MIMO Radar Target Detection Using Low-Complexity Receiver

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Background

- A large number of transmitters are often available.
- Processing additional transmitters requires additional hardware or software complexity at the receivers.
- It is necessary to design the receivers wisely to control the complexity cost.

Received Signal Model

- The received target-present signal at the n-th receiver:

$$r_n(t) = \sum_{m=1}^{M} \frac{\beta_{mn}\sqrt{E_m}}{R_{t,m}R_{r,n}} s_m(t-\tau_{mn}) + w_n(t),$$

- β_{mn}, τ_{mn}-reflection coefficient and time delay for mn-th path. β_{mn} is correlated complex Gaussian;
- $R_{t,m}$, $R_{r,n}$- distance between the m-th transmitter/n-th receiver and target;
- E_m, $s_m(t)$ - transmitted energy, known waveform from m-th transmitter. Assume direct path antenna gives accurate $s_m(t)$;
- $w_n(t)$ - zero-mean complex Gaussian temporally white clutter-plus-noise such that $\mathbb{E}\{w_i(t)w_j^*(u)\} = N_{ij}\delta(t-u)$.

Detection Problem

- Hypothesis Testing Problem

$$\mathcal{H}_0 : r_n(t) = w_n(t)$$

$$\mathcal{H}_1 : r_n(t) = \sum_{m=1}^{M} \xi_{mn} s_m(t-\tau_{mn}) + w_n(t).$$

- $\xi_{mn} = \beta_{mn}\sqrt{E_m}/(R_{t,m}R_{r,n})$

- Optimal Test Statistic (TS)

$$\mathcal{T} = \boldsymbol{x}^H(\boldsymbol{\Sigma}_0^{-1} - \boldsymbol{\Sigma}_1^{-1})\boldsymbol{x}$$

$$\boldsymbol{x} = [\boldsymbol{x}_1^T, ..., \boldsymbol{x}_N^T]^T$$

where

$$\boldsymbol{x}_n = [x_{11n}, ..., x_{MNn}]^T$$

in which

$$x_{mn'n} = \int_{\mathcal{T}_m} s_m^*(t-\tau_{mn'}) r_n(t) dt$$

is the output of the mn'-th matched filter (MF) at receiver n. TS depends on MF outputs vector. We use them in receiver design problem.

- The terms

$$\boldsymbol{\Sigma}_0 = \boldsymbol{N} \otimes \boldsymbol{\Xi}$$

$$\boldsymbol{\Sigma}_1 = \boldsymbol{N} \otimes \boldsymbol{\Xi} + \boldsymbol{\Psi}\boldsymbol{\Lambda}\boldsymbol{\Psi}^H$$

are the covariance matrices of $\boldsymbol{x}$ under $\mathcal{H}_0$ and $\mathcal{H}_1$ respectively;

- $\boldsymbol{\Lambda} = \mathbb{E}\{\boldsymbol{\xi}\boldsymbol{\xi}^H\}$ is the covariance matrix of $\boldsymbol{\xi} = [\xi_{11}, ...\xi_{MN}]^T$;
- $\boldsymbol{\Xi}$ is an $MN \times MN$ matrix and

$$\boldsymbol{\Xi}_{(n_1-1)M+m_1,(n_2-1)M+m_2} = \int_{\mathcal{T}_{m_1}} s_{m_1}^*(t-\tau_{m_1n_1}) s_{m_2}(t-\tau_{m_2n_2}) dt;$$

- $\boldsymbol{\Psi} = \mathrm{Diag}\{\boldsymbol{\Psi}_1, ..., \boldsymbol{\Psi}_N\}$, where $\boldsymbol{\Psi}_n$ is an $MN \times M$ matrix whose i-th column is the $((n-1)M+i)$-th column of $\boldsymbol{\Xi}$.

Receiver Design Method

- **Define:** $\boldsymbol{a} \triangleq [\boldsymbol{a}_1^T, \boldsymbol{a}_2^T, ..., \boldsymbol{a}_N^T]^T$, where $\boldsymbol{a}_n = [\boldsymbol{a}_{11n}, ..., \boldsymbol{a}_{MNn}]^T$ and $a_{mn'n} \in \{1, 0\}$ indicating whether or not the (m, n')-th MF is selected at the n-th receiver and a selection vector $\boldsymbol{J}_n(\boldsymbol{a}_n)$, where $\boldsymbol{J}_n(\boldsymbol{a}_n)$ has one unit element per row. The other elements are zeroes.
- **Example:**

$$\boldsymbol{a}_1 = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix}^T$$

$$\boldsymbol{J}_1(\boldsymbol{a}_1) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\boldsymbol{J}_1(\boldsymbol{a}_1) \begin{bmatrix} x_{111} \\ x_{121} \\ x_{211} \\ x_{221} \end{bmatrix} = \begin{bmatrix} x_{111} \\ x_{211} \end{bmatrix}$$

Put "$\boldsymbol{a}$" on diagonal and removal all 0 rows.

- The MF output vector at receiver n is reduced from $\boldsymbol{x}_n$ to $\boldsymbol{J}_n(\boldsymbol{x}_n)$.
- The TS becomes

$$T_s = (\boldsymbol{J}(\boldsymbol{a})\boldsymbol{x})^H \left(\left(\boldsymbol{J}(\boldsymbol{a})\boldsymbol{\Sigma}_0\boldsymbol{J}^T(\boldsymbol{a})\right)^{-1} - \left(\boldsymbol{J}(\boldsymbol{a})\boldsymbol{\Sigma}_1\boldsymbol{J}^T(\boldsymbol{a})\right)^{-1} \right) \boldsymbol{J}(\boldsymbol{a})\boldsymbol{x}$$

- The receiver design can be described in the following problem:

$$\mathbf{P}_1 \begin{cases} \max\limits_{\boldsymbol{a}\in\{0,1\}^{MN^2}} & Pr(T_s > \gamma(P_{FA}; \boldsymbol{a}), \mathcal{H}_1) \\ s.t. & 1 \le \|\boldsymbol{a}_n\|_0 \le A_n, n = 1, 2, ..., N \end{cases}$$

Transmitter selection

- Special Case
  - Spatially white reflection coefficients and clutter-plus-noise $\boldsymbol{\Lambda} = \mathrm{Diag}\{\boldsymbol{\Lambda}_1, ..., \boldsymbol{\Lambda}_N\}$, where $\boldsymbol{\Lambda}_n = \mathrm{diag}\ \{\sigma_{1n}^2 E_1/(R_{t,1}R_{r,n})^2, ..., \sigma_{Mn}^2 E_M/(R_{t,M}R_{r,n})^2\}$ and $\boldsymbol{N} = N_0\boldsymbol{I}_N$.
  - Orthogonal waveforms

$$\int_{\mathcal{T}_m} s_m^*(t-\tau) s_{m'}(t) dt \approx 0 \quad \text{for } m \neq m'.$$

- Test Statistic
  - The TS becomes

$$T_s = \sum_{n=1}^{N}\sum_{m=1}^{M} \frac{E_m\sigma_{mn}^2}{N_0(E_m\sigma_{mn}^2 + N_0(R_{t,m}R_{r,n})^2)} |x_{mn}|^2$$

  - Only need MN MFs. Redefine an $MN \times 1$ selection vector $\boldsymbol{a} = [\boldsymbol{a}_1^T, ..., \boldsymbol{a}_N^T]^T$, where $\boldsymbol{a}_n = [a_{1n}, ..., a_{Mn}]^T$.
  - The TS after selection becomes

$$T_s = \sum_{n=1}^{N}\sum_{m=1}^{M} \frac{E_m a_{mn}\sigma_{mn}^2}{N_0(E_m\sigma_{mn}^2 + N_0(R_{t,m}R_{r,n})^2)} |x_{mn}|^2$$

MSCNR-based Selection

- **Define** the SCNR of the (m,n)-th path as

$$\eta_{mn} = \frac{E_m\sigma_{mn}^2}{N_0(R_{t,m}R_{r,n})^2}.$$

The TS can be rewritten as

$$T_s = \sum_{n=1}^{N}\sum_{m=1}^{M} \zeta_{mn},$$

where $\zeta_{mn} = \frac{\rho_{mn}}{N_0(\rho_{mn}+1)|x_{mn}|^2}$ and $\rho_{mn} = \eta_{mn}a_{mn}$.

Let $\boldsymbol{\eta} = [\eta_{11}, ..., \eta_{MN}]$, $\boldsymbol{\rho} = [\rho_{11}, ...\rho_{MN}]^T$.

Lemma: Denote by $\rho_{(1)}, \rho_{(2)}..., \rho_{(K)}$ the decreasing sequence of nonnegative $\rho_{11}, \rho_{21}, ..., \rho_{MN}$ and define $\boldsymbol{\rho}_{(K)}=[\rho_{(1)}, \rho_{(2)} ..., \rho_{(K)}]^T$, where $K = MN$. Let $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ be two feasible solutions for $\mathbf{P}_1$, and correspondingly $\boldsymbol{\rho_\alpha} = \boldsymbol{\alpha} \odot \boldsymbol{\eta}$ and $\boldsymbol{\rho_\beta} = \boldsymbol{\beta} \odot \boldsymbol{\eta}$. If $\boldsymbol{\rho}_{\boldsymbol{\alpha}(K)} \succeq \boldsymbol{\rho}_{\boldsymbol{\beta}(K)}$, then $P_D(\boldsymbol{\rho_\alpha}) \geqslant P_D(\boldsymbol{\rho_\beta})$.

Theorem: If the corresponding SCNRs of the selected transmitters at receiver n are the largest A_n values in $\{\eta_{1n}, ..., \eta_{Mn}\}$, we can obtain the optimal solution of $\mathbf{P}_1$.

Numerical Examples

- Parameter Set up:
  - The PSD of noise $N_i i = 1, i = 1, ..., N$, the transmitted energy $E_m = 10^{13}$, the variance of the mn-th path reflection coefficient σ_{mn}^2=1 for all m, n.
  - Target located at $(0, 0)$ km.
  - The transmitted waveform

$$s_m(t) = \frac{1}{\sqrt{T}}\exp(j2\pi f_m t), 0 < t < T$$

where $T = 1ms$ for all m and f_m is the transmitted frequency of m-th transmitter. Define $\boldsymbol{f} = [f_1, ..., f_M]$ the transmitted frequency vector.

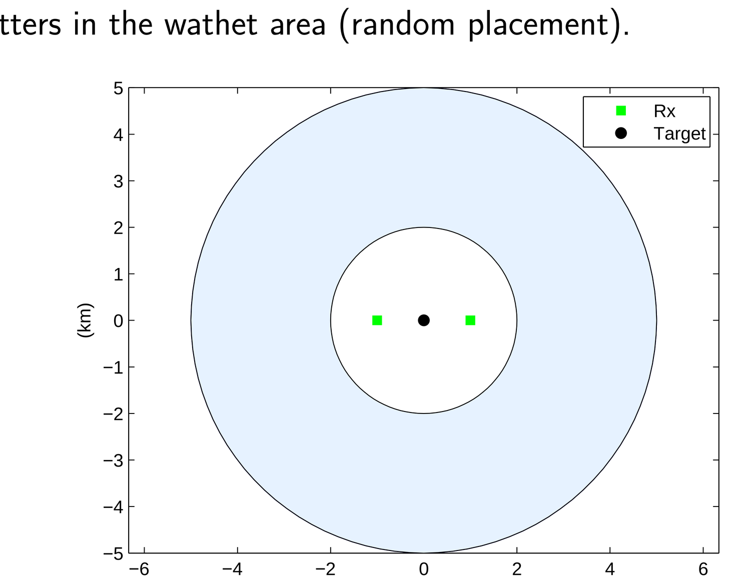
- Example 1: Special case
  - Transmitters: $(x_{t,1}, y_{t,1})$=(0,1) km, $(x_{t,2}, y_{t,2})$= (0, 2) km, and $(x_{t,3}, y_{t,3})$ =(0, 3) km;
  - Receivers: $(x_{r,1}, y_{r,2})$=(-1, 0) km and $(x_{r,2}, y_{r,2})$=(1, 0) km;
  - Frequency vector $\boldsymbol{f} = [\frac{10}{T}, \frac{20}{T}, \frac{30}{T}]$ and $P_{FA} = 10^{-2}$.

Table 1: Detection probability of different selections

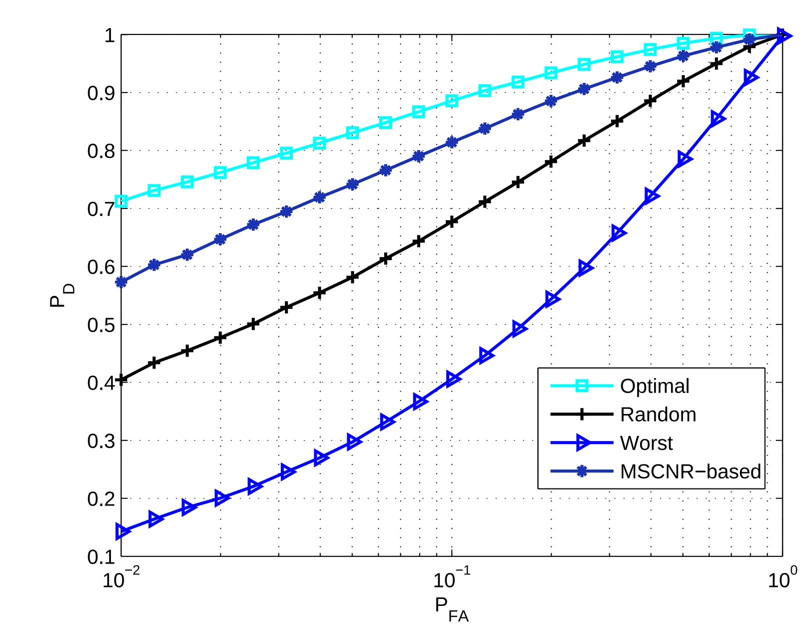
Selection Combination	SCNR (dB)	P_D
$\{<1,1>, <1,2>\}$	{10,10}	0.8798
$\{<1,1>, <2,2>\}$	{10,3.98}	0.7286
$\{<1,1>, <3,2>\}$	{10,0.46}	0.6868
$\{<2,1>, <1,2>\}$	{3.98,10}	0.7422
$\{<2,1>, <2,2>\}$	{3.98,3.98}	0.4354
$\{<2,1>, <3,2>\}$	{3.98,0.46}	0.3196
$\{<3,1>, <1,2>\}$	{0.46,10}	0.6785
$\{<3,1>, <2,2>\}$	{0.46,3.98}	0.3117
$\{<3,1>, <3,2>\}$	{0.46,0.46}	0.1880

  - $<m, n>$ means the m-th transmitter being selected at the n-th receiver.

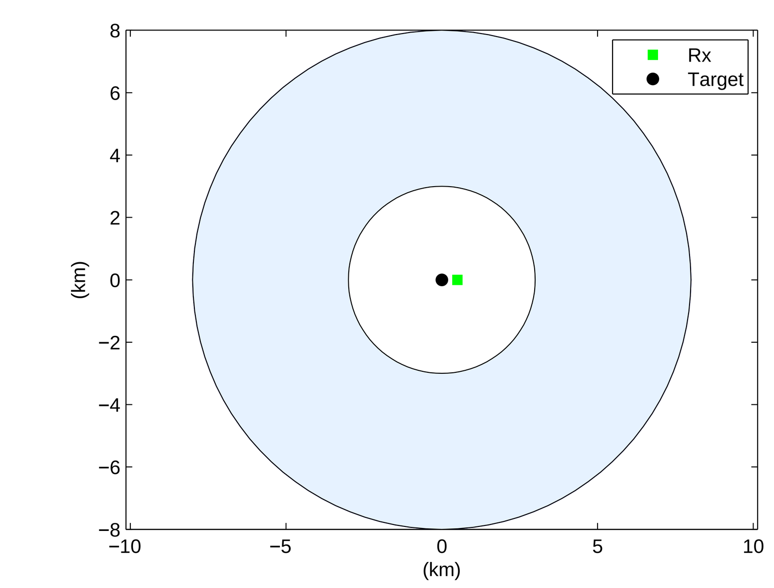
- Example 2: General case
  - Frequency vector $\boldsymbol{f} = [\frac{10}{T}, \frac{20}{T}, \frac{30}{T}]$, $P_{FA} = 10^{-2}$;
  - Correlation of clutter-plus-noise N_{ij}=0.1, $i, j = 1, 2, i \neq j$;
  - Correlation of reflection coefficients $\mathbb{E}\{\beta_{m_1n_1}\beta_{m_2n_2}^*\}$=0.1, $m_1, m_2 = 1, ...M$, n_1, n_2=$1, ..., N$, $m_1 \neq m_2$ or $n_1 \neq n_2$;
  - 8 transmitters in the wathet area (random placement).



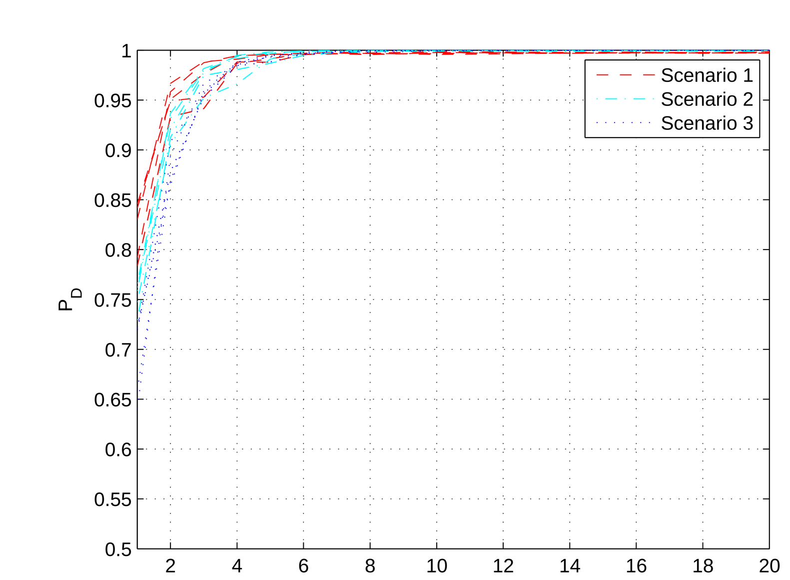
- Different selections



- Example 3: Large number of transmitters
  - Frequency vector $\boldsymbol{f}$ is $[\frac{3}{50T}, \frac{6}{50T}, ..., \frac{3}{T}]$ (scenario 1), $[\frac{6}{50T}, \frac{12}{50T}, ..., \frac{6}{T}]$ (scenario 2) and $[\frac{9}{50T}, \frac{18}{50T}, ..., \frac{9}{T}]$ (scenario 3).
  - False alarm probability is 10^{-2}.
  - 50 transmitters in the wathet area (random placement).



- P_D vs. number of selected trasnmitters



Conclusion

- Studied low-complexity receiver design for MIMO radar by selecting a limited number of MFs at each receiver due to cost considerations.
- For the case of uncorrelated clutter-plus-noise, uncorrelated reflection coefficients, and orthogonal waveforms, at each receiver, maximum P_D can be achieved if we select the transmitters corresponding to the largest SCNRs.
- Selecting a few transmitters can lead to detection performance very close to using all transmitters.

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