Manifold-based analysis of natural stochastic textures with application in texture synthesis

Ido Zachevsky* and Yehoshua Y. Zeevi

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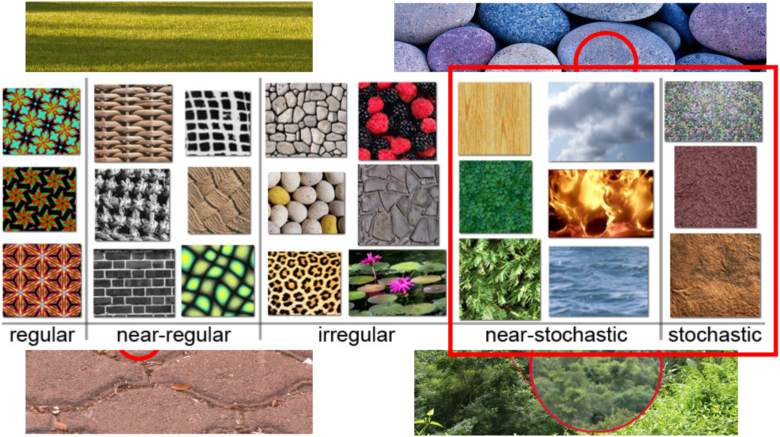
Natural stochastic textures (NST)

What are natural stochastic textures? [Zachevsky and Zeevi, 2014]



[Lin et al., 2004]

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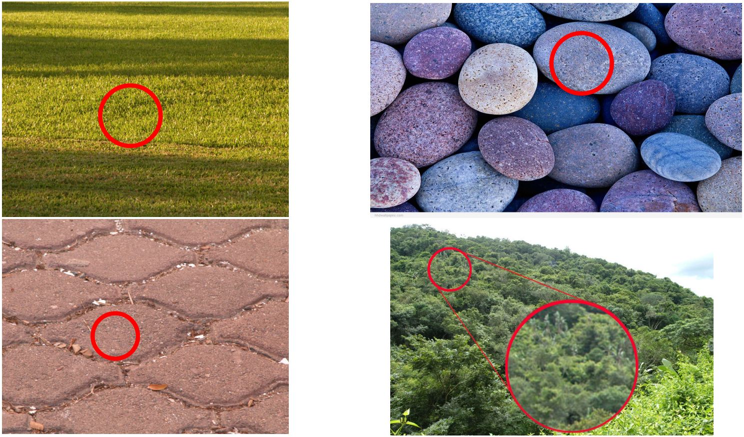


The goal of this work

Find a method that embeds NST in a manifold endowed with geometrical properties, that also allows sampling, or synthesis, of new textures.

Why an NST dedicated approach?

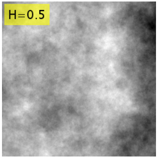
Manifolds – describing high dimensional data by a low dimensional representation

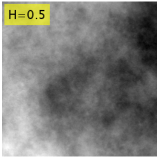
- Riemannian manifolds [Kimmel et al., 2000], e.g. Laplacian Eigenmaps [Belkin and Niyogi, 2003]
- Variational autoencoders [Kingma and Welling, 2013]

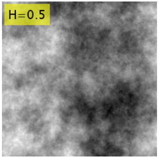
What makes NST special?

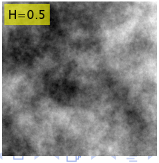
- They are inherently random but also exhibit deterministic characteristics.

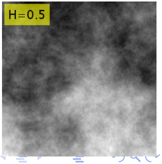
$\leftrightarrow$











Caveat

What this work is *not* about

- Manifold extraction for general signals
  - Variational autoencoders [Kingma and Welling, 2013]
  - Laplacian eigenmaps [Belkin and Niyogi, 2003]
  - Diffusion maps [Coifman and Lafon, 2006]
- Synthesis of general images or textures by a high dimensional descriptor
  - Generative adversarial networks [Goodfellow, 2016]

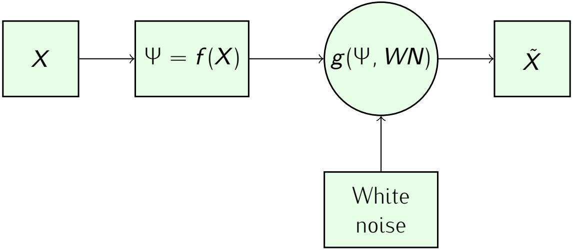
How to obtain a suitable analysis method?

Requirement: extraction of deterministic features, keeping in mind random data.

Analysis by synthesis

We propose a framework for analysis and synthesis of textures, based on [Gatys et al., 2015]

Texture synthesis frameworks [Portilla and Simoncelli, 2000, Gatys et al., 2015]



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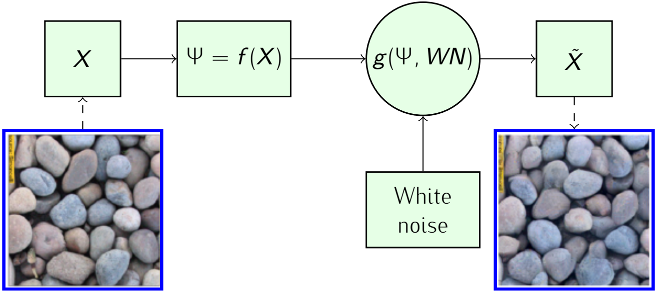


[Gatys et al., 2015]

Ψ: $\sim 890K$ parameters.

Texture analysis by synthesis

Framework description:

1. Extract features Θ
   Approximate dimension: square root of original size
2. Perform PCA on each coefficient set (Φ)

Analysis:

- Observe PCA components

Synthesis:

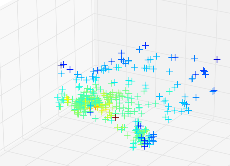
- Sample in PCA space to create new variables, $\tilde{\Phi}$ and reconstruct $\tilde{\Theta}$
- Perform synthesis with updated, $\tilde{G}$

$$\boxed{\Psi_1, \Psi_2, \ldots, \Psi_n} \rightarrow \boxed{\Theta_1, \Theta_2, \ldots, \Theta_n}$$

$$\downarrow$$

$$\boxed{\Phi_1, \Phi_2, \ldots, \Phi_n}$$

$$\downarrow$$



$$\downarrow$$

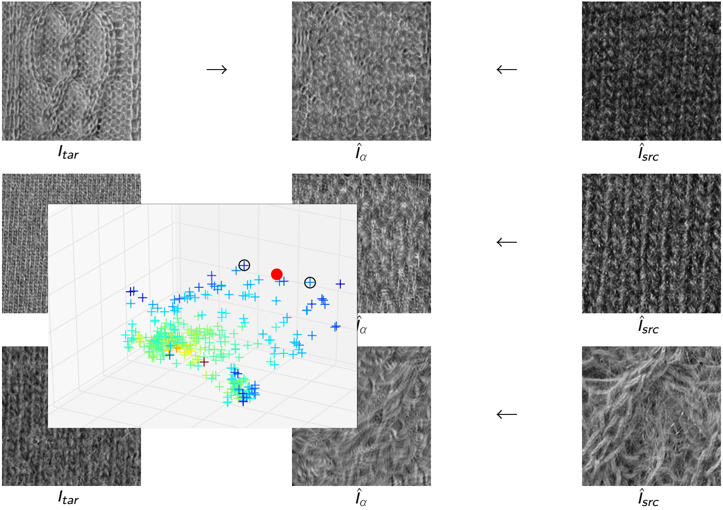
$$\boxed{\tilde{\Phi}_1, \tilde{\Phi}_2, \ldots, \tilde{\Phi}_n}$$

$$\downarrow$$

$$\boxed{\tilde{\Psi}_1, \tilde{\Psi}_2, \ldots, \tilde{\Psi}_n} \leftarrow \boxed{\tilde{\Theta}_1, \tilde{\Theta}_2, \ldots, \tilde{\Theta}_n}$$

Examples: texture synthesis

Samping by interpolation in PCA space



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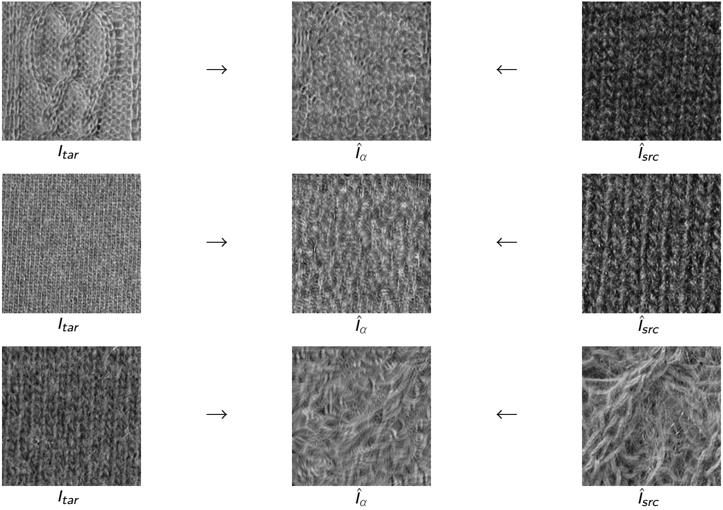


Texture parameter analysis

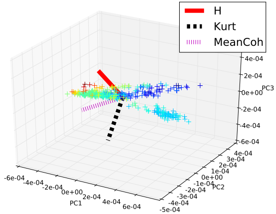
NST have several main features [Zachevsky and Zeevi, 2016]

1. Stochastic features:
   - Fractal properties (Hurst)
   - Gaussianity
2. Structural features:
   - Local coherence, via local phase or the coherence metric

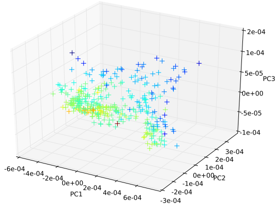
Are these features *fundamental*, or arbitrary, in texture?

Texture parameter analysis

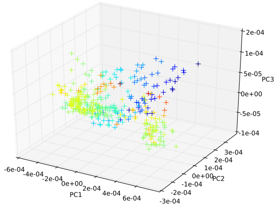
Imparting color identity by ***H***, Kurtosis and local coherence



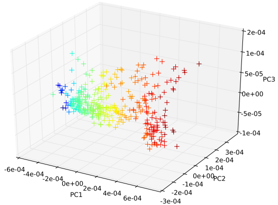
Layer 0: Hurst



Layer 0: Kurtosis



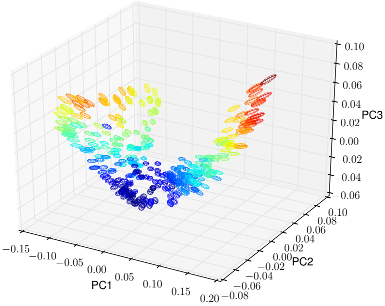
Layer 0: Mean coh.



Layer 0: Std. coh.

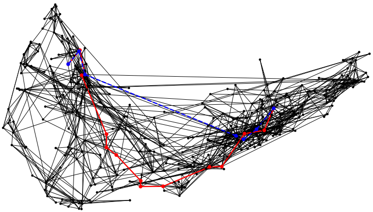
Intrinsic geometry

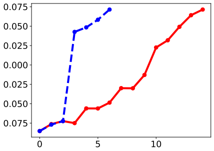
- Defining data points on a graph
- Graph Laplacian approximates Laplace Beltrami operator on graphs
- Assume original data embedded in $(\mathcal{M}, \boldsymbol{g})$
- Intrinsic geometry obtained by *pushforward metric* [Perrault-joncas et al., 2006]
- Given new embedding, $\mathcal{N} = \boldsymbol{f}(\mathcal{M})$, find $\boldsymbol{h}$ such that $(\mathcal{M}, \boldsymbol{g})$ is isometric to $(\mathcal{N}, \boldsymbol{h})$
- $(\mathcal{N}, \boldsymbol{h})$ manifold with transformation $\boldsymbol{f}()$ and local geometry $\boldsymbol{h}$.



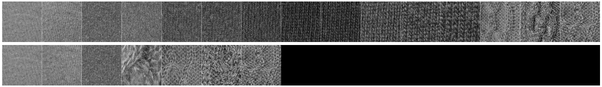
Intrinsic geometry

- Path between two points, Euclidean vs. intrinsic





- Textures along the manifold trajectory:



Further research

- Use nonlinear manifold extraction methods
- Sample by using the manifold geometry
- Obtain manifold-based algorithms, e.g. clustering

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Thank you

Bibliography

Mikhail Belkin and Partha Niyogi. Laplacian Eigenmaps for Dimensionality Reduction and Data Representation. *Neural Comput.*, 15(6):1373–1396, 2003. ISSN 0899-7667. doi: 10.1162/089976603321780317.

Ronald R. Coifman and Stéphane Lafon. Diffusion maps. *Appl. Comput. Harmon. Anal.*, 21(1):5–30, jul 2006. ISSN 10635203. doi: 10.1016/j.acha.2006.04.006. URL `http://linkinghub.elsevier.com/retrieve/pii/S1063520306000546`.

Leon A. Gatys, Alexander S. Ecker, and Matthias Bethge. Texture Synthesis Using Convolutional Neural Networks. *Neural Image Process. Syst.*, pages 1–10, 2015. ISSN 10495258. doi: 10.1109/CVPR.2016.265. URL `http://arxiv.org/abs/1505.07376`.

Ian Goodfellow. NIPS 2016 Tutorial: Generative Adversarial Networks. 2016. ISSN 0253-0465. doi: 10.1001/jamainternmed.2016.8245. URL `http://arxiv.org/abs/1701.00160`.

Ron Kimmel, Ravikanth Malladi, and Nir Sochen. Images as Embedded Maps and Minimal Surfaces: Movies, Color , Texture, and Volumetric Medical Images. *Int. J. Comput. Vis.*, 39(2):111–129, 2000.

Diederik P Kingma and Max Welling. Auto-Encoding Variational Bayes. 2013. ISSN 1312.6114v10. doi: 10.1051/0004-6361/201527329. URL `http://arxiv.org/abs/1312.6114`.

Wen-Chieh Lin, James Hays, Chenyu Wu, Vivek Kwatra, and Yanxi Liu. A comparison study of four texture synthesis algorithms on regular and near-regular textures. Technical report, Carnegie Mellon University, 2004.

Dominique Perrault-joncas, Marina Meil, Lk Saul, Kq Weinberger, Jh Ham, F Sha, and Chang Wang. Metric Learning of Manifolds. *Semisupervised Learn.*, 1(September):293–306, 2006. doi: 10.1234/12345678. URL `http://www-all.cs.umass.edu/{~}chwang/ChangThesis.pdf{%}5Cnhttp://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.85.4537{&}rep=rep1{&}type=pdf`.

Javier Portilla and Eero P Simoncelli. A Parametric Texture Model Based on Joint Statistics of Complex Wavelet Coefficients. *Int. J. Comput. Vis.*, 40(1):49–71, 2000.

Ido Zachevsky and Yehoshua Y. Zeevi. On the statistics of Natural Stochastic Textures. *CCIT Report. EE Pub, Tech. Isr. Inst. Technol.*, 862(1819), 2014.

Ido Zachevsky and Yehoshua Y Zeevi. Statistics of Natural Stochastic Textures and Their Application in Image Denoising. *IEEE Trans. Image Process.*, 25(5):2130–2145, 2016.