Energy-Aware Sensor Selection in Field Reconstruction

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Outline

Sensor Selection

- Context: limited communication, bandwidth, and sensor battery life; not desirable to have all sensors report their measurements at all time-instants
- Objective: seek an optimal tradeoff between sensor activations (over space and time) and estimation accuracy
- Research scope: offline selection algorithm, field estimation

Related Work

- Formulation I: Minimum estimation error subject to a constraint on total number of sensor activations [1]-[2]
- Formulation II: Minimum sensor activation subject to a constraint on estimation performance [3]-[4]
- Formulation III (sparsity promoting): Mininum estimation error while *simultaneously penalizing* number of sensor activations [5]-[6]

[1] S. Joshi and S. Boyd, *IEEE TSP*, 2009
[2] Y. Mo, R. Ambrosino, and B. Sinopoli, *Automatica*, 2011
[3] S. P. Chepuri and G. Leus, *IEEE TSP*, 2015
[4] H. Godrich, A. P. Petropulu, and H. V. Poor, *IEEE TSP*, 2015
[5] E. Masazade, M. Fardad, and P. K. Varshney, *IEEE SPL*, 2012
[6] S. Liu, M. Fardad, E. Masazade, and P. K. Varshney, *IEEE TSP*, 2014

Related Work (Cont.)

- Sparsity-promoting formulation [Liu *et al.*, 2015]:

$$\text{minimize} \quad \underbrace{J(\mathbf{w})}_{\text{estimation error}} + \underbrace{\gamma h(\mathbf{w})}_{\text{sparsity promoting term}}$$

  - $\gamma > 0$: regularization parameter
  - $h(\mathbf{w})$: *total* number of sensor activations, analogous to *cumulative* energy constraint in Formulation I

- Limitations of prior formulation:

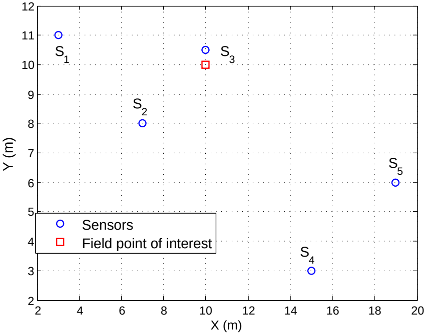
  myopic (single-time) sensor scheduling, leads to successive selections of sensors, excluding individual sensor energy constraints

- Our contribution:

  development of sparsity-promoting approach for non-myopic (multi-time) scheduling, balanced use of individual sensor energy

Sparsity-Promoting Field Estimation

- Field estimation with 5 sensors and 1 field point of interest



- Estimate the field intensity at the location of our interest
- Linear estimator: $\hat{\mathbf{f}} = \mathbf{W}\mathbf{y}$

  $\mathbf{W}$: estimator coefficient matrix

  $\mathbf{y}$: measurement vector

- Linear estimator: $\hat{\mathbf{f}} = \mathbf{W}\mathbf{y}$
- Distortion: $J(\mathbf{W}) = \mathbb{E}[(\hat{\mathbf{f}} - \mathbf{f})^T(\hat{\mathbf{f}} - \mathbf{f})]$
- Nonzero columns of $\mathbf{W} \implies$ active sensors

$$\mathbf{W}\mathbf{y} = \begin{bmatrix} \mathbf{W}_1 & \mathbf{W}_2 & \dots & \mathbf{W}_N \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \mathbf{W}_1 y_1 + \mathbf{W}_2 y_2 + \cdots + \mathbf{W}_N y_N$$

Column-cardinality of $\mathbf{W}$: total number of sensor activations

$$h(\mathbf{W}) \triangleq \mathrm{card}\left(\begin{bmatrix} \|\mathbf{W}_1\|_{\ell_1} & \|\mathbf{W}_2\|_{\ell_1} & \cdots & \|\mathbf{W}_N\|_{\ell_1} \end{bmatrix} \right)$$

- Conventional sparsity-promoting framework [Liu *et al.*, TSP'15]

$$\underset{\mathbf{W}}{\text{minimize}} \quad \frac{1}{2} J(\mathbf{W}) + \gamma h(\mathbf{W})$$

larger γ promotes sparser sensor schedule

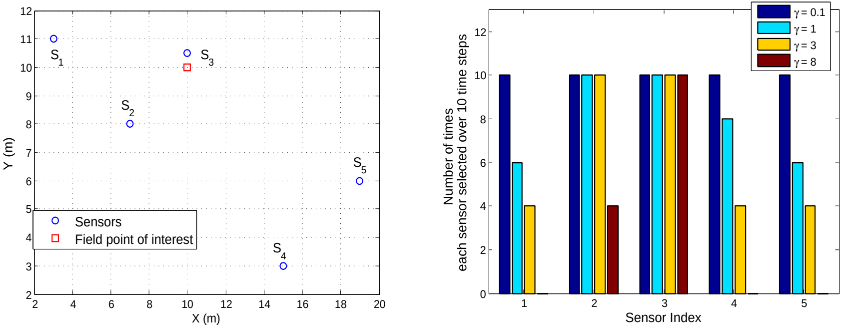


Figure: (a) Sensor network and field point; (b) Sensor schedules by varying γ.

- Imbalance of energy usage: successive selections of most informative sensors, e.g., S_3 largest spatial correlation with field point

How to strike a balance between sparsity of sensor activations and balanced sensor energy usage?

Generalized Sparsity Promoting

Generalized sparsity-promoting optimization framework

$$\underset{\mathbf{W}}{\text{minimize}} \quad \frac{1}{2}J(\mathbf{W}) + \gamma h(\mathbf{W}) + \eta g(\mathbf{W})$$

γ, η: positive regularization parameters

- $J(\mathbf{W})$: trace of error covariance
- $h(\mathbf{W})$: *total* number of sensor activations *(conventional)*

$$h(\mathbf{W}) \triangleq \text{card}\left(\left[\|\mathbf{W}_1\|_{\ell_1} \quad \|\mathbf{W}_2\|_{\ell_1} \quad \cdots \quad \|\mathbf{W}_{KM}\|_{\ell_1} \right] \right)$$

K : length of time horizon; M: number of sensors

- $g(\mathbf{W})$: *proposed* sparsity-promoting penalty function that can discourage successive selections of the same sensors

How to define $g(\mathbf{W})$? Relationship with $h(\mathbf{W})$?

$$g(\mathbf{W}) = \sum_m \left(\sum_k \text{card}(\|\mathbf{W}_{k,m}\|_{\ell_1}) \right)^2$$

- $\mathbf{W}_{k,m}$: column of $\mathbf{W}$ corresponding to observation of the mth sensor at time k
- $\text{card}(x) = \begin{cases} 1 & x \neq 0, \\ 0 & x = 0 \end{cases}$
- $\kappa_m = \sum_{k=1}^{K} \text{card}(\|\mathbf{W}_{k,m}\|_{\ell_1})$: number of times sensor m selected over K time steps
- $g(\mathbf{W}) = \kappa_m^2$: quadratic penalty, leads to large penalty when sensors are successively selected

Example of two sensors: $(4^2 + 0^2) > (2^2 + 2^2)$

Proposed Sensor Selection Problem

$$\underset{\mathbf{W}}{\text{minimize}} \quad \frac{1}{2}J(\mathbf{W}) + \gamma h(\mathbf{W}) + \eta g(\mathbf{W})$$

- $J(\mathbf{W})$: MSE of linear estimator, convex quadratic
- $h(\mathbf{W})$: governs the total number of sensor activations
- $g(\mathbf{W})$: characterizes the cost of successive selections
- Solution: a) sensor selection schemes (column-sparsity of $\mathbf{W}$)
  b) optimal linear estimator for field reconstruction
- Presence of cardinality function (ℓ_0 norm), nonconvex

Convexification

- Vectorization: $\mathbf{W} \Longrightarrow$ columnwise vector $\mathbf{w}$
- reweighted ℓ_1 relaxation:

$$\text{card}(\|\mathbf{w}_{k,m}\|_{\ell_1}) \Longrightarrow \alpha_{k,m}\|\mathbf{w}_{k,m}\|_{\ell_1}$$

- ℓ_1 optimization problem:

$$\underset{\mathbf{w}}{\text{minimize}} \quad J(\mathbf{w}) + \gamma \sum_{m=1}^{M} \sum_{k=1}^{K} h_{k,m}(\mathbf{w}) + \eta \sum_{m=1}^{M} g_m(\mathbf{w}),$$

$$h_{k,m}(\mathbf{w}) := \alpha_{k,m}\|\mathbf{w}_{k,m}\|_{\ell_1},\ g_m(\mathbf{w}) := \left(\sum_{k=1}^{K} \alpha_{k,m}\|\mathbf{w}_{k,m}\|_{\ell_1}\right)^2$$

- [Prop. 1]: Problem is equivalent to convex quadratic program

$$\begin{array}{ll} \underset{\mathbf{x}}{\text{minimize}} & \frac{1}{2}\mathbf{x}^T\mathbf{H}\mathbf{x} - \mathbf{x}^T\mathbf{h} \\ \text{subject to} & \mathbf{x} \leq 0, \end{array}$$

$\mathbf{w} = \mathbf{A}\mathbf{x}$, $\mathbf{A}$, $\mathbf{H}$, $\mathbf{h}$ known appropriate matrices

Fast Algorithm for QP

- Quadratic program (QP):

$$\begin{array}{ll} \underset{\mathbf{x}}{\text{minimize}} & \frac{1}{2}\mathbf{x}^T\mathbf{H}\mathbf{x} - \mathbf{x}^T\mathbf{h} \\ \text{subject to} & \mathbf{x} \leq 0, \end{array}$$

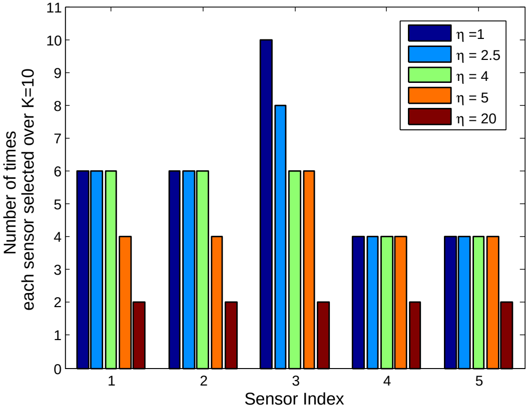
$$\Updownarrow$$

$$\begin{array}{ll} \underset{\mathbf{x}}{\text{minimize}} & \phi(\mathbf{x}) + \psi(\mathbf{x}), \\ & \phi(\mathbf{x}) = \frac{1}{2}\mathbf{x}^T\mathbf{H}\mathbf{x} - \mathbf{x}^T\mathbf{h} \\ & \psi(\mathbf{x}) = \begin{cases} 0 & \text{if } \mathbf{x} \leq 0 \\ +\infty & \text{otherwise.} \end{cases} \end{array}$$

- Efficient algorithm:
  1. alternating direction method of multipliers (ADMM)
  2. accelerated proximal gradient algorithm (APG)

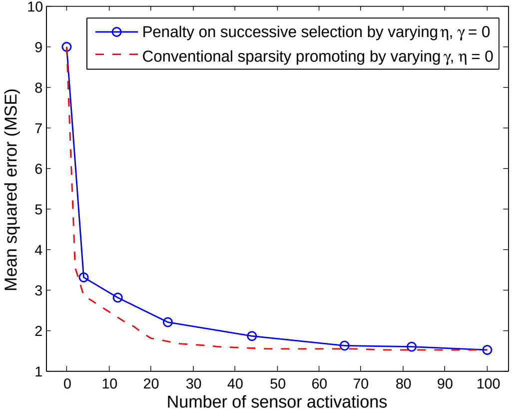
Numerical Results

- $M = 5$ sensors, $K = 10$ time steps
- Sensor schedules from the proposed sparsity-promoting framework $(\gamma = 1)$



Discourage successive selections of the same sensors for large η

Approach with and without avoiding successive selections



Higher MSE, since successive selections of the most informative sensors are prevented for balanced energy usage

Conclusion

Summary

- Novel sparsity-promoting penalty function that discourages successive selection of the same group of sensors
- Convexity analysis, QP with ADMM or APG

Future work

- Study on the choice of sparsity-promoting parameters for achieving desired sparsity levels
- Decentralized sensor scheduling framework