Identification of Bilinear Forms with the Kalman Filter

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Outline

Introduction

- **Kalman filter** - used before in system identification
  $\rightarrow$ reference (desired) signal:

$$d(t) = \mathbf{h}^T(t)\mathbf{x}(t) + v(t)$$

  h - unknown system of length L
  $\mathbf{x}(t) = [x(t)x(t-1)\ldots x(n-L+1)]^T$ - input signal
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- Our approach - identification of **bilinear forms**
  $\rightarrow$ reference (desired) signal:

$$d(t) = \mathbf{h}^T(t)\mathbf{X}(t)\mathbf{g}(t) + v(t)$$

  h, **g**: unknown systems of lengths L, M
  $\mathbf{X}(t) = [\mathbf{x}_1(t) \quad \mathbf{x}_2(t) \quad \ldots \quad \mathbf{x}_M(t)]$ - input signal matrix
  $\mathbf{x}_m(t) = [x_m(t) \quad x_m(t-1) \quad \ldots \quad x_m(t-L+1)]^T \quad m = 1, 2, \ldots, M$

Motivation

- **Target:** a Kalman algorithm for the identification of bilinear forms
- Timely topic
- Numerous applications:
  $\rightarrow$ nonlinear acoustic echo cancellation
  $\rightarrow$ identification of Hammerstein systems
  $\rightarrow$ tensor algebra - Big Data

System Model

- **Signal model:**
  $d(t) = \mathbf{h}^T(t)\mathbf{X}(t)\mathbf{g}(t) + v(t)$
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$$\mathbf{h}(t) = \mathbf{h}(t-1) + \mathbf{w_h}(t) \qquad \mathbf{g}(t) = \mathbf{g}(t-1) + \mathbf{w_g}(t)$$

$\mathbf{w_h}(t)$, $\mathbf{w_g}(t)$: zero-mean WGN

$$\mathbf{R}_{\mathbf{w_h}}(t) = \sigma^2_{w_\mathbf{h}}\mathbf{I}_L \qquad \mathbf{R}_{\mathbf{w_g}}(t) = \sigma^2_{w_\mathbf{g}}\mathbf{I}_M$$

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- **Equivalent model:**
  $d(t) = \mathbf{f}^T(t)\widetilde{\mathbf{x}}(t) + v(t)$

$\mathbf{f}(t) = \mathbf{g}(t) \otimes \mathbf{h}(t)$ – Kronecker product of length ML

$$\widetilde{\mathbf{x}}(t) = \mathrm{vec}[\mathbf{X}(t)] = [\mathbf{x}_1^T(t) \quad \mathbf{x}_2^T(t) \quad \ldots \quad \mathbf{x}_M^T(t)]^T$$

Scaling Ambiguity

- $\mathbf{f}(t) = \mathbf{g}(t) \otimes \mathbf{h}(t) = [\eta \mathbf{g}(t)] \otimes \left[\frac{1}{\eta}\mathbf{h}(t)\right] \quad \eta \in \mathcal{R}^*$ - scaling factor

$$\left[\frac{1}{\eta}\mathbf{h}(t)\right]^T \mathbf{X}(t)\,[\eta \mathbf{g}(t)] = \mathbf{h}^T(t)\mathbf{X}(t)\mathbf{g}(t)$$

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$$\left[\frac{1}{\eta}\mathbf{h}(t)\right]^T \mathbf{X}(t)\,[\eta \mathbf{g}(t)] = \mathbf{h}^T(t)\mathbf{X}(t)\mathbf{g}(t) \quad \Rightarrow \quad \begin{array}{l} \widehat{\mathbf{h}}(t) \rightarrow \frac{1}{\eta}\mathbf{h}(t) \\ \widehat{\mathbf{g}}(t) \rightarrow \eta \mathbf{g}(t) \\ \widehat{\mathbf{f}}(t) \rightarrow \mathbf{f}(t) \end{array}$$

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- Normalized projection misalignment (NPM) [1]

$$\mathrm{NPM}[\mathbf{h}(t), \widehat{\mathbf{h}}(t)] = 1 - \left[\frac{\mathbf{h}^T(t)\widehat{\mathbf{h}}(t)}{||\mathbf{h}(t)||\,||\widehat{\mathbf{h}}(t)||}\right]^2 \quad \mathrm{NPM}[\mathbf{g}(t), \widehat{\mathbf{g}}(t)] = 1 - \left[\frac{\mathbf{g}^T(t)\widehat{\mathbf{g}}(t)}{||\mathbf{g}(t)||\,||\widehat{\mathbf{g}}(t)||}\right]^2$$

- Normalized misalignment (NM)

$$\mathrm{NM}[\mathbf{f}(t), \widehat{\mathbf{f}}(t)] = \|\mathbf{f}(t) - \widehat{\mathbf{f}}(t)\|^2 / \|\mathbf{f}(t)\|^2$$

[1] [Morgan et al., IEEE Signal Processing Letters, July 1998]

Kalman Filter for Bilinear Forms (**KF - BF**)

- estimated output signal: $\widehat{y}(t) = \widehat{\mathbf{h}}^T(t-1)\mathbf{X}(t)\widehat{\mathbf{g}}(t-1)$

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$$\begin{aligned} e(t) &= d(t) - \widehat{\mathbf{f}}^T(t-1)\widetilde{\mathbf{x}}(t) \\ &= d(t) - \widehat{\mathbf{h}}^T(t-1)\widetilde{\mathbf{x}}_{\widehat{\mathbf{g}}}(t) \leftarrow e_{\widehat{\mathbf{g}}}(t) \\ &= d(t) - \widehat{\mathbf{g}}^T(t-1)\widetilde{\mathbf{x}}_{\widehat{\mathbf{h}}}(t) \leftarrow e_{\widehat{\mathbf{h}}}(t) \end{aligned}$$

$$\widetilde{\mathbf{x}}_{\widehat{\mathbf{g}}}(t) = [\widehat{\mathbf{g}}(t-1) \otimes \mathbf{I}_L]^T \widetilde{\mathbf{x}}(t) \qquad \widetilde{\mathbf{x}}_{\widehat{\mathbf{h}}}(t) = [\mathbf{I}_M \otimes \widehat{\mathbf{h}}(t-1)]^T \widetilde{\mathbf{x}}(t)$$

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- optimal estimates of the state vectors:

$$\widehat{\mathbf{h}}(t) = \widehat{\mathbf{h}}(t-1) + \mathbf{k}_{\mathbf{h}}(t)e(t) \qquad \widehat{\mathbf{g}}(t) = \widehat{\mathbf{g}}(t-1) + \mathbf{k}_{\mathbf{g}}(t)e(t)$$

$\mathbf{k}_{\mathbf{h}}(t)$, $\mathbf{k}_{\mathbf{g}}(t)$: Kalman gain vectors

- a posteriori misalignments:

$$\boldsymbol{\mu}_{\mathbf{h}}(t) = \mathbf{h}(t)/\eta - \widehat{\mathbf{h}}(t) \qquad \boldsymbol{\mu}_{\mathbf{g}}(t) = \eta\mathbf{g}(t) - \widehat{\mathbf{g}}(t)$$

- a priori misalignments:

$$\begin{aligned}\mathbf{m}_{\mathbf{h}}(t) &= \mathbf{h}(t)/\eta - \widehat{\mathbf{h}}(t-1) \\ &= \boldsymbol{\mu}_{\mathbf{h}}(t-1) + \mathbf{w}_{\mathbf{h}}(t)/\eta\end{aligned} \qquad \begin{aligned}\mathbf{m}_{\mathbf{g}}(t) &= \eta\mathbf{g}(t) - \widehat{\mathbf{g}}(t-1) \\ &= \boldsymbol{\mu}_{\mathbf{g}}(t-1) + \eta\mathbf{w}_{\mathbf{g}}(t)\end{aligned}$$

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- simplifying notations:

$$\begin{aligned}\overline{\mathbf{w}}_{\mathbf{h}}(t) &= \mathbf{w}_{\mathbf{h}}(t)/\eta \\ \mathbf{R}_{\mathbf{m}_{\mathbf{h}}}(t) &= \mathbf{R}_{\boldsymbol{\mu}_{\mathbf{h}}}(t-1) + \sigma^2_{\overline{w}_{\mathbf{h}}}\mathbf{I}_L\end{aligned} \qquad \begin{aligned}\overline{\mathbf{w}}_{\mathbf{g}}(t) &= \eta\mathbf{w}_{\mathbf{g}}(t) \\ \mathbf{R}_{\mathbf{m}_{\mathbf{g}}}(t) &= \mathbf{R}_{\boldsymbol{\mu}_{\mathbf{g}}}(t-1) + \sigma^2_{\overline{w}_{\mathbf{g}}}\mathbf{I}_M\end{aligned}$$

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- minimizing $(1/L)\mathrm{tr}\left[\mathbf{R}_{\boldsymbol{\mu}_{\mathbf{h}}}(t)\right]$, $(1/M)\mathrm{tr}\left[\mathbf{R}_{\boldsymbol{\mu}_{\mathbf{g}}}(t)\right]$ yields:

$$\mathbf{k}_{\mathbf{h}}(t) = \mathbf{R}_{\mathbf{m}_{\mathbf{h}}}(t)\widetilde{\mathbf{x}}_{\widehat{\mathbf{g}}}(t)[\widetilde{\mathbf{x}}^T_{\widehat{\mathbf{g}}}(t)\mathbf{R}_{\mathbf{m}_{\mathbf{h}}}(t)\widetilde{\mathbf{x}}_{\widehat{\mathbf{g}}}(t) + \sigma^2_v]^{-1}$$
$$\mathbf{k}_{\mathbf{g}}(t) = \mathbf{R}_{\mathbf{m}_{\mathbf{g}}}(t)\widetilde{\mathbf{x}}_{\widehat{\mathbf{h}}}(t)[\widetilde{\mathbf{x}}^T_{\widehat{\mathbf{h}}}(t)\mathbf{R}_{\mathbf{m}_{\mathbf{g}}}(t)\widetilde{\mathbf{x}}_{\widehat{\mathbf{h}}}(t) + \sigma^2_v]^{-1}$$

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- misalignments of the individual coefficients: uncorrelated $\rightarrow$

$\rightarrow$ approximate:

$$\mathbf{I}_L - \mathbf{k_h}(t)\widetilde{\mathbf{x}}^T_{\widehat{\mathbf{g}}}(t) \approx \left[1 - \frac{1}{L}\mathbf{k}^T_{\mathbf{h}}(t)\widetilde{\mathbf{x}}_{\widehat{\mathbf{g}}}(t)\right]\mathbf{I}_L$$

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$\Rightarrow$ **Simplified Kalman Filter for bilinear forms (SKF - BF)**

Practical Considerations

- The parameters related to uncertainties in **h**, **g**: $\sigma^2_{w_\mathbf{h}}$, $\sigma^2_{w_\mathbf{g}}$:
  - small $\Rightarrow$ good misalignment, poor tracking
  - large (i.e., high uncertainty in the systems) $\Rightarrow$
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- A good compromise is needed!
- In practice $\rightarrow$ some a priori information (e.g., if **g** - time-invariant $\Rightarrow \sigma^2_{w_\mathbf{g}} = 0$)
- By applying the ℓ_2 norm on the state equation:

$$\widehat{\sigma}^2_{w_\mathbf{h}}(t) = \frac{1}{L}\left\|\widehat{\mathbf{h}}(t) - \widehat{\mathbf{h}}(t-1)\right\|_2^2$$

Simulation Setup

Conditions

- input signals $x_m(t), m = 1, 2, \ldots, M$ - independent WGN, respectively AR(1) generated by filtering a white Gaussian noise through a first-order system $1/\left(1 - 0.8z^{-1}\right)$
- **h**, **g** - Gaussian, randomly generated, of lengths $L = 64$, $M = 8$
- $v(t)$ - independent white Gaussian noise signal

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Compared algorithms

- KF-BF and KF
- SKF-BF and SKF when $\sigma^2_{w_\mathbf{h}} = \sigma^2_{w_\mathbf{g}} = \sigma^2_{w} = 10^{-9}$
- SKF-BF and SKF when $\sigma^2_{w_\mathbf{g}} = 0$ and $\widehat{\sigma}^2_{w_\mathbf{h}}(t) = \frac{\left\|\widehat{\mathbf{h}}(t) - \widehat{\mathbf{h}}(t-1)\right\|_2^2}{L}$

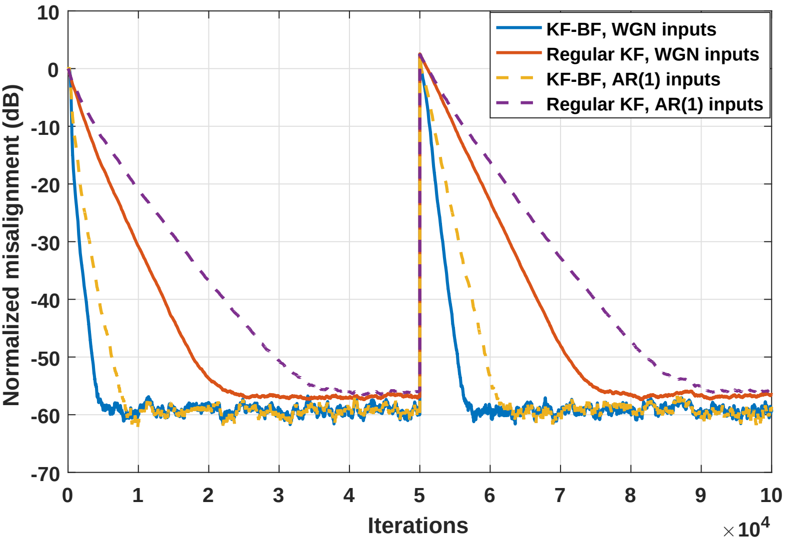

Figure 1: Normalized misalignment of the KF-BF and regular KF for different types of input signals. $ML = 512$, $\sigma_v^2 = 0.01$, $\sigma_{w_\mathbf{h}}^2 = \sigma_{w_\mathbf{g}}^2 = \sigma_w^2 = 10^{-9}$, and $\epsilon = 10^{-5}$.

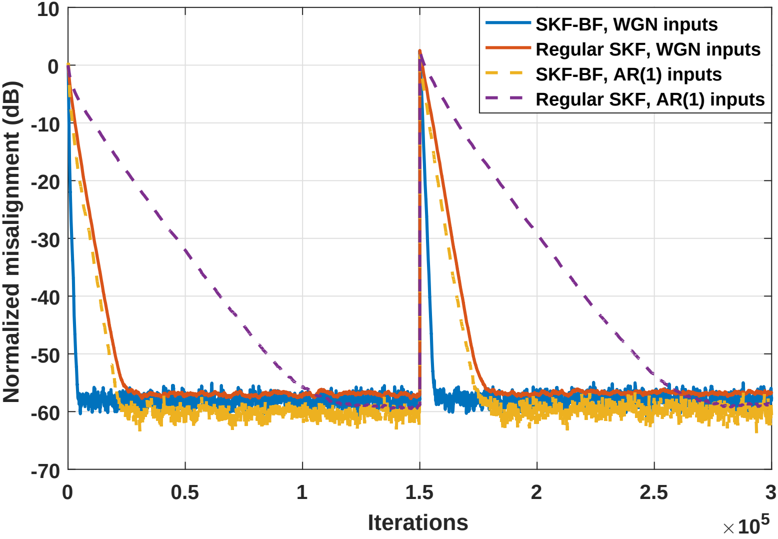

Figure 2: Normalized misalignment of the SKF-BF and regular SKF for different types of input signals. Other conditions are the same as in Fig. 1.

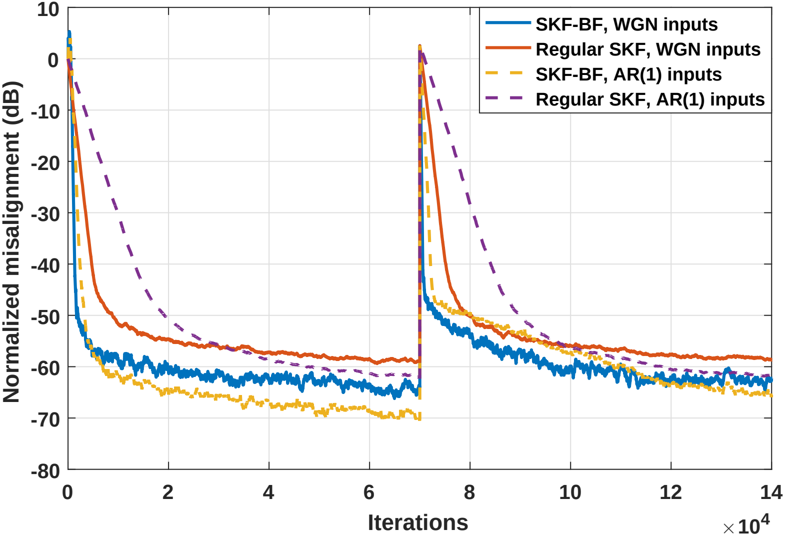

Figure 3: Normalized misalignment of the SKF-BF and regular SKF for different types of input signals, using the recursive estimates $\widehat{\sigma}^2_{\overline{w}_h}(t)$ and $\widehat{\sigma}^2_{w}(t)$, respectively. $ML = 512$, $\sigma_v^2 = 0.01$, $\sigma^2_{\overline{w}_a} = 0$, and $\epsilon = 10^{-5}$.

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  - reduced computational complexity , but also
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  with respect to KF-BF
- The experimental results indicate the good performance of the proposed algorithms

Thank you!