FDD Channel Spatial Covariance Conversion Using Projection Methods

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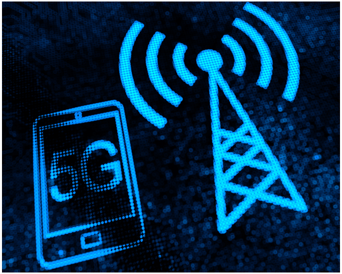
Introduction

- Problem: DL channel estimation in FDD Massive MIMO systems.
  - Training overhead as one of the main performance bottlenecks.
  - Classical designs, which impose a training time equal to the number of BS antennas, are unfeasible.
- Available solutions rely on lower dimensional representation of the channel vectors.
  1. Compressed sensing based.
  2. Second-order statistics based.
- Focus on the second category.
  - Knowledge of the DL channel spatial covariance matrix $\mathbf{R}^d$ is crucial.



Main Contribution

In the following we propose an effective algorithm to obtain an estimate of $\mathbf{R}^d$ from UL measurements only, by converting it from the UL spatial covariance $\mathbf{R}^u$.

Why UL to DL Spatial Covariance Conversion?

- Compared to traditional feedback based approaches (e.g. DL sample covariance), continuous covariance feedback from the UE is eliminated.
- If long term beamforming based on $\mathbf{R}^d$ is applied, DL training could be completely eliminated.
- Operators can immediately apply the proposed scheme to boost the already implemented beamforming and CSI acquisition algorithms in perfect compliance with current standards.
  - The proposed mechanism for DL covariance estimation is completely transparent to the UEs.
  - Example: Boost the codebook based CSI acquisition techniques, by projecting the selected codeword onto the estimated channel subspace.

System Model

- $\boldsymbol{h} := \begin{bmatrix} h_1 & h_2 & \cdots & h_N \end{bmatrix}^T$.

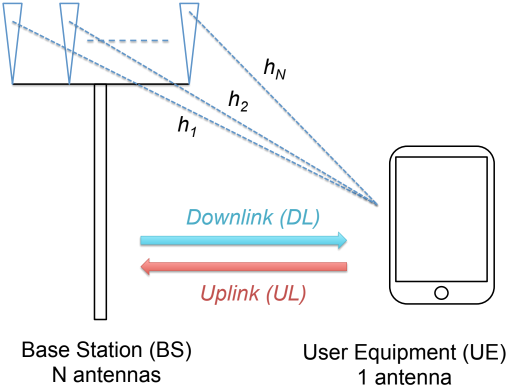


Figure: Flat-fading channel between a massive MIMO BS and a single-antenna UE.

Channel Model

By assuming for simplicity 2D propagation and unpolarized antennas:

Channel spatial covariance matrix

$$\mathbf{R}(f) := \mathbb{E}[\mathbf{h}(f)\mathbf{h}^H(f)] = \int_{-\pi}^{\pi} \rho(\theta)\mathbf{a}(\theta, f)\mathbf{a}^H(\theta, f) d\theta$$

- $\mathbf{h} \in \mathbb{C}^{N\times 1}$ is the channel vector.
- $\mathbf{a} : [-\pi, \pi] \times \mathbb{R}^+ \rightarrow \mathbb{C}^{N\times 1}$ is the frequency dependent BS antenna array response.
- $\rho : [-\pi, \pi] \rightarrow \mathbb{R}^+$ is the angular power spectrum (APS), describing the channel average power density in the angular domain.

Main assumptions

- Angular reciprocity: ρ is assumed to be frequency invariant for reasonable duplex gaps (order of 100 MHz).
- Windowed-WSS assumption: $\mathbf{R}$ is assumed to be constant for a sufficiently long time frame T_{WSS} (typical values $1 - 10$ s).

Algorithm Overview

Goal: UL to DL covariance conversion.

UL and DL covariance matrices

$$\mathbf{R}^{u} = \int_{-\pi}^{\pi} \rho(\theta)\mathbf{a}^{u}(\theta)\mathbf{a}^{u}(\theta)^{H} d\theta \quad (1)$$

$$\mathbf{R}^{d} = \int_{-\pi}^{\pi} \rho(\theta)\mathbf{a}^{d}(\theta)\mathbf{a}^{d}(\theta)^{H} d\theta \quad (2)$$

The proposed scheme can be summarized into two steps as follows:

1. We obtain an estimate $\hat{\rho}$ of the APS ρ based on the knowledge of $\mathbf{R}^{\mathbf{u}}$, the equality in (1), and known properties of ρ.
2. We compute an estimate of $\mathbf{R}^{\mathbf{d}}$ from (2), by substituting ρ with its estimate $\hat{\rho}$.

We assume perfect knowledge of the array responses.

Core idea

Unlike related studies, we formalize the APS estimation problem as a convex feasibility problem, so that we can apply very effective solutions based on projection methods in an infinite-dimensional Hilbert space.

Algorithm 1 - Projection onto a Linear Variety I

- Let us consider the Hilbert space $\mathcal{H}$ of real functions in $L^2[-\pi, \pi]$ equipped with the inner product $\langle f, g \rangle := \int_{-\pi}^{\pi} f(\theta) g(\theta) d\theta$.
- We can rewrite

$$\mathbf{R}^u = \int_{-\pi}^{\pi} \rho(\theta) \mathbf{a}^u(\theta) \mathbf{a}^u(\theta)^H d\theta$$

as a system of equations of the form

$$r_m^u = \langle \rho, g_m^u \rangle$$

  - $r_m^u \in \mathbb{R}$ is the mth element of $\mathbf{r}^u := \text{vec}\left(\begin{bmatrix} \Re\{\mathbf{R}^u\} & \Im\{\mathbf{R}^u\} \end{bmatrix}\right)$.
  - $g_m^u : [-\pi, \pi] \longrightarrow \mathbb{R}$ is the mth element of $\text{vec}\left(\begin{bmatrix} \Re\{\mathbf{a}^u(\theta)\mathbf{a}^u(\theta)^H\} & \Im\{\mathbf{a}^u(\theta)\mathbf{a}^u(\theta)^H\} \end{bmatrix}\right)$.

Convex Feasibility Problem

$$\text{find } \rho^* \in V := \bigcap_{m=1}^{M} V_m,$$

where $V_m := \{\rho \in \mathcal{H} : \langle \rho, g_m^u \rangle = r_m^u\}$ are hyperplanes in $\mathcal{H}$.

Algorithm 1 - Projection onto a Linear Variety II

- Among all the possible solutions of the feasibility problem, all equivalent based only on the information we have, we choose the minimum norm solution

$$\hat{\rho}(\theta) = \arg\min_{\rho^* \in V} \|\rho^*\| = \sum_{m=1}^{M} \alpha_m g_m^u(\theta),$$

where $\boldsymbol{\alpha} := [\alpha_1 \dots \alpha_M]$ is a solution to the linear system

$$\mathbf{r}^u = \mathbf{G}^u \boldsymbol{\alpha},$$

$$\mathbf{G}^u = \begin{bmatrix} \langle g_1^u, g_1^u \rangle & \langle g_1^u, g_2^u \rangle & \dots & \langle g_1^u, g_M^u \rangle \\ \langle g_2^u, g_1^u \rangle & \langle g_2^u, g_2^u \rangle & \dots & \langle g_2^u, g_M^u \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle g_M^u, g_1^u \rangle & \langle g_M^u, g_2^u \rangle & \dots & \langle g_M^u, g_M^u \rangle \end{bmatrix},$$

which corresponds to the orthogonal projection $P_V(0)$ of the zero vector onto the linear variety V.

Algorithm 1 - Projection onto a Linear Variety III

- We finally obtain an estimate of $\mathbf{R^d}$ by replacing ρ in the DL covariance expression with its estimate $\hat{\rho}$:

$$\hat{r}_m^d = \langle \hat{\rho}, g_m^d \rangle = \sum_{l=1}^{M} \alpha_l \langle g_l^u, g_m^d \rangle \quad m = 1, \ldots, M,$$

which can be rewritten in matrix form as

$$\hat{\mathbf{r}}^d = \mathbf{Q}\boldsymbol{\alpha},$$

where $\hat{\mathbf{r}}^d$ is an estimate of the vector $\mathbf{r}^d := \text{vec}(\left[\Re\{\mathbf{R}^d\}\ \Im\{\mathbf{R}^d\}\right])$, $\boldsymbol{\alpha}$ is a solution to the linear system $\mathbf{r}^u = \mathbf{G}^u\boldsymbol{\alpha}$, and

$$\mathbf{Q} = \begin{bmatrix} \langle g_1^d, g_1^u \rangle & \langle g_1^d, g_2^u \rangle & \cdots & \langle g_1^d, g_M^u \rangle \\ \langle g_2^d, g_1^u \rangle & \langle g_2^d, g_2^u \rangle & \cdots & \langle g_2^d, g_M^u \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle g_M^d, g_1^u \rangle & \langle g_M^d, g_2^u \rangle & \cdots & \langle g_M^d, g_M^u \rangle \end{bmatrix}.$$

Summary

In summary, the algorithm can be implemented as follows:

Algorithm 1

1. $\mathbf{r}^u := \text{vec}\left(\begin{bmatrix}\Re\{\mathbf{R}^u\} & \Im\{\mathbf{R}^u\}\end{bmatrix}\right)$
2. $\hat{\mathbf{r}}^d = \mathbf{Q}(\mathbf{G}^u)^\dagger \mathbf{r}^u$
3. $\hat{\mathbf{R}}^d = \text{vec}^{-1}(\hat{\mathbf{r}}^d)$

where $(\mathbf{G}^u)^\dagger$ is the Moore-Penrose pseudoinverse of $\mathbf{G}^u$.

Note:

- $\mathbf{Q}(\mathbf{G}^u)^\dagger$ need to be computed just once for the entire system lifetime.

Algorithm 2 - Enforcing the Positivity of the APS I

- We recall that ρ is a real and non-negative function.
  - The real constraint is already taken into account by Algorithm 1.
  - But not the non-negativity!

Problem

$$\text{find } \rho^* \in C := V \cap Z,$$

where $Z = \{\rho \in \mathcal{H} : \forall\theta \in [-\pi, \pi] \quad \rho(\theta) \geq 0\}$ is the closed convex set of non-negative functions in $\mathcal{H}$, and V is the linear variety considered before.

- Wide literature of iterative projection methods for solving this class of convex feasibility problems.
  - We adopt a fast method called *extrapolated alternating projection method* (EAPM), a particular case of the *adaptive projected subgradient method*.

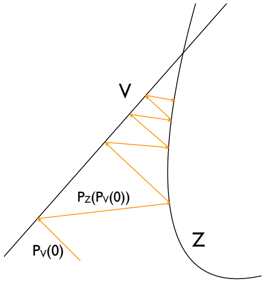


Figure: A simple and popular example of iterative projection method: *projections onto convex sets* (POCS).

Algorithm 2 - Enforcing the Positivity of the APS II

- The projection $P_V : \mathcal{H} \to \mathcal{H}$ onto the set V is given by

$$P_V(x) = x - \sum_{m=1}^{M} \beta_m g_m^u + P_V(0),$$

with $\boldsymbol{\beta} := [\beta_1 \dots \beta_M]$ being a solution to the linear system $\mathbf{b} = \mathbf{G}^u \boldsymbol{\beta}$ where the mth element of $\mathbf{b}$ is given by $b_m = \langle x, g_m^u \rangle$.

- The projection $P_Z : \mathcal{H} \to \mathcal{H}$ is given by

$$P_Z(x) = \begin{cases} x(\theta), & \text{if } x(\theta) \geq 0 \\ 0, & \text{otherwise} \end{cases}.$$

- An estimate of $\mathbf{R^d}$ can be finally obtained by evaluating

$$\hat{r}_m^d = \langle \hat{\rho}, g_m^d \rangle \quad m = 1, \dots, M.$$

where, unlike Algorithm 1, $\hat{\rho}$ is here computed explicitly with the given iterative projection method.

  - Algorithm 2 is more complex: it requires the online evaluation of the inner products, i.e. integrals of the form $\int_{-\pi}^{\pi} x(\theta) d\theta$.

Simulation Details

- BS array: ULA, $f^u = 1.8$ GHz, $f^d = 1.9$ GHz, half-wavelength inter-antenna spacing.
  - Analytical expression for $\mathbf{G}^u$ and $\mathbf{Q}$ is available.
- $\mathbf{R}^u$ and $\mathbf{R}^d$ randomly drawn, based on the following GSCM-like channel model:

$$\rho(\theta) = \sum_{q=1}^{Q} f_q(\theta)\alpha_q, \quad f_q \sim \mathcal{N}\left(\phi_q, \Delta_q^2\right), \quad \alpha_q \geq 0 \text{ s.t. } \sum_q \alpha_q = 1.$$

- The BS has access to a sample covariance $\hat{\mathbf{R}}^u$ obtained from 1000 noisy channels samples, with SNR randomly drawn from $[10, 30]$ dB.
- Comparison with state-of-the-art techniques for UL to DL spatial covariance conversion. The DL sample covariance, obtained with the same technique and parameters as for the UL, is used as a baseline.

Simulation Results

- Normalized Euclidean distance

$$\text{MSE} := \mathbb{E}\left[\frac{\|\mathbf{R}-\hat{\mathbf{R}}\|_F^2}{\|\mathbf{R}\|_F^2}\right].$$

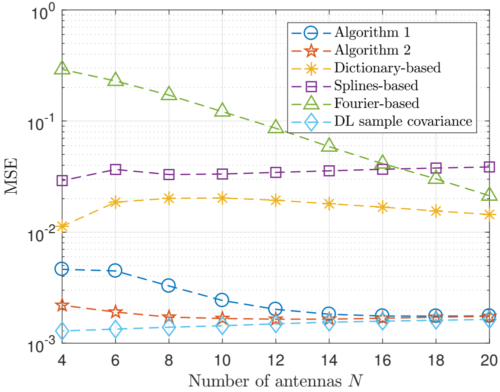


Figure: Comparison of different DL covariance estimators vs number of BS antennas N.

Advantages of the Proposed DL Covariance Estimation Scheme

- No particular geometry of the array response in assumed.
- No training set is required.
- The performance approaches the baseline given by the DL sample covariance.
  - Furthermore, it can be shown that this holds also when applied to some practical CSI aquisition techniques.
- Algorithm 1 is extremely simple (a matrix-vector multiplication).
  - Furthermore, the performances in terms of rate in practical applications are already close to the bound given by the DL sample covariance.
- Due to its generality, it can be shown that the proposed scheme can be extended also to more complex channel models that take into account 3D propagation and polarization effects.