. 2013.

Distributed Compressed Sensing

unknown sparse vector $\boldsymbol{x} \in \mathbb{R}^N$



communications link

node k

Application [4]

- ✦ sensor network
- ✦ video coding
- ✦ image fusion

$$\left(\sum_{k=1}^{K} M_k < N\right)$$

measurement matrix: $\boldsymbol{A}_k \in \mathbb{R}^{M_k \times N}$

measurement vector: $\boldsymbol{y}_k = \boldsymbol{A}_k \boldsymbol{x} + \boldsymbol{v}_k \in \mathbb{R}^{M_k}$

reconstruct $\boldsymbol{x}$ from $\boldsymbol{y}_k, \boldsymbol{A}_k \ (k = 1, \ldots, K)$

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[4] H. Yin, J. Li, Y. Chai, and S. X. Yang, "A survey on distributed compressed sensing: theory and applications," *Frontiers of Computer Science*, vol. 8, no. 6, pp. 893–904, Dec. 2014.

Conventional Methods (1/2)

✦ D-LASSO [5]

(**D**istributed-**L**east **A**bsolute **S**hrinkage and **S**election **O**perator)

✦ D-ADMM [6]

(**D**istributed-**A**lternating **D**irection **M**ethod of **M**ultipliers)

- The computational complexity might be large

✦ D-IHT [7]

(**D**istributed-**I**terative **H**ard **T**hresholding)

✓ Each node performs simple calculations

- The sparsity level is required

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[5] J. A. Bazerque and G. B. Giannakis, "Distributed spectrum sensing for cognitive radio networks by exploiting sparsity," *IEEE Trans. Signal Process.*, vol. 58, no. 3, pp. 1847–1862, Mar. 2010.

[6] J. F. C. Mota, J. M. F. Xavier, P. M. Q. Aguiar, and M. Püschel, "Distributed basis pursuit," *IEEE Trans. Signal Process.*, vol. 60, no. 4, pp. 1942–1956, Apr. 2012.

[7] S. Patterson, Y. C. Eldar, and I. Keidar, "Distributed sparse signal recovery for sensor networks," in *Proc. IEEE ICASSP*, May 2013.

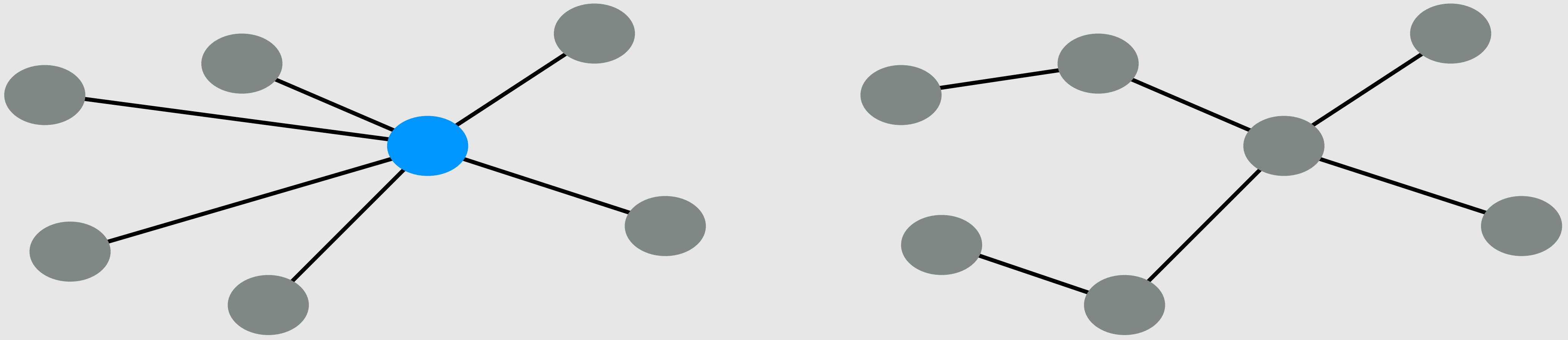
Conventional Methods (2/2)

✦ Distributed AMP [8], Multi-processor AMP [9]

(**A**pproximate **M**essage **P**assing)

✓ Each node performs simple calculations
✓ The sparsity level is **not** required
- A **fusion node** communicating with all nodes is required

- much energy at fusion node
- vulnerable to failure of node



applicable **not applicable**

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[8] P. Han, R. Niu, M. Ren, and Y. C. Eldar, "Distributed approximate message passing for sparse signal recovery," in *Proc. IEEE GlobalSIP*, Dec. 2014.

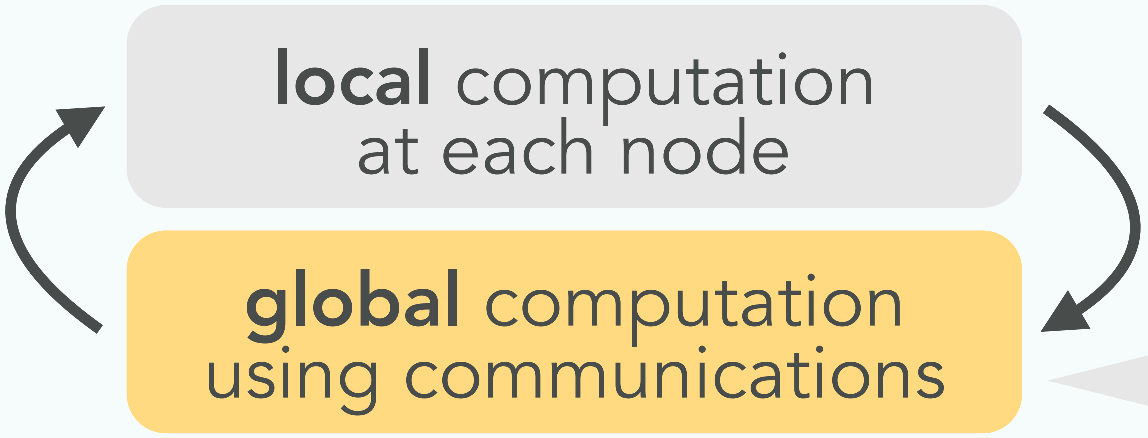
[9] J. Zhu, R. Pilgrim, and D. Baron, "An overview of multi-processor approximate message passing," in *Proc. IEEE CISS*, Mar. 2017.

Summary of This Study

Purpose of This Study

propose a **fully distributed** AMP algorithm,
which does not require any fusion node

① obtain update equations of the AMP algorithm
for distributed measurements



local computation
at each node

global computation
using communications

② propose **summation propagation**
for the global computation

③ show the validity of the proposed algorithm via computer simulation

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AMP Algorithm (1/2)

$\boldsymbol{x} \in \mathbb{R}^N$: unknown sparse vector

$\boldsymbol{A} \in \mathbb{R}^{M \times N}$: measurement matrix $(M < N)$
i.i.d. elements with zero mean and unit variance

$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{v} \in \mathbb{R}^M$: measurement vector

ℓ_1 optimization

$$\hat{\boldsymbol{x}} = \arg \min_{\boldsymbol{z} \in \mathbb{R}^N} \|\boldsymbol{z}\|_1 \text{ subject to } \boldsymbol{y} = \boldsymbol{A}\boldsymbol{z}$$

approximate belief propagation for $\hat{\boldsymbol{x}}$ [10]

AMP Algorithm [10, 11]
(**A**pproximate **M**essage **P**assing)

- ✓ low complexity (no matrix inversion, $O(MN)$)
- ✓ asymptotic analysis

[10] D. L. Donoho, A. Maleki, and A. Montanari, "Message passing algorithms for compressed sensing: I. motivation and construction," in *Proc. IEEE Inf. Theory Workshop*, Jan. 2010.

[11] D. L. Donoho, A. Maleki, and A. Montanari, "Message passing algorithms for compressed sensing," *Proc. Nat. Acad. Sci.*, vol. 106, no. 45, pp. 18 914–18 919, Nov. 2009.

AMP Algorithm (2/2)

$\boldsymbol{x} \in \mathbb{R}^N$: unknown sparse vector

$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{v} \in \mathbb{R}^M$: measurement vector

① Initialization: $t = 1, \hat{\boldsymbol{x}}(1) = \mathbf{0}, \boldsymbol{s}(0) = \mathbf{0}, \boldsymbol{r}(0) = \mathbf{0}, \hat{\sigma}^2(0) = 0$

② $\boldsymbol{s}(t) = \boldsymbol{y} - \boldsymbol{A}\hat{\boldsymbol{x}}(t) + \frac{1}{\Delta}\boldsymbol{s}(t-1)\left\langle \eta'\left(\boldsymbol{r}(t-1); \hat{\sigma}^2(t-1)\right)\right\rangle$

$t \leftarrow t+1$

$\Delta = M/N$: measurement ratio

$\langle \cdot \rangle$: mean

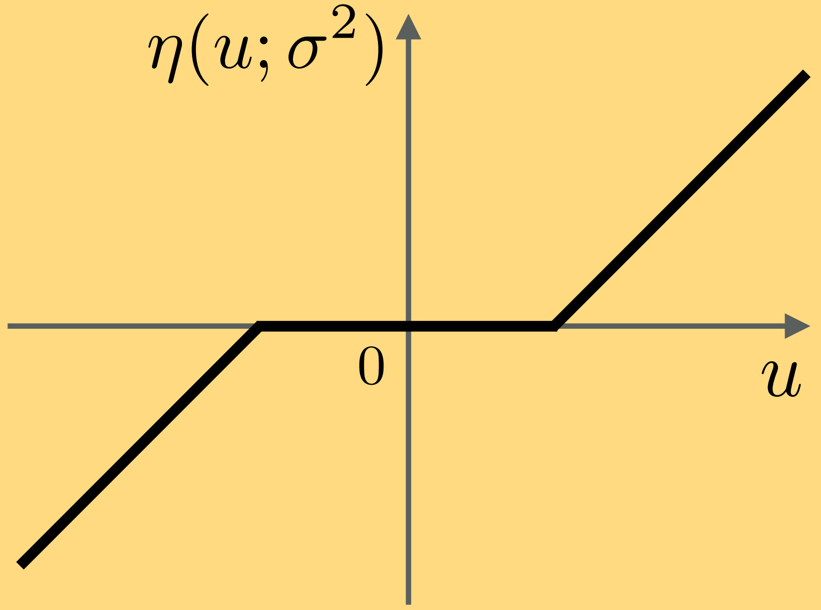
③ $\boldsymbol{r}(t) = \hat{\boldsymbol{x}}(t) + \frac{1}{M}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{s}(t)$

④ $\hat{\sigma}^2(t) = \frac{\|\boldsymbol{s}(t)\|_2^2}{MN}$

⑤ $\hat{\boldsymbol{x}}(t+1) = \eta\left(\boldsymbol{r}(t); \hat{\sigma}^2(t)\right)$

estimate of $\boldsymbol{x}$

example of $\eta(\cdot;\cdot)$: soft thresholding



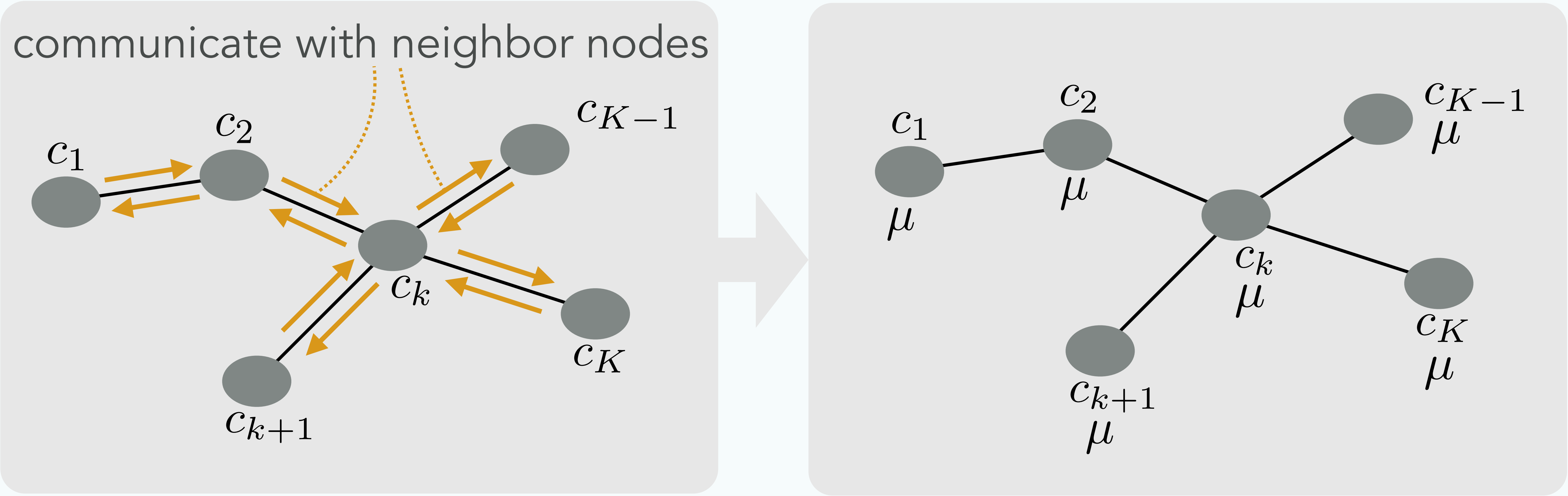
Consensus Propagation [12] (1/2)

A distributed algorithm for **average consensus** on undirected graphs

All nodes obtain the mean $\mu = \frac{1}{K}\sum_{k=1}^{K} c_k$

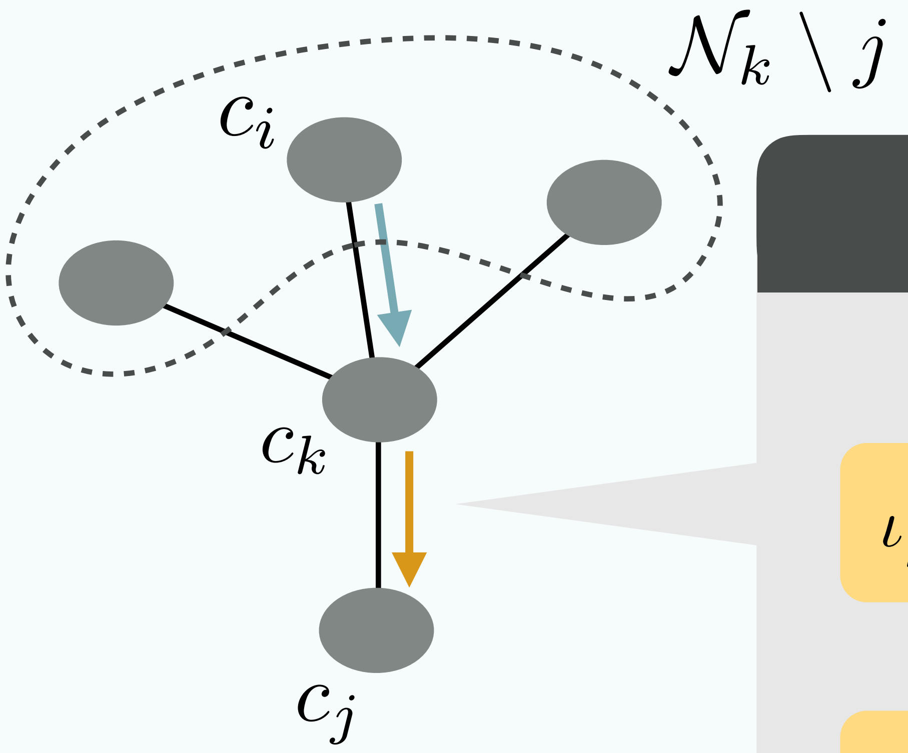
number of nodes

initial value at node k


communicate with neighbor nodes

[12] C. C. Moallemi and B. V. Roy, "Consensus propagation," *IEEE Trans. Inf. Theory*, vol. 52, no. 11, pp. 4753–4766, Nov. 2006.

Consensus Propagation [12] (2/2)



message from node k to node j

message from other nodes at the previous iteration

$$\iota_{k\to j}^{(t')} = 1 + \sum_{i\in\mathcal{N}_k\setminus j} \iota_{i\to k}^{(t'-1)} \qquad \left(\iota_{k\to j}^{(0)} = 0\right)$$

$$\nu_{k\to j}^{(t')} = \frac{c_k + \sum_{i\in\mathcal{N}_k\setminus j}\iota_{i\to k}^{(t'-1)}\nu_{i\to k}^{(t'-1)}}{1+\sum_{i\in\mathcal{N}_k\setminus j}\iota_{i\to k}^{(t'-1)}} \qquad \left(\nu_{k\to j}^{(0)} = 0\right)$$

- The graph is a tree
- # of iterations $\geq$ graph diameter

→ average consensus is achieved

[12] C. C. Moallemi and B. V. Roy, "Consensus propagation," *IEEE Trans. Inf. Theory*, vol. 52, no. 11, pp. 4753–4766, Nov. 2006.

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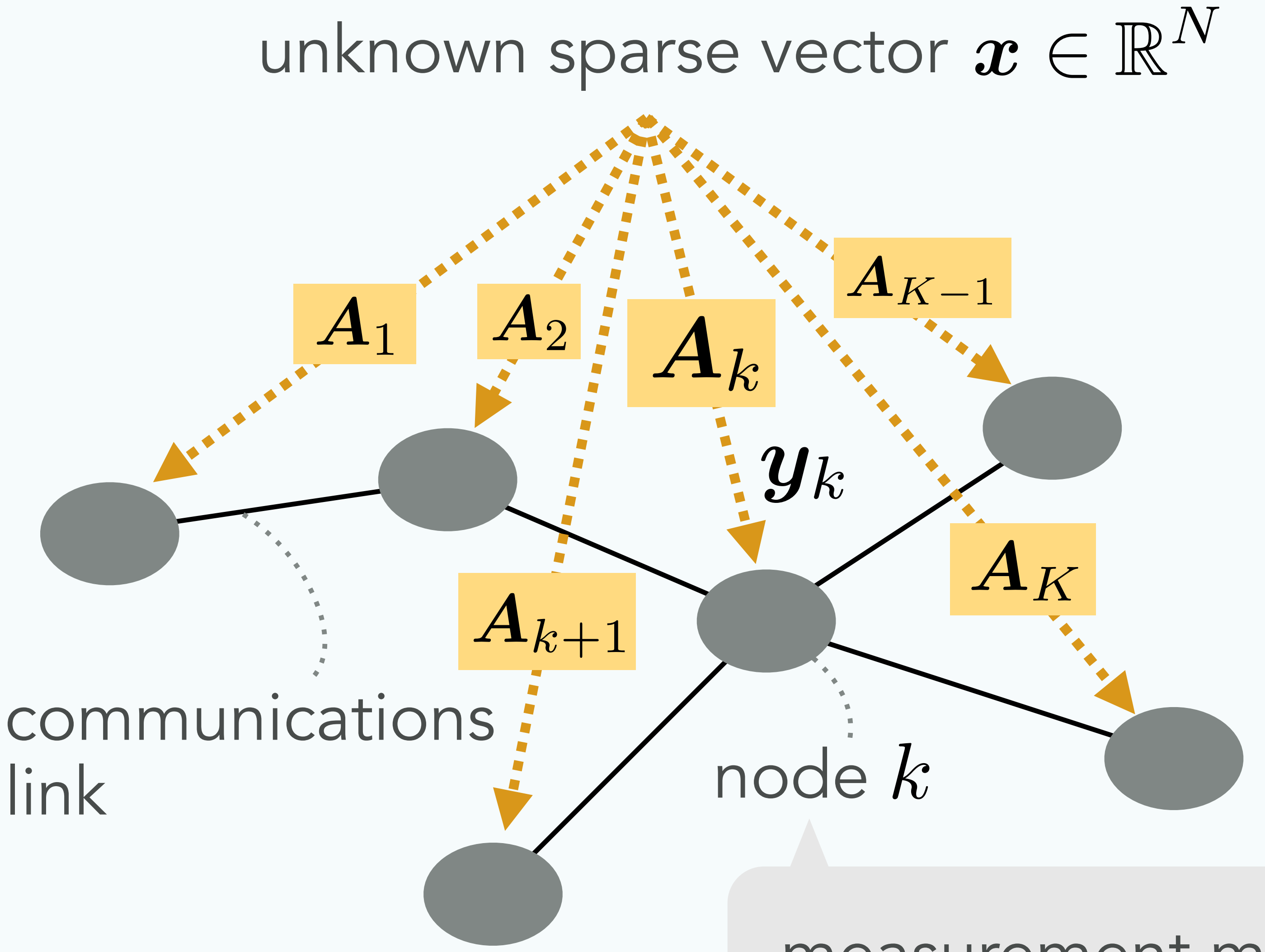
Distributed Compressed Sensing



All measurements $\boldsymbol{y}_1, \dots, \boldsymbol{y}_K$ can be combined as

$$\begin{bmatrix} \boldsymbol{y}_1 \\ \vdots \\ \boldsymbol{y}_K \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_1 \\ \vdots \\ \boldsymbol{A}_K \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} \boldsymbol{v}_1 \\ \vdots \\ \boldsymbol{v}_K \end{bmatrix}$$

measurement matrix: $\boldsymbol{A}_k \in \mathbb{R}^{M_k \times N}$

measurement vector: $\boldsymbol{y}_k = \boldsymbol{A}_k \boldsymbol{x} + \boldsymbol{v}_k \in \mathbb{R}^{M_k}$

reconstruct $\boldsymbol{x}$ from $\boldsymbol{y}_k, \boldsymbol{A}_k \ (k = 1, \dots, K)$

AMP Algorithm for Distributed Model (1/2)

centralized model

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{v}$$

AMP algorithm

$$\boldsymbol{s}(t) = \boldsymbol{y} - \boldsymbol{A}\hat{\boldsymbol{x}}(t) + \frac{1}{\Delta}\boldsymbol{s}(t-1)\left\langle \eta'\left(\boldsymbol{r}(t-1);\hat{\sigma}^2(t-1)\right)\right\rangle$$

$$\boldsymbol{r}(t) = \hat{\boldsymbol{x}}(t) + \frac{1}{M}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{s}(t)$$

$$\hat{\sigma}^2(t) = \frac{\|\boldsymbol{s}(t)\|_2^2}{MN}$$

$$\hat{\boldsymbol{x}}(t+1) = \eta\left(\boldsymbol{r}(t);\hat{\sigma}^2(t)\right)$$

distributed model

$$\begin{bmatrix} \boldsymbol{y}_1 \\ \vdots \\ \boldsymbol{y}_K \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_1 \\ \vdots \\ \boldsymbol{A}_K \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} \boldsymbol{v}_1 \\ \vdots \\ \boldsymbol{v}_K \end{bmatrix}$$

AMP Algorithm

$$\boldsymbol{s}_k(t) = \boldsymbol{y}_k - \boldsymbol{A}_k\hat{\boldsymbol{x}}(t) + \frac{1}{\Delta}\boldsymbol{s}_k(t-1)\left\langle \eta'\left(\boldsymbol{r}(t-1);\hat{\sigma}^2(t-1)\right)\right\rangle$$

$$\boldsymbol{r}(t) = \sum_{k=1}^{K}\left(\frac{1}{K}\hat{\boldsymbol{x}}(t) + \frac{1}{M}\boldsymbol{A}_k^{\mathrm{T}}\boldsymbol{s}_k(t)\right)$$

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AMP Algorithm for Distributed Model (1/2)

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$$\hat{\sigma}^2(t) = \sum_{k=1}^{K}\frac{\|\boldsymbol{s}_k(t)\|_2^2}{MN}$$

$$\hat{\boldsymbol{x}}(t+1) = \eta\left(\boldsymbol{r}(t);\hat{\sigma}^2(t)\right)$$

AMP Algorithm for Distributed Model (2/2)

centralized model

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{v}$$

AMP algorithm

$$\boldsymbol{s}(t) = \boldsymbol{y} - \boldsymbol{A}\hat{\boldsymbol{x}}(t) + \frac{1}{\Delta}\boldsymbol{s}(t-1)\left\langle \eta'\left(\boldsymbol{r}(t-1); \hat{\sigma}^2(t-1)\right)\right\rangle$$

$$\boldsymbol{r}(t) = \hat{\boldsymbol{x}}(t) + \frac{1}{}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{s}(t)$$

cannot be computed locally

$$\hat{\sigma}^2(t) = \frac{\|\boldsymbol{s}(t)\|_2}{MN}$$

$$\hat{\boldsymbol{x}}(t+1) = \eta\left(\boldsymbol{r}(t); \hat{\sigma}^2(t)\right)$$

distributed model

$$\begin{bmatrix} \boldsymbol{y}_1 \\ \vdots \\ \boldsymbol{y}_K \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_1 \\ \vdots \\ \boldsymbol{A}_K \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} \boldsymbol{v}_1 \\ \vdots \\ \boldsymbol{v}_K \end{bmatrix}$$

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$$\hat{\boldsymbol{x}}(t+1) = \eta\left(\boldsymbol{r}(t); \hat{\sigma}^2(t)\right)$$

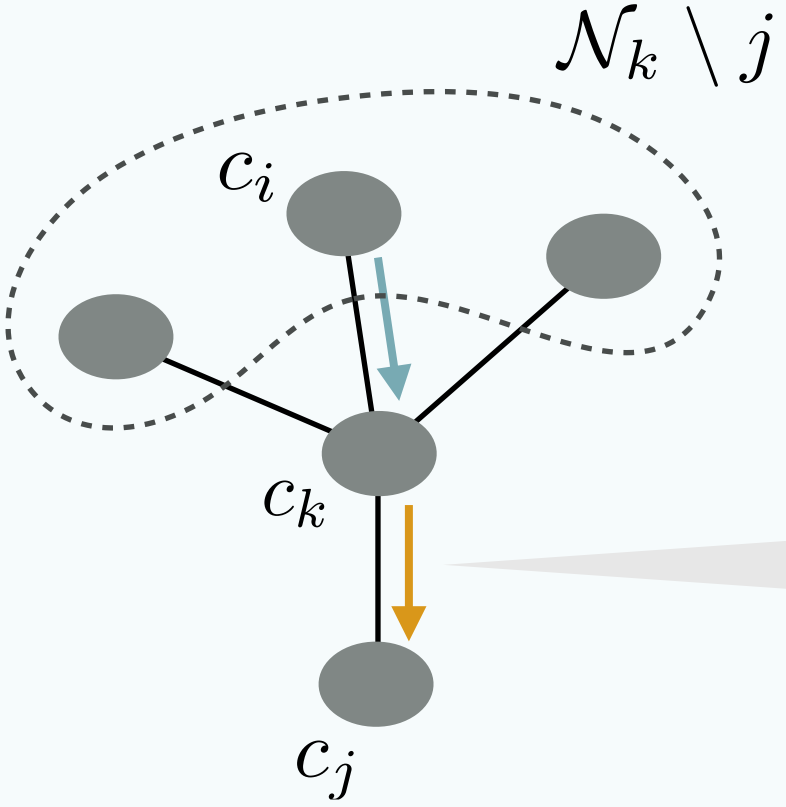
Proposed Method: Distributed AMP Algorithm

Summation Propagation

We propose **summation propagation** to compute

$$\boldsymbol{r}(t) = \sum_{k=1}^{K} \left(\frac{1}{K} \hat{\boldsymbol{x}}(t) + \frac{1}{M} \boldsymbol{A}_k^{\mathrm{T}} \boldsymbol{s}_k(t) \right) \qquad \hat{\sigma}^2(t) = \sum_{k=1}^{K} \frac{\|\boldsymbol{s}_k(t)\|_2^2}{MN}$$

by using the idea of consensus propagation



summation propagation

send the **summation** of c_k and messages from other nodes

$$\boxed{\xi_{k\to j}^{(t')}} = c_k + \sum_{i\in\mathcal{N}_k\setminus j} \boxed{\xi_{i\to k}^{(t'-1)}} \qquad \left(\xi_{k\to j}^{(0)} = 0\right)$$

- The graph is a tree
- # of iterations $\geq$ graph diameter

➡ All nodes obtain the summation $\sum_{k=1}^{K} c_k$

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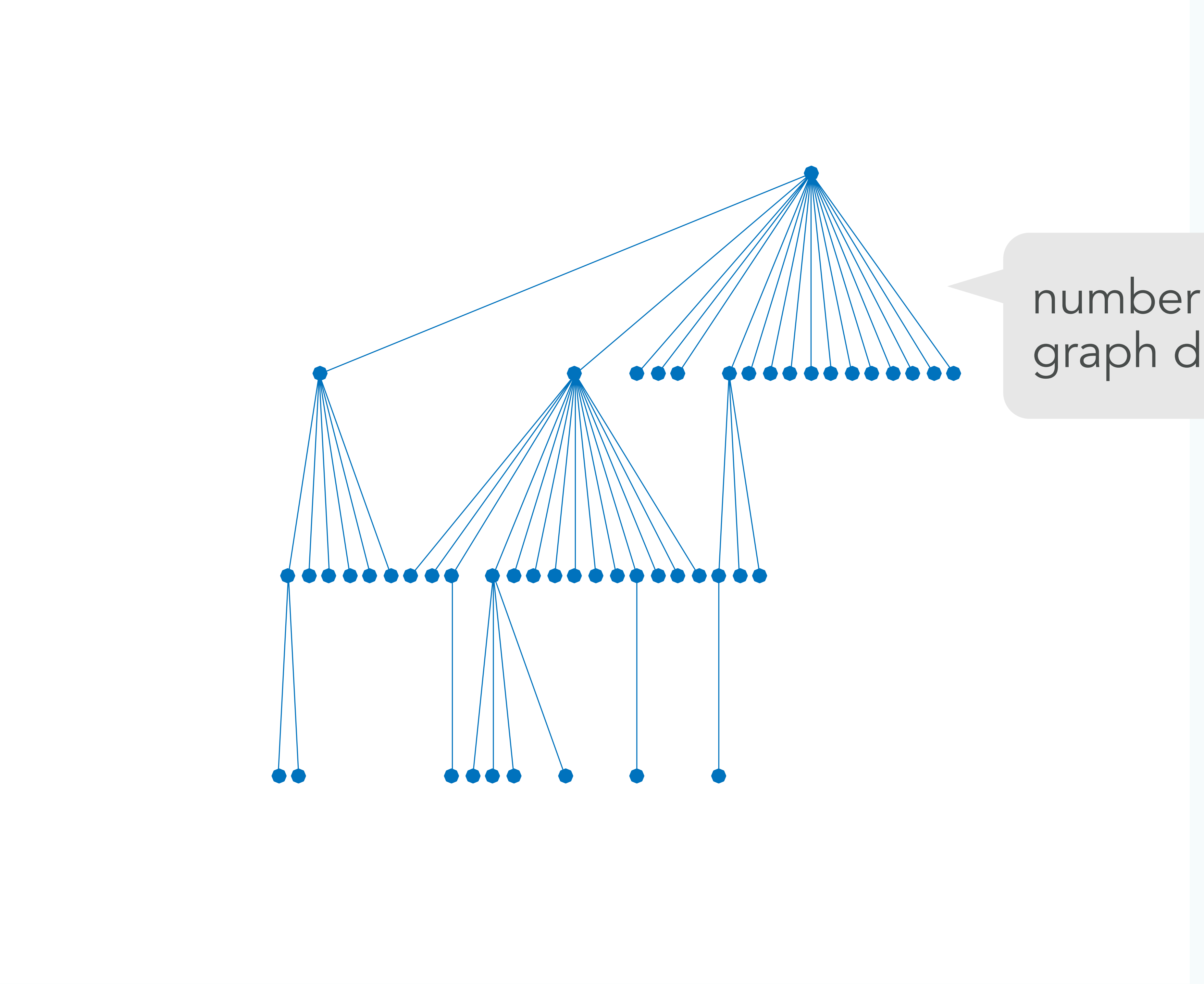
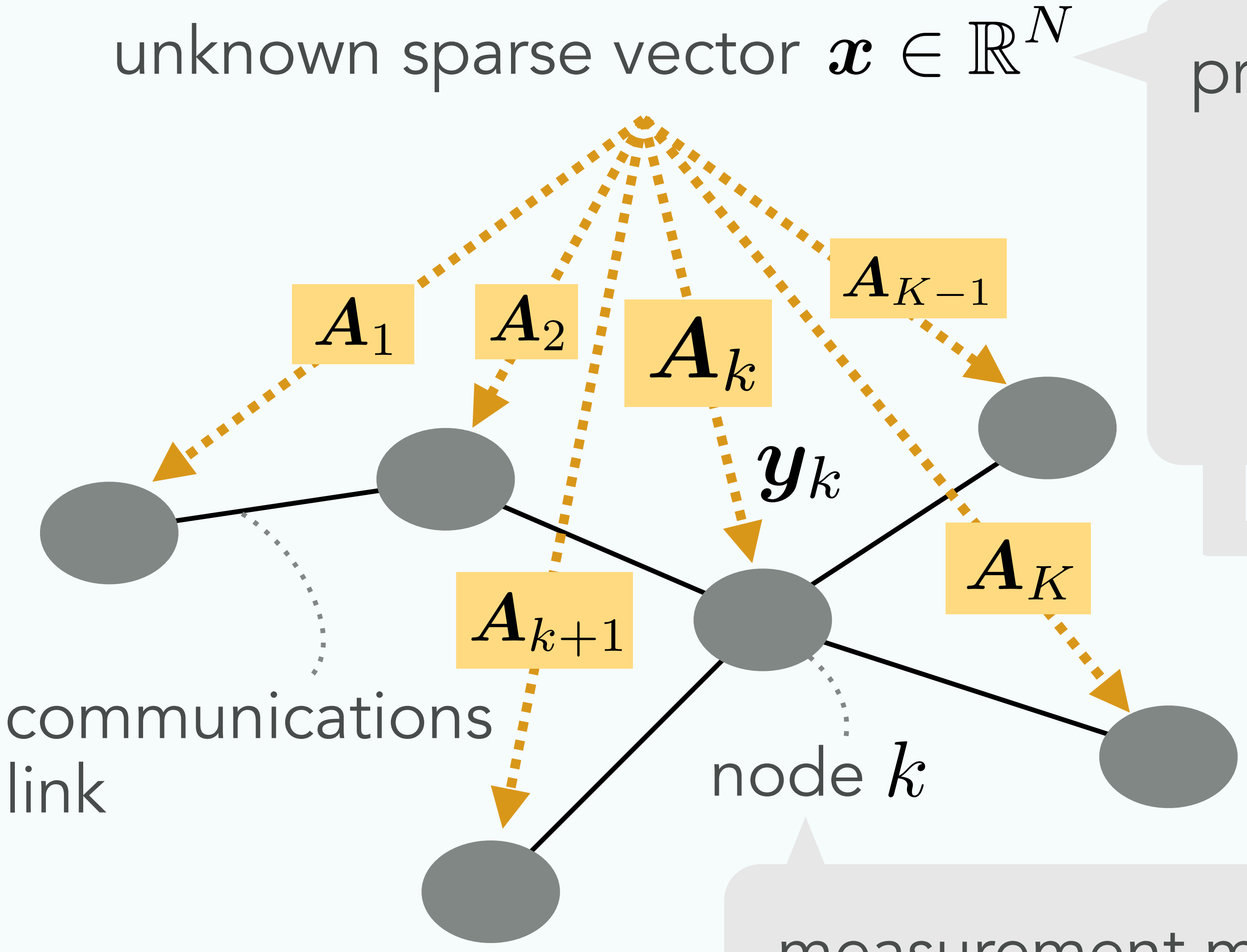
Simulation Result

Graph Structure



number of nodes: $K = 50$
graph diameter: 6

Problem Settings (Sparse Vector Reconstruction)



probability distribution (unknown):

$$p(x_n) = \underbrace{q}_{\text{parameter}}\delta(x_n) + (1-q)\underbrace{\phi}_{\text{probability density function of standard Gaussian distribution}}(x_n)$$

⇒ $\boldsymbol{x}$ is sparse when q is large

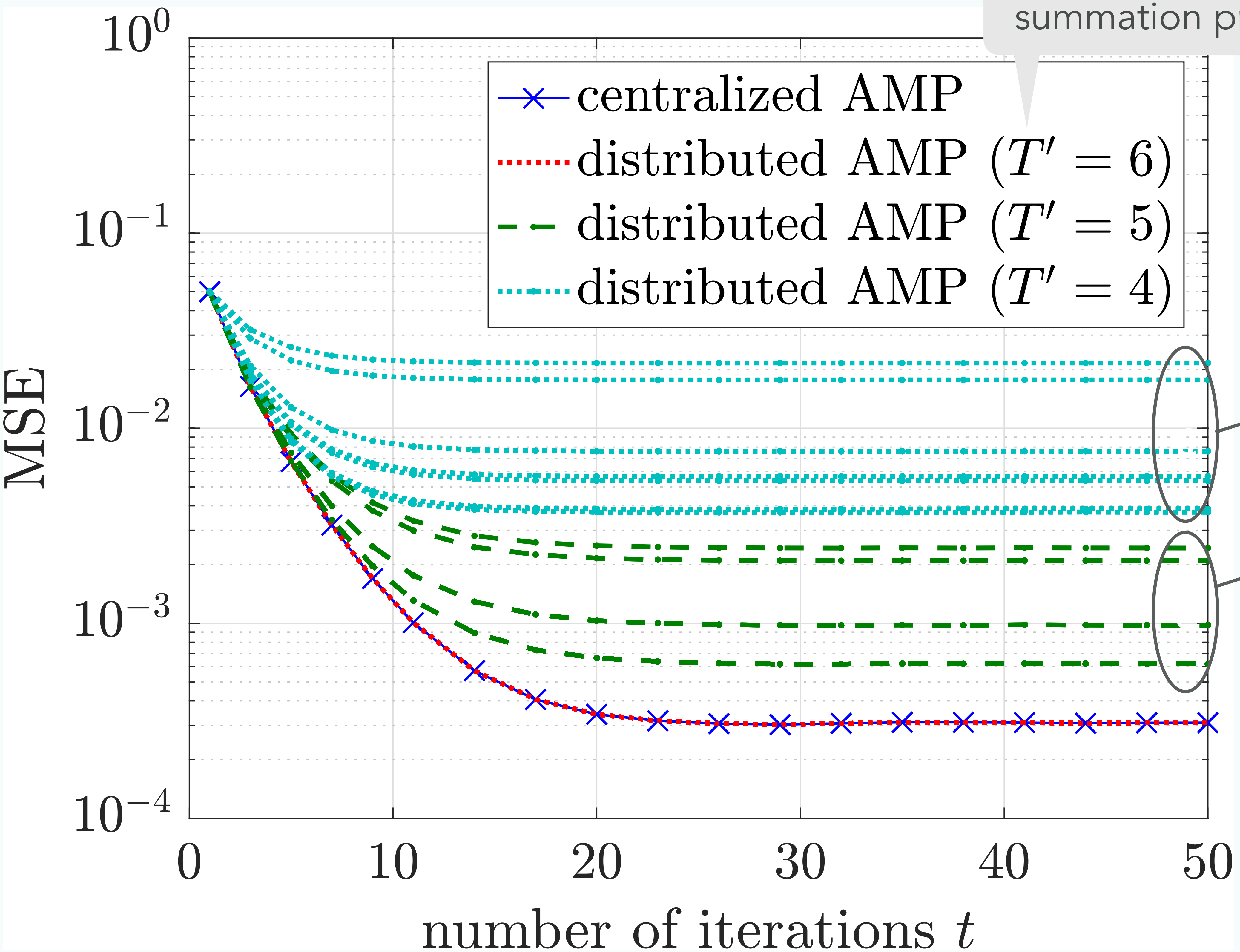
measurement matrix: $\boldsymbol{A}_k \in \mathbb{R}^{M_k \times N}$

measurement vector: $\boldsymbol{y}_k = \boldsymbol{A}_k \boldsymbol{x} + \boldsymbol{v}_k \in \mathbb{R}^{M_k}$

Simulation Result

MSE for Sparse Vector Reconstruction

(Mean-Square-Error)



\# of iterations in summation propagation

$N = 1000$
$M_k = 6$
$\sigma_v^2 = 0.1$
$q = 0.95$

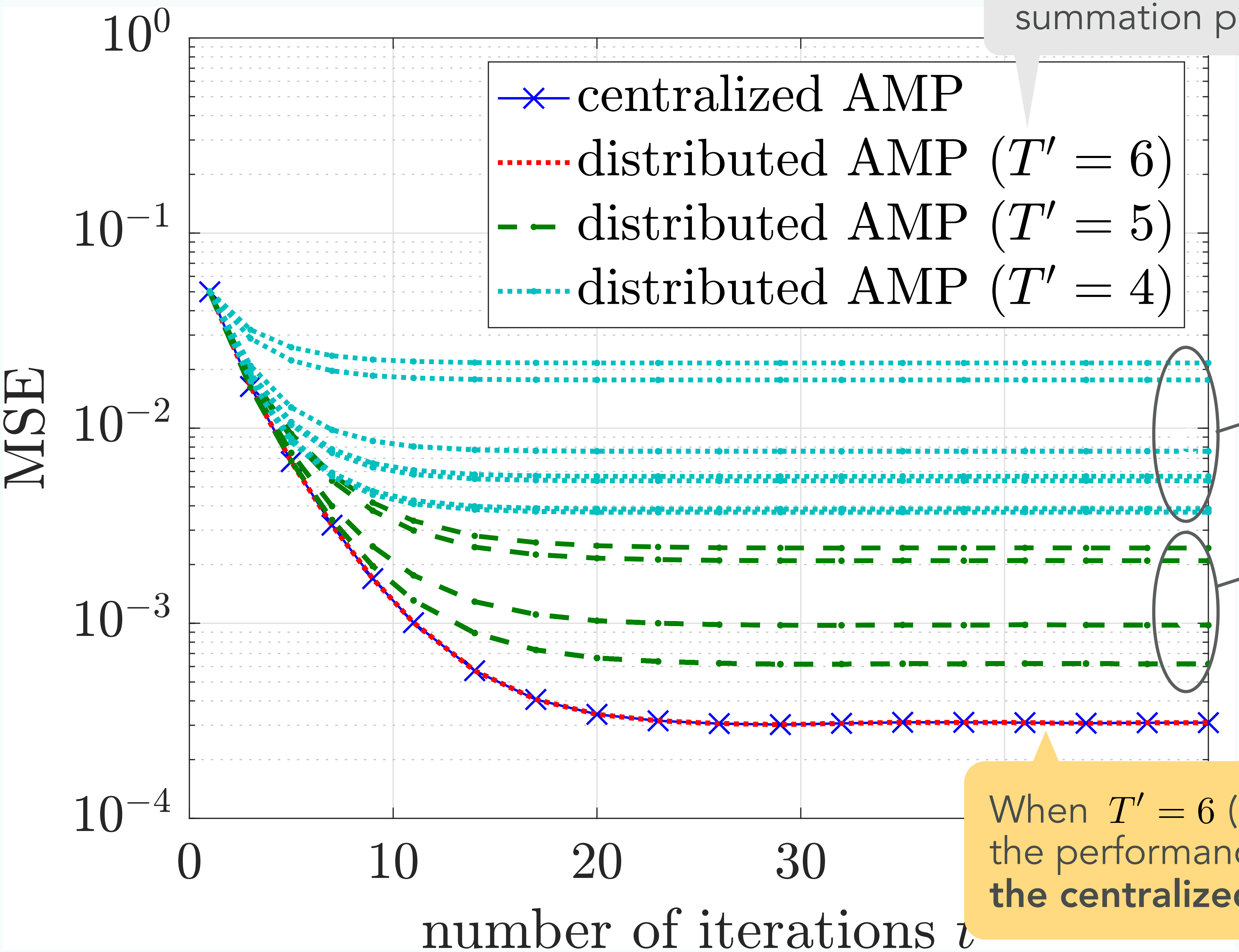
MSEs at $K = 50$ nodes when $T' = 4$

MSEs at $K = 50$ nodes when $T' = 5$

Simulation Result

MSE for Sparse Vector Reconstruction

(Mean-Square-Error)



\# of iterations in summation propagation

$N = 1000$
$M_k = 6$
$\sigma_v^2 = 0.1$
$q = 0.95$

MSEs at $K = 50$ nodes when $T' = 4$

MSEs at $K = 50$ nodes when $T' = 5$

When $T' = 6$ (graph diameter), the performance is **the same as the centralized AMP algorithm**

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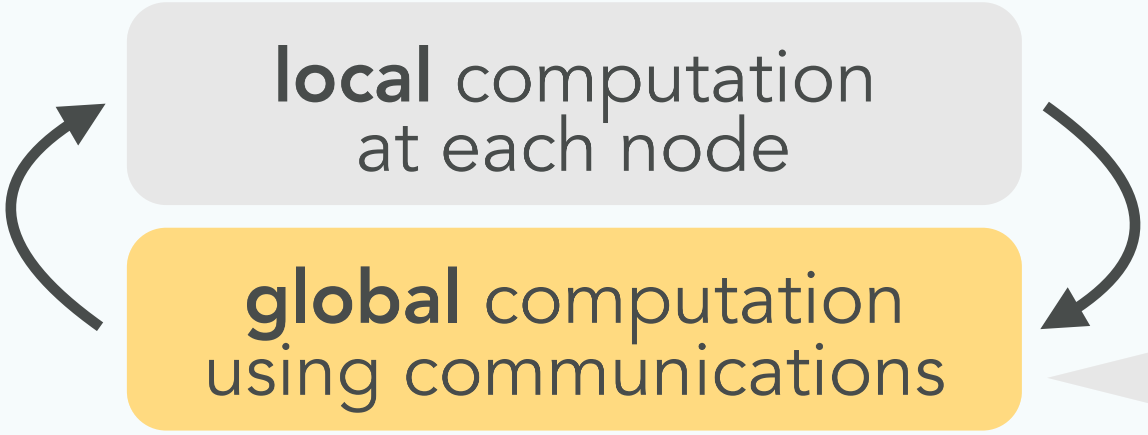
Conclusion

Purpose of This Study

propose a **fully distributed** AMP algorithm,
which does not require any fusion node

① obtain update equations of the AMP algorithm for distributed measurements


local computation at each node

global computation using communications

② propose **summation propagation** for the global computation

③ show the validity of the proposed algorithm via computer simulation

Future Work

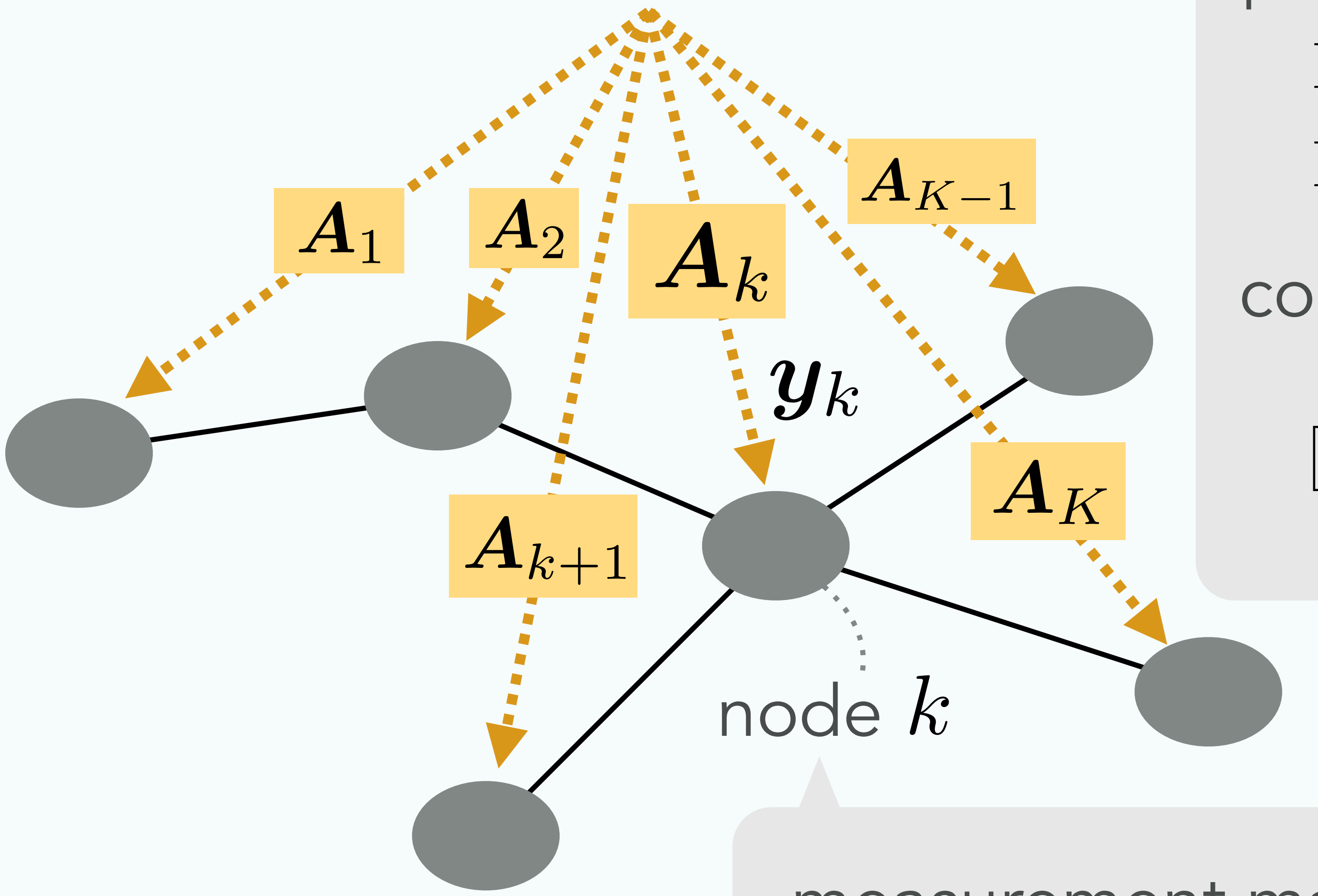
- ✦ extension for generalized AMP algorithm
- ✦ comparison with conventional methods

Appendix

Problem Settings (Binary Vector Reconstruction)

We can apply the AMP algorithm for binary vector reconstruction by using another function as $\eta(\cdot;\cdot)$

unknown vector $\boldsymbol{x} \in \{0,1\}^N$



probability distribution (known):

$$\Pr(x_n = 0) = p_1$$

$$\Pr(x_n = 1) = p_2$$

corresponding $\eta(\cdot;\cdot)$:

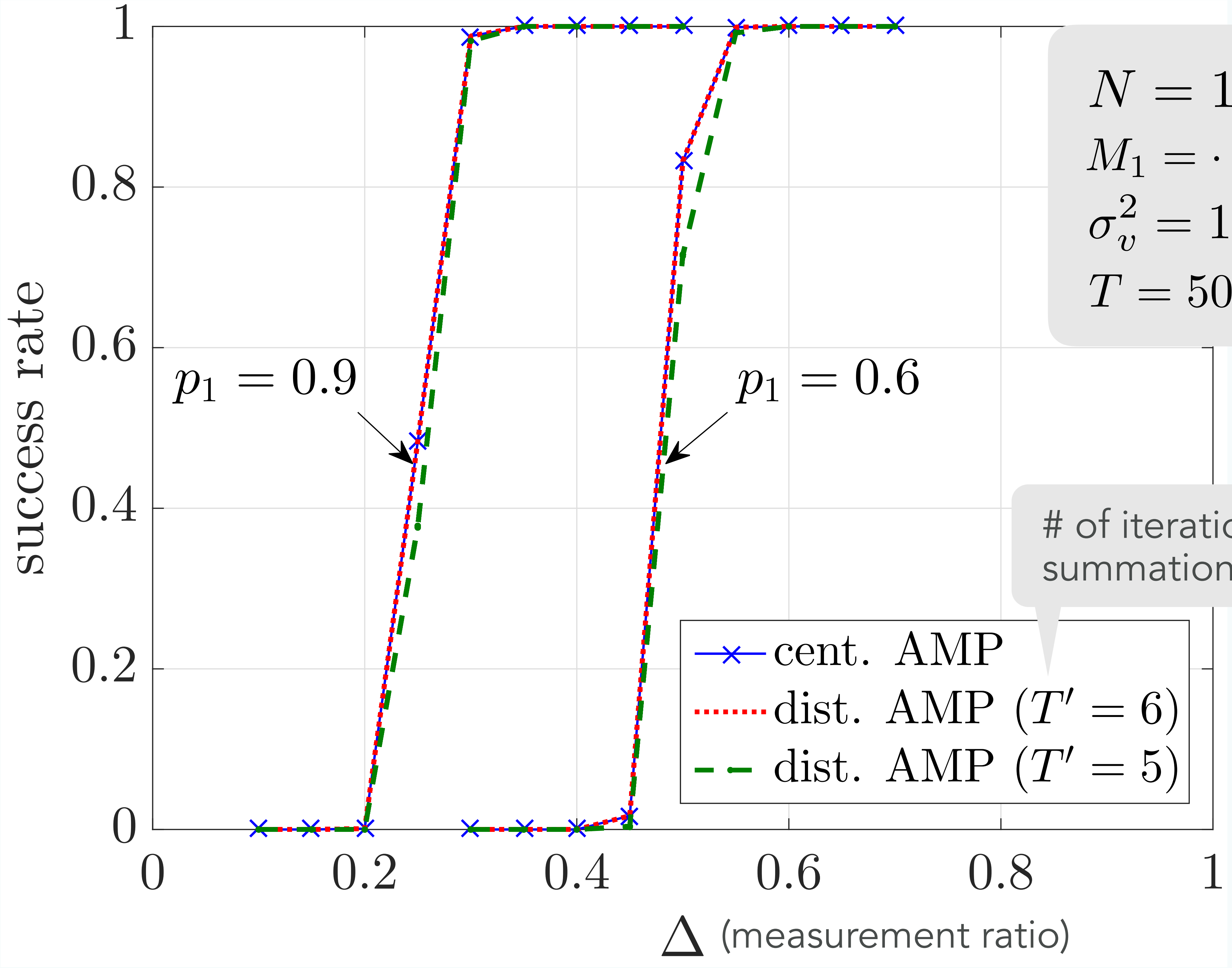
$$[\eta(\boldsymbol{r};\sigma^2)]_n = \frac{p_2\phi\left(\frac{\sqrt{\Delta}}{\sigma}(r_n-1)\right)}{p_1\phi\left(\frac{\sqrt{\Delta}}{\sigma}r_n\right) + p_2\phi\left(\frac{\sqrt{\Delta}}{\sigma}(r_n-1)\right)}$$

measurement matrix: $\boldsymbol{A}_k \in \mathbb{R}^{M_k \times N}$

measurement vector: $\boldsymbol{y}_k = \boldsymbol{A}_k\boldsymbol{x} + \boldsymbol{v}_k \in \mathbb{R}^{M_k}$

Appendix

Success Rate for Binary Vector Reconstruction



$N = 1000$

$M_1 = \cdots = M_K = \dfrac{\Delta N}{K}$

$\sigma_v^2 = 1$

$T = 50$

# of iterations in summation propagation