IMAGE UNMIXING SUCCESS ESTIMATION IN SPATIALLY VARYING SYSTEMS

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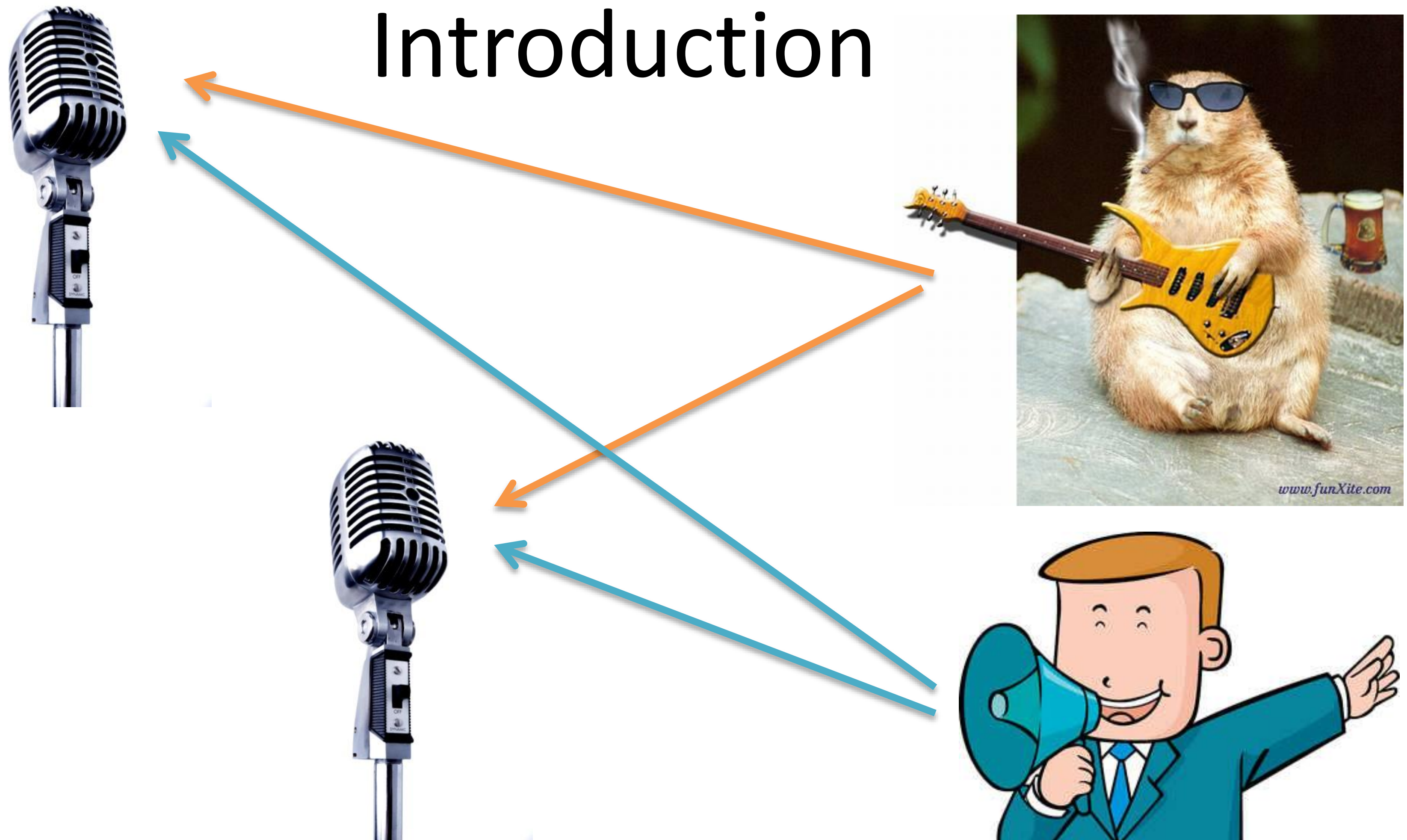
Introduction

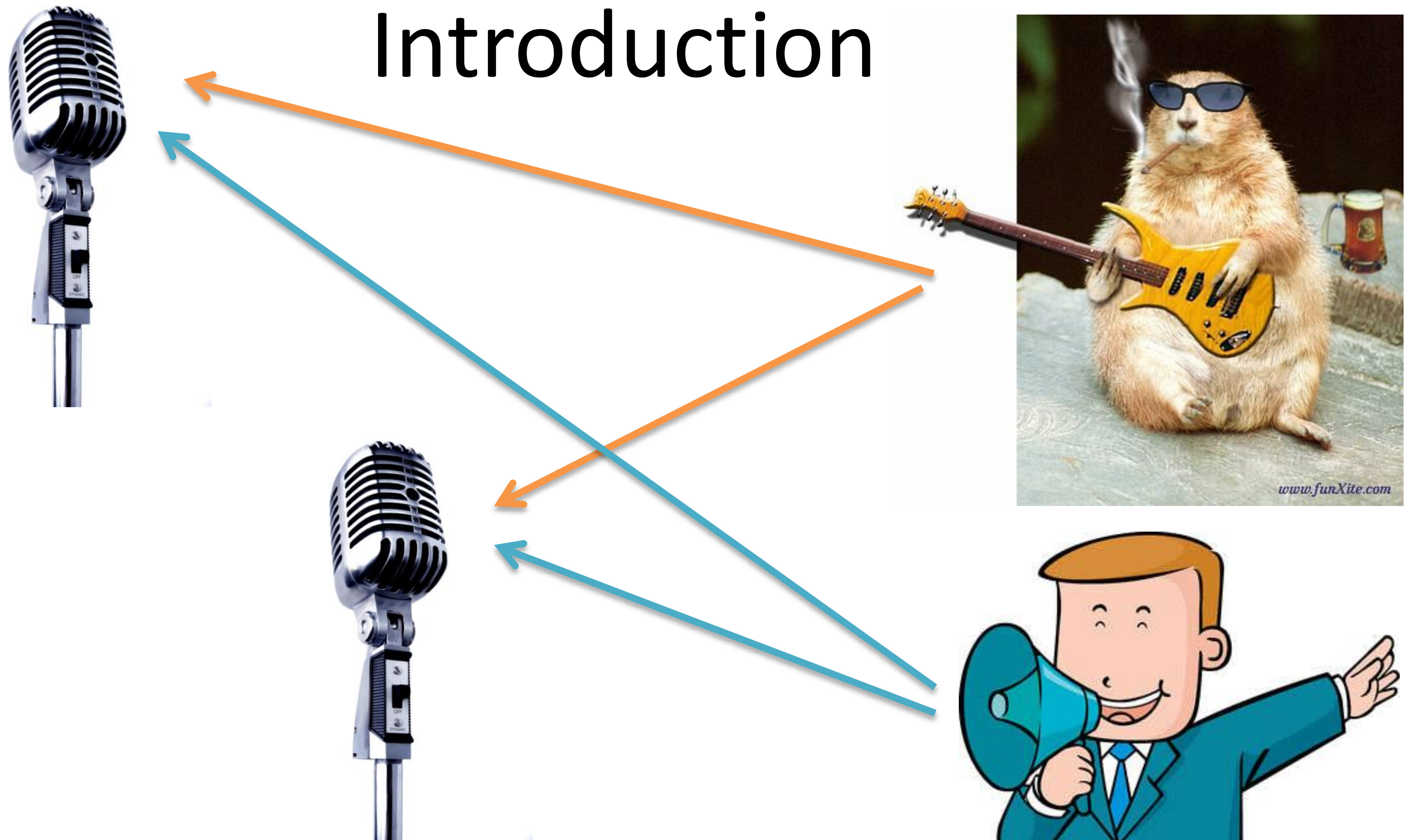
- The problem of recovering N_s source signals from N_z mixtures with only limited knowledge of the mixing process.
- Motivation: Speech signals separation, Image reflection unmixing, Medical Signals Anaysis (MRI, ECG), Communication signals unmixing,…
- Example: “Cocktail party problem”

Introduction



$$\underbrace{\begin{bmatrix} z_1 \\ z_2 \end{bmatrix}}_{Z} = \underbrace{\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}}_{H} \underbrace{\begin{bmatrix} s_1 \\ s_2 \end{bmatrix}}_{S}$$

Introduction



$$\underbrace{\begin{bmatrix} z_1 \\ z_2 \end{bmatrix}}_{Z} = \underbrace{\begin{bmatrix} 1 & 1 \\ h_{21} / h_{11} & h_{22} / h_{12} \end{bmatrix}}_{H'} \underbrace{\begin{bmatrix} s_1' \\ s_2' \end{bmatrix}}_{S'}$$

$$\begin{cases} s_1' = h_{11} s_1 \\ s_2' = h_{12} s_2 \end{cases}$$

R.Kaftory, Y.Zeevi 2009

SSCA

(Staged Sparse Component Analysis)

Space invariant Instantaneous example:

$$\underbrace{\begin{bmatrix} z_1 \\ z_2 \end{bmatrix}}_{Z} = \underbrace{\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}}_{H} \underbrace{\begin{bmatrix} s_1 \\ s_2 \end{bmatrix}}_{S}$$

Stage 1:
Estimate H based on sparseness of the data.

Stage 2:
Use $\hat{H}$ in order to estimate $\hat{S}$.

s_1

z_1

s_2

z_2

SSCA

(Staged Sparse Component Analysis)

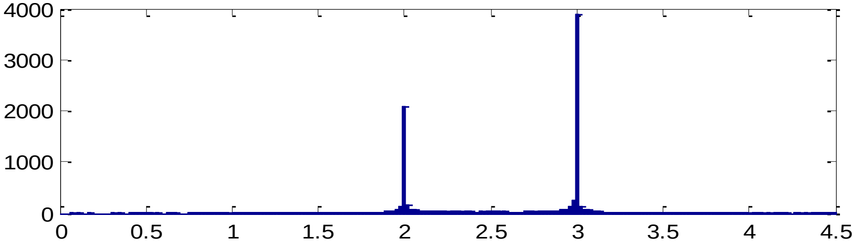
Stage 1:

$$r('active') = \frac{z_2}{z_1}('active')$$

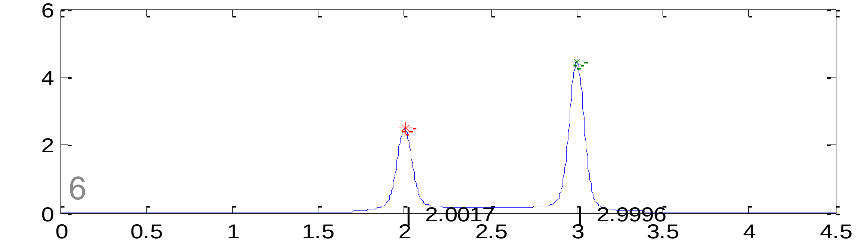
$$= \frac{2s_1 + 3s_2}{s_1 + s_2} \in h_{21}, h_{22} \quad (*)$$

$$(*) \qquad s_1 = 0 \ \text{ either } \ s_2 = 0$$

r_1 hist



r_1 smooth by gaussian kernal with σ=0.040241



s_1

z_1

s_2

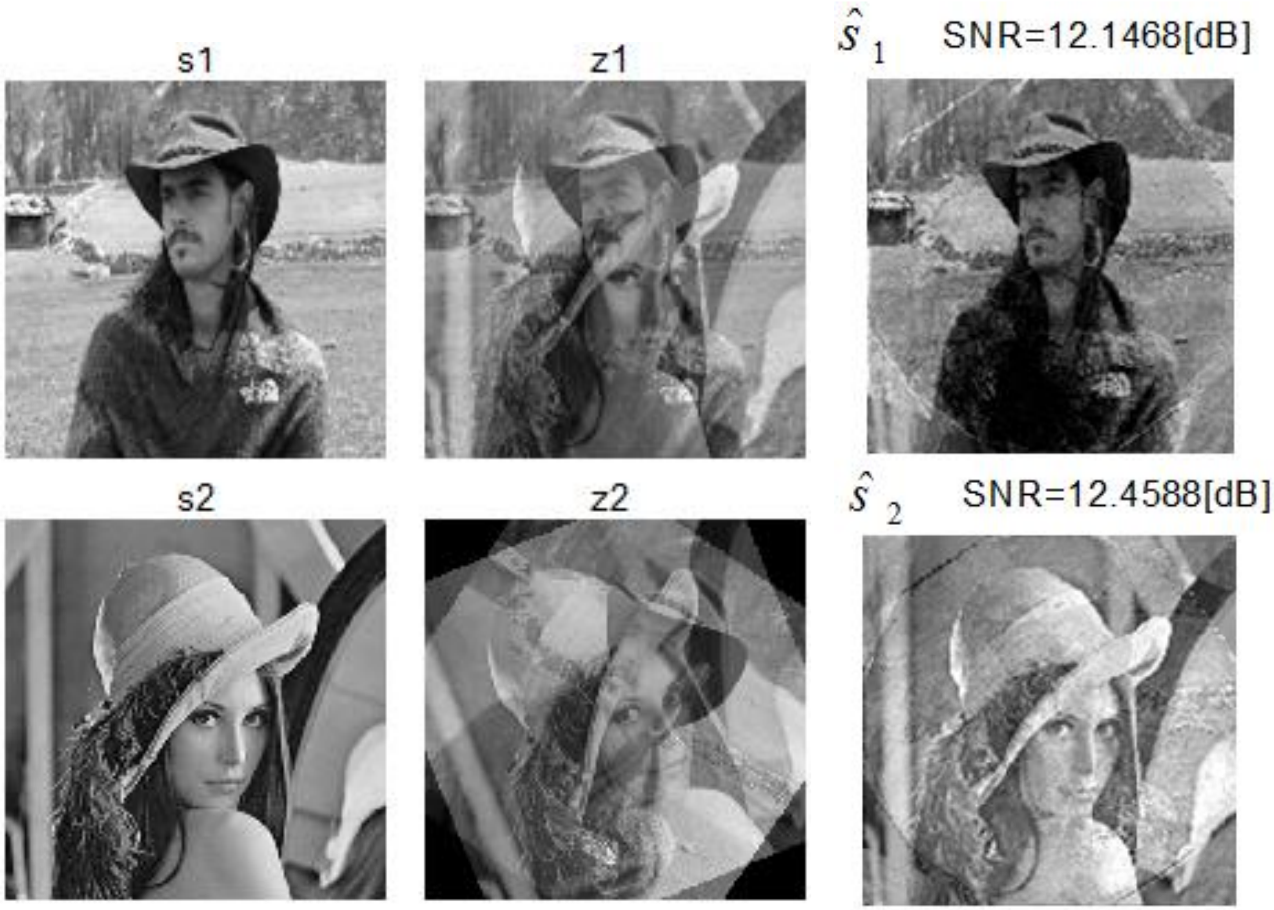
z_2

Non-Sparse signals ⇨ Sparsify

$\mathfrak{I}$ ⇨

Signals Estimation

$$\hat{S} = \underset{\hat{S}}{\operatorname{argmin}} \quad \left\| Z - H\left(\hat{\theta}\right)\hat{S} \right\| + \operatorname{Reg}\left\{\hat{S}\right\}$$



Success Estimation Method (SEM)

$$\varphi\left(\hat{S}\left(\hat{\theta}\right)\right)$$

* Function Demands:

- Local Minima when $\hat{\theta}=\theta:$ $\varphi\left(\hat{S}\left(\hat{\theta}=\theta\right)\right)=\min_{\hat{\theta}}\varphi\left(\hat{S}\left(\hat{\theta}\right)\right)$

- Smooth, Convex around true parameters.

* Known Methods demand prior knowledge :

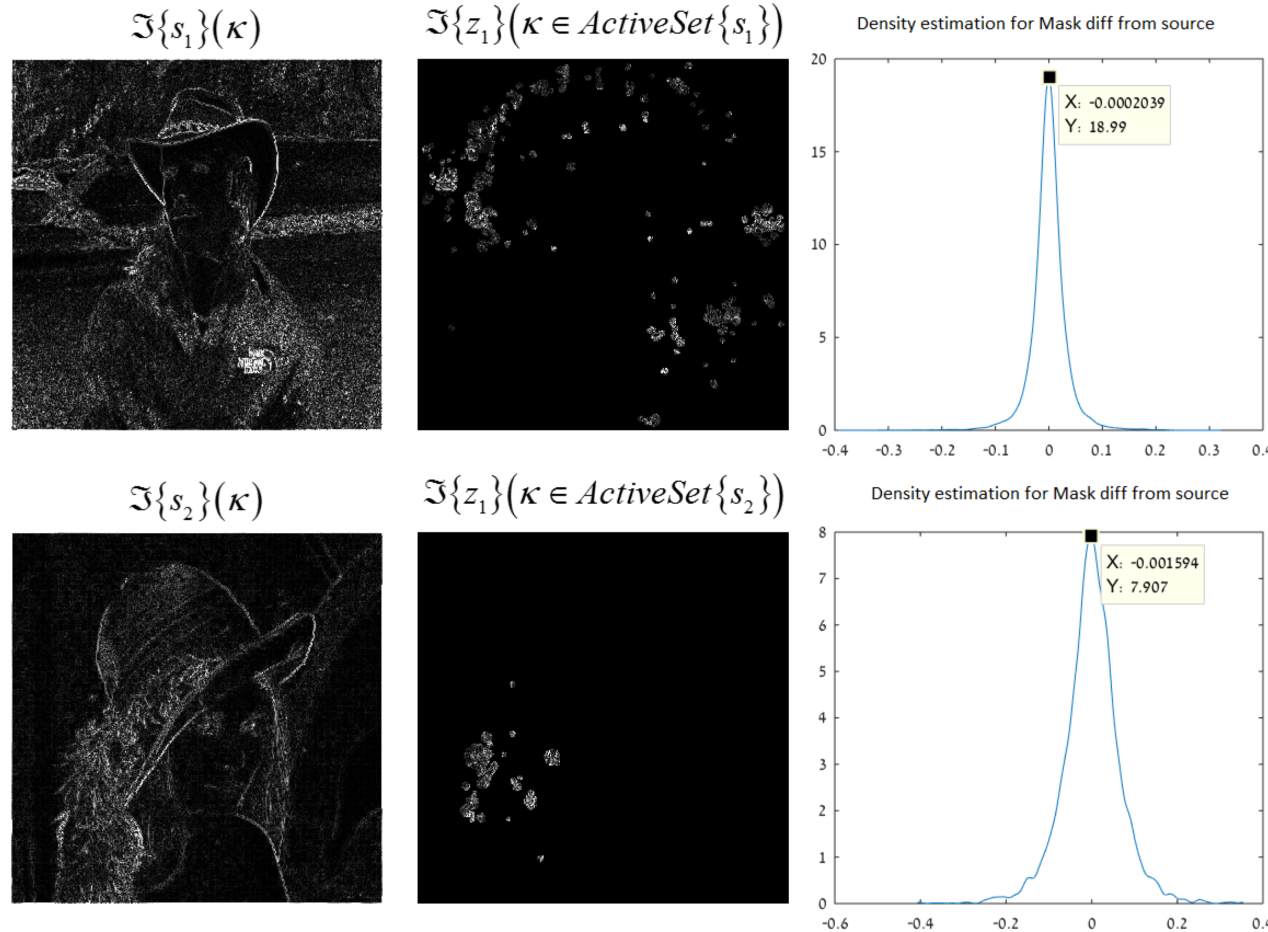
- Signals independence on one another ($MI\left(\hat{s}_1,\hat{s}_2\right)$).
- Signals sparseness on some domain.

($\sum PC\left(\hat{s}_i\right)^{0.3},\ \sum Edge\left(\hat{s}_i\right),\ \sum\left|Grad\left(\hat{s}_i\right)\right|^{0.2},\ \sum\left|SIFT\left(\hat{s}_i\right)\right|$)

A.Achtenberg Thesis 2011

Proposed Success Estimation Method (SEM)

Use additional knowledge of active set coefficients to estimate success.



Proposed Success Estimation Method

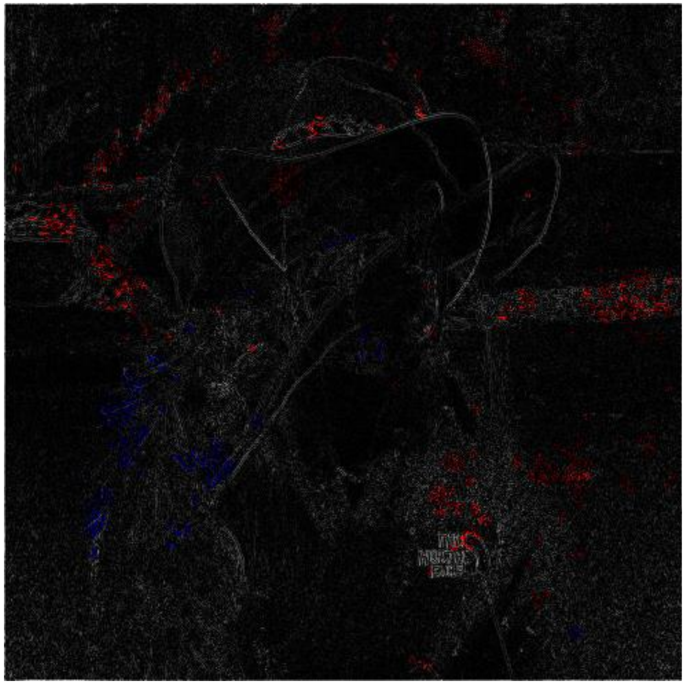
$$\varphi_p\left(\hat{\theta}\right) = \varphi_p\left(\hat{s}_1, \hat{s}_2, z_1, z_2, active\ \ set\ \ s_1, active\ \ set\ \ s_2\right) =$$

$$\sum_{\kappa \in active\ \ set\ \ s_1} \left\| \Im\left\{z_1\right\}(\kappa) - \Im\left\{\hat{s}_1\right\}(\kappa) \right\|$$

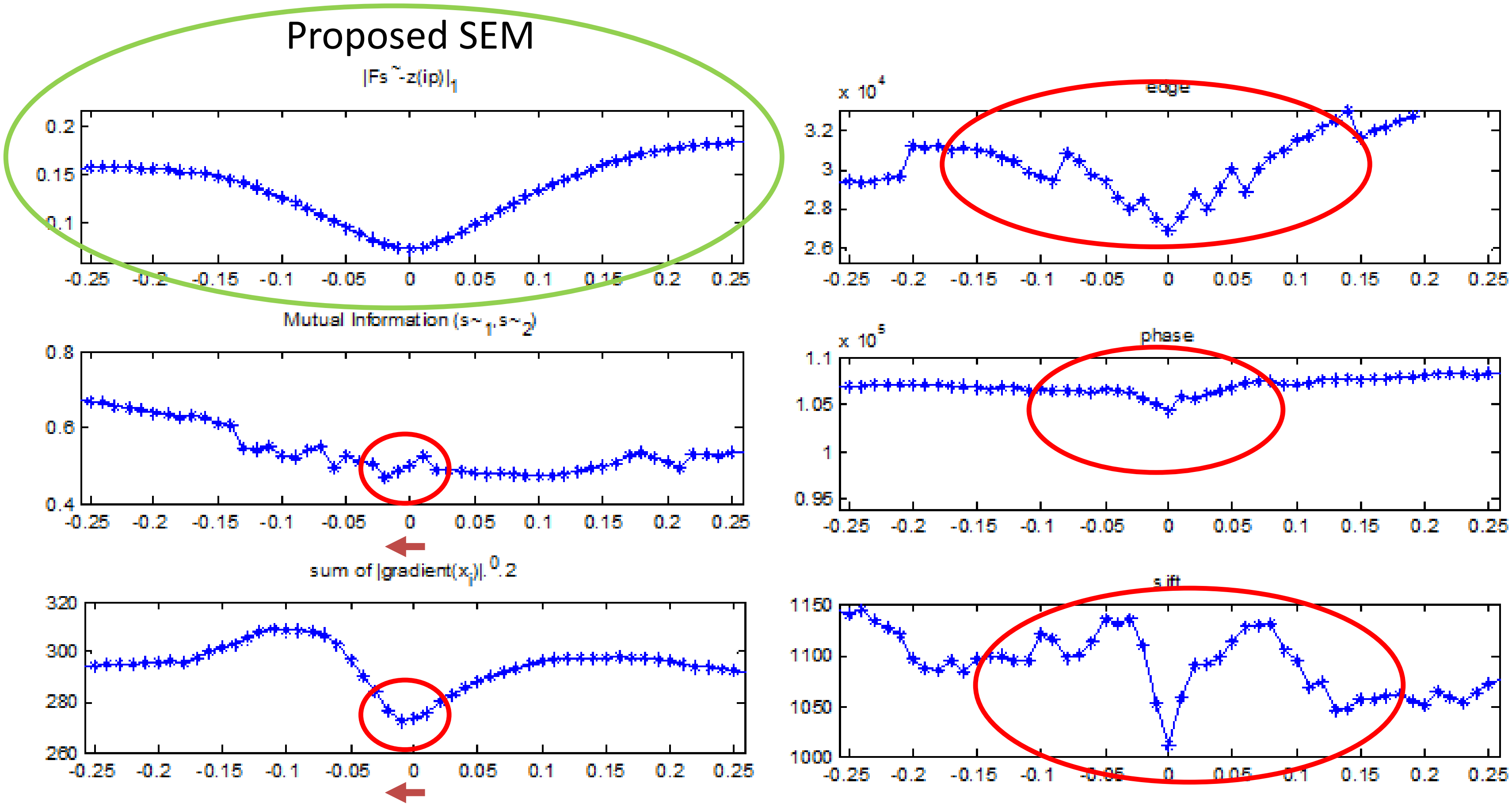
$$+ \quad \sum_{\kappa \in active\ \ set\ \ s_2} \left\| \Im\left\{z_1\right\}(\kappa) - \Im\left\{\hat{s}_2\right\}(\kappa) \right\|$$

z_1

$\Im\left\{z_1\right\}(\kappa)$



Success Estimation Methods (SEM)

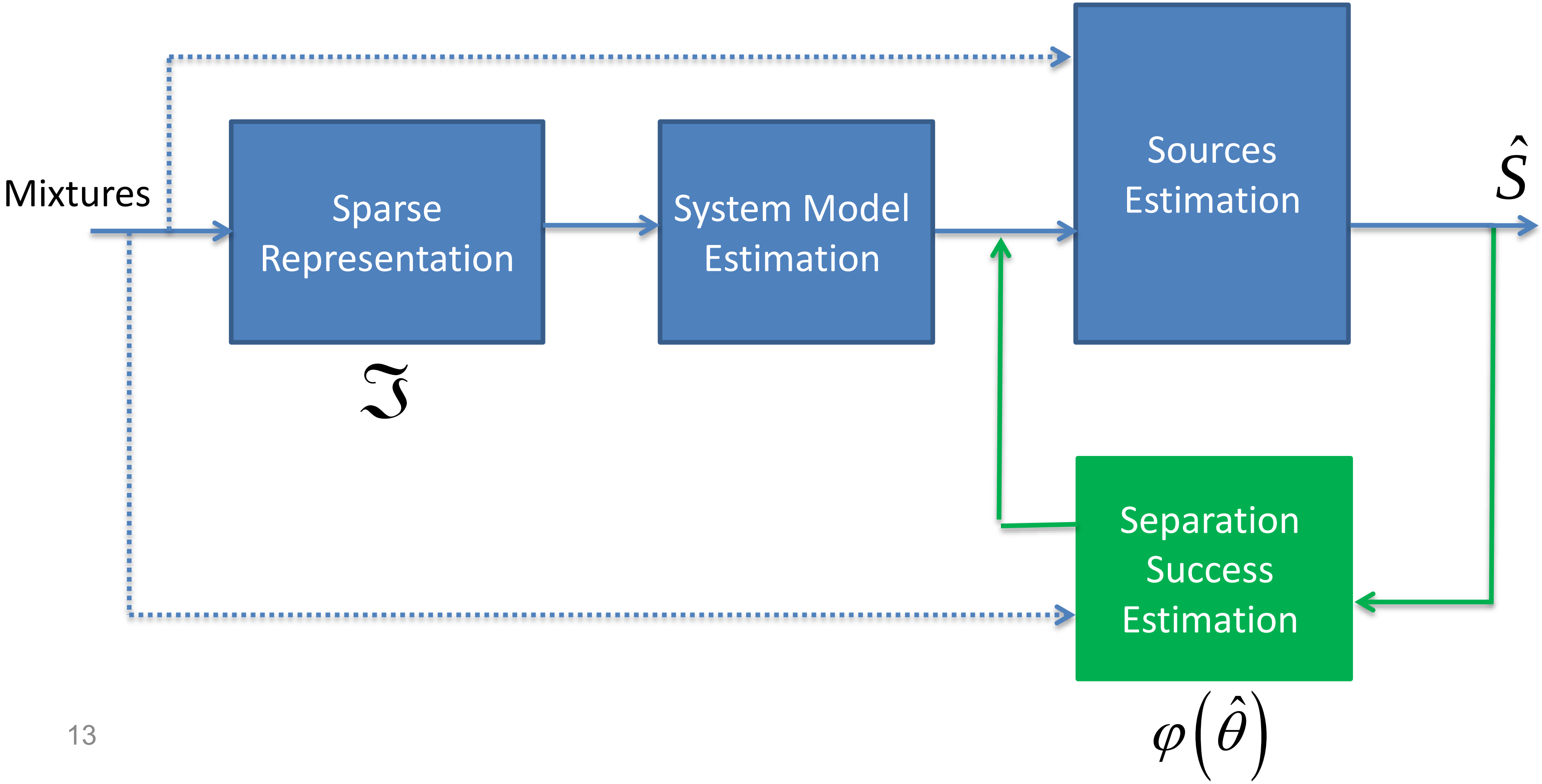


- Success versus deviation in mixing system parameters estimation.
- Other methods do not satisfy function demands.

Feedback Approach to Signal Estimation

$$\min_{\hat{\theta}} \varphi\left(\hat{S}\left(\hat{\theta}\right)\right)$$

$$s.t. \qquad \hat{S} = \underset{\hat{S}}{\operatorname{argmin}} \quad \left\| Z - H\left(\hat{\theta}\right)\hat{S} \right\| + \operatorname{Reg}\left\{\hat{S}\right\}$$



Feedback Approach : An Example

Original Signals

s1

s2

Mixtures

z1

z2

Estimated Signals using SSCA

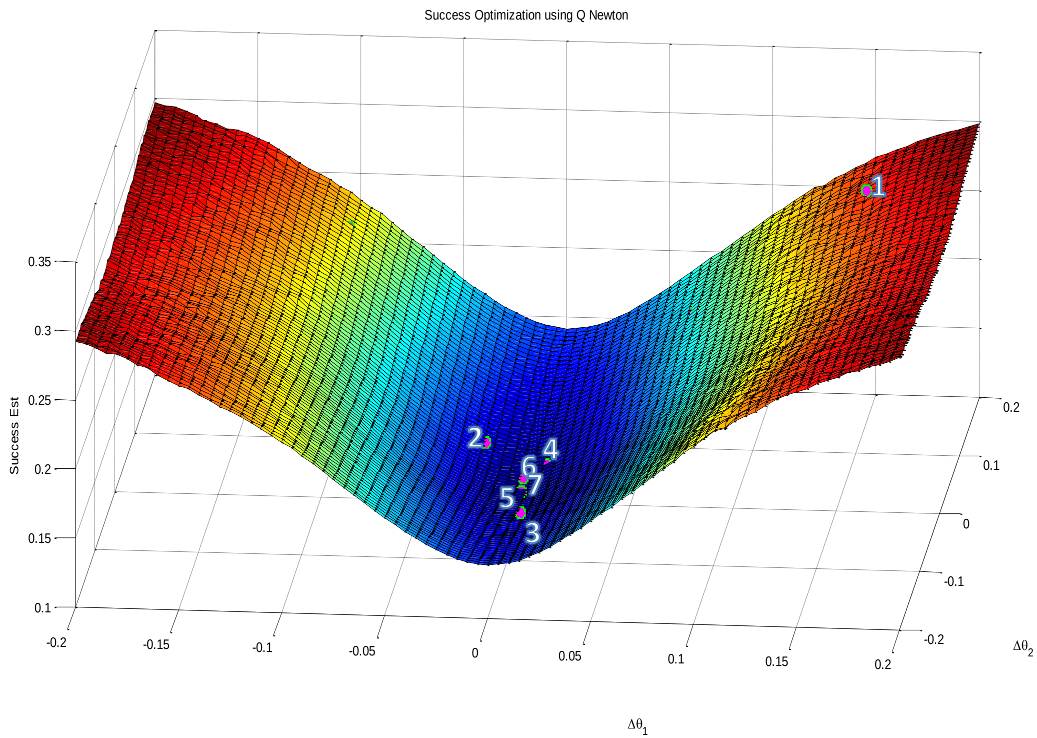
s_1 gal (lsqlin),MSE=0.00026189, SNR=12.1468[dB]

s_2 gal (lsqlin),MSE=0.00023082, SNR=12.4588[dB]

Feedback Approach : An Example

$$\min_{\hat{\theta}} \varphi\left(\hat{S}\left(\hat{\theta}\right)\right)$$

$$s.t. \qquad \hat{S} = \underset{\hat{S}}{\operatorname{argmin}} \quad \left\| Z - H\left(\hat{\theta}\right)\hat{S} \right\| + \mathrm{Reg}\left\{\hat{S}\right\}$$



Optimization of success estimation function.

Feedback Approach : An Example

Without Feedback With Feedback

Summary

- Staged Sparse Component Analysis Method.
- Success Estimation Method For Signal and Image un-mixing.
- Feedback Approach for improving Sources Reconstruction.

The End…

Mixing kernels

- Instantaneous time/space invariant

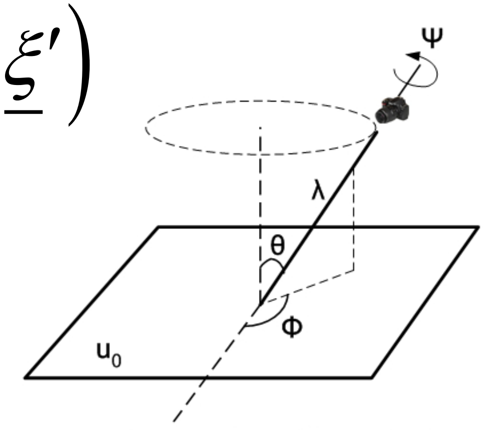
$$h_{ij}\left(\underline{\xi},\underline{\xi}'\right)=a_{ij}\delta\left(\underline{\xi}-\underline{\xi}'\right)$$

- Instantaneous time/space variant

$$h_{ij}\left(\underline{\xi},\underline{\xi}'\right)=a_{ij}\left(\underline{\xi}\right)\delta\left(\underline{\xi}-\underline{\xi}'\right)$$

- Attenuation and shift time/space variant

$$h_{ij}\left(\underline{\xi},\underline{\xi}'\right)=a_{ij}\left(\underline{\xi}\right)\delta\left(T_{ij}\left(\underline{\xi}\right)-\underline{\xi}'\right)$$



$u_{0:}$ frontal view of flat object

Sparsification $\Im\{\cdot\}$

- Commutative

$$\Im\{Z\} = \Im\{H \star S\} \approx H \star \Im\{S\}$$

- One 'Active' Source

$$activeset \triangleq \left\{\kappa \ \mid \ \min_{i=1\dots N_z} \left|\Im\{z_i\}(\kappa)\right| \geq th\right\}$$

$$\forall \kappa \in activeset: \ \ \exists j(\kappa) \in \{1..N_s\}, \ \ \forall \{1..N_z\}:$$

$$\Im\{z_i\}(\kappa) \approx h^{\vec{\theta}}_{ij(\kappa)}\left(\underline{\xi}, \underline{\xi}'\right) \star \Im\left\{s_{j(\kappa)}\right\}(\kappa)$$

Proposed Success Estimation Method

$$\varphi_p\left(\hat{\theta}\right) = \varphi_p\left(\hat{s}_1, \hat{s}_2, z_1, z_2, p_k^{12}\right) =$$

$$\sum_{\left\{k,\ \ p_k^{12} \leftrightarrow q_k^{12} \in s_1\right.} \left\| \Im\left\{z_1\left(\cdot\right)\right\}\left(p_k^{12}\right) - \Im\left\{\hat{s}_1\left(\cdot\right)\right\}\left(p_k^{12}\right) \right\|_p$$

$$+ \sum_{\left\{k,\ \ p_k^{12} \leftrightarrow q_k^{12} \in s_2\right.} \left\| \Im\left\{z_1\left(\cdot\right)\right\}\left(p_k^{12}\right) - \Im\left\{\hat{s}_2\left(\cdot\right)\right\}\left(p_k^{12}\right) \right\|_p$$

$$= \sum_{i=1..2} \sum_{\left\{k,\ \ p_k^{12} \leftrightarrow q_k^{12} \in s_i\right.} \left\| \Im\left\{z_1\left(\cdot\right)\right\}\left(p_k^{12}\right) - \Im\left\{\hat{s}_i\left(\cdot\right)\right\}\left(p_k^{12}\right) \right\|_p$$

$$= \sum_{i=1..2} \sum_{\left\{k,\ \ p_k^{12} \leftrightarrow q_k^{12} \in s_i\right.} \left\| \Im\left\{s_1\left(\cdot\right) + s_2\left(\cdot\right)\right\}\left(p_k^{12}\right) - \Im\left\{\hat{s}_i\left(\cdot\right)\right\}\left(p_k^{12}\right) \right\|_p$$

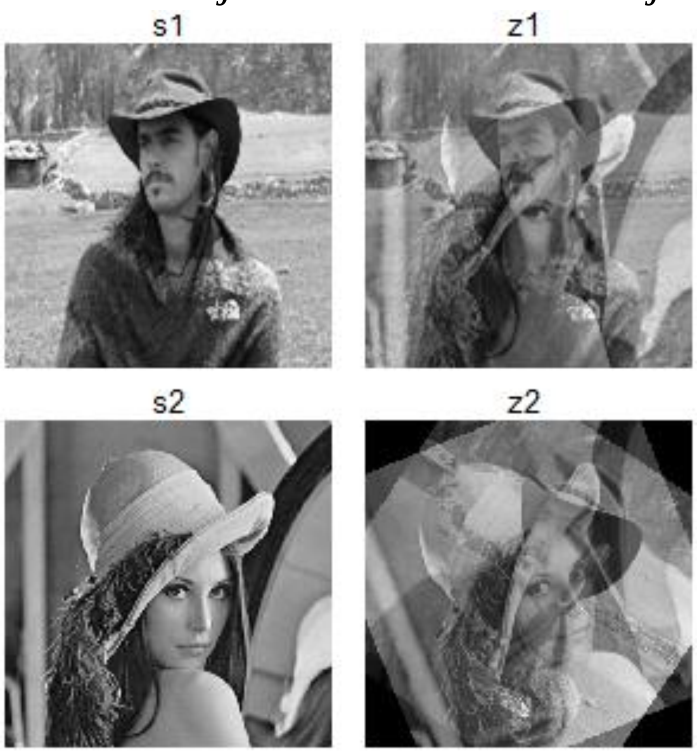
$$\approx \sum_{i=1..2} \sum_{\left\{k,\ \ p_k^{12} \leftrightarrow q_k^{12} \in s_i\right.} \left\| \Im\left\{s_1\left(\cdot\right)\right\}\left(p_k^{12}\right) + \Im\left\{s_2\left(\cdot\right)\right\}\left(p_k^{12}\right) - \Im\left\{\hat{s}_i\left(\cdot\right)\right\}\left(p_k^{12}\right) \right\|_p$$

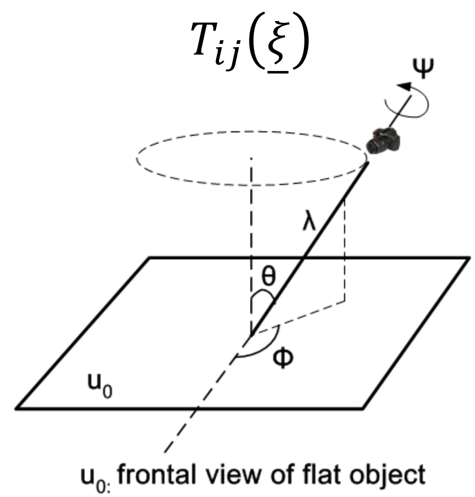
$$\approx \sum_{i=1..2} \sum_{\left\{k,\ \ p_k^{12} \leftrightarrow q_k^{12} \in s_i\right.} \left\| \Im\left\{s_i\left(\cdot\right)\right\}\left(p_k^{12}\right) - \Im\left\{\hat{s}_i\left(\cdot\right)\right\}\left(p_k^{12}\right) \right\|_p \underset{\hat{\theta}=\theta}{=} 0$$

Single path mixtures

$$h_{ij}\left(\underline{\xi},\underline{\xi}'\right)=a_{ij}\left(\underline{\xi}\right)\delta\left(T_{ij}\left(\underline{\xi}\right)-\underline{\xi}'\right)$$

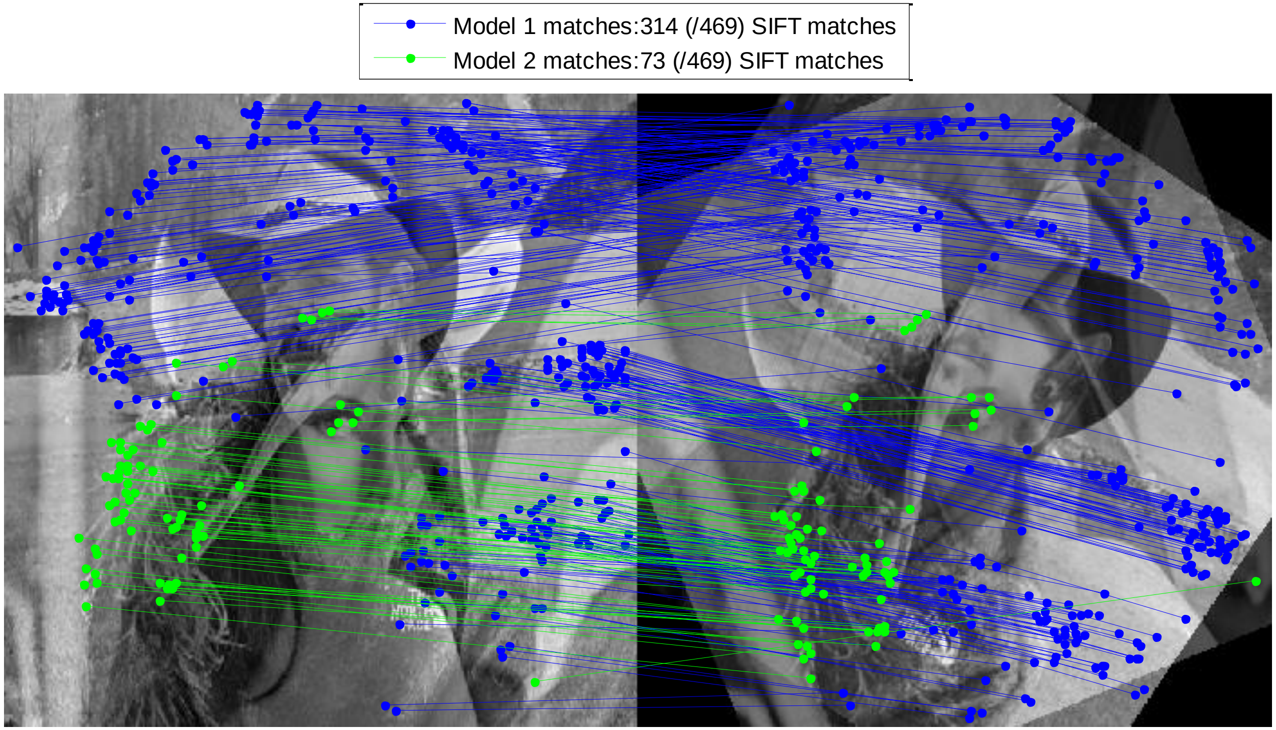
$$z_i\left(x,y\right)=\sum_j w_{ij}\left(x,y\right)=\sum_j a_{ij}\left(x,y\right)s_j\left(T_{ij}^x\left(x,y\right),T_{ij}^y\left(x,y\right)\right)$$





Not Sparse ? Sparsify

- Single path spatial distortion system
  -> SIFT (for spatial transform)



$$activeset \triangleq \left\{ \kappa \mid \min_{i=1\ldots N_z} \left| \Im\left\{ z^{aligned}{}_i \right\}(\kappa) \right| \geq th \right\}$$

Not Sparse ? Sparsify

- Single path spatial distortion system
  -> SIFT (for spatial transform) + Alignment

z_1

z_2 aligned

Not Sparse ? Sparsify

- Single path spatial distortion system

  -> SIFT (for spatial transform) + Alignment + Wavelet Transform (For attenuation model)