Low Rank Fourier Ptychography

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Problem Setup

Objective: To design a principled algorithmic approach for Fourier ptychographic imaging of dynamic, time-varying targets.

Main Challenges

- No existing measurement framework to solve this problem
- Large number of samples required for frame-by-frame recovery

Our Contribution

We design a low-rank ptychography algorithm that:

- Models the video signals as a *low-rank* matrix
- Works for the Fourier ptychographic measurement setup
- Involves a non-convex, iterative estimation procedure with a novel initialization mechanism
- Uses two novel under-sampling strategies that can reduce the sample complexity of video Fourier ptychography
- Shows better performance in terms of sample complexity as compared to existing "single-frame" methods

Concept of Fourier Ptychography

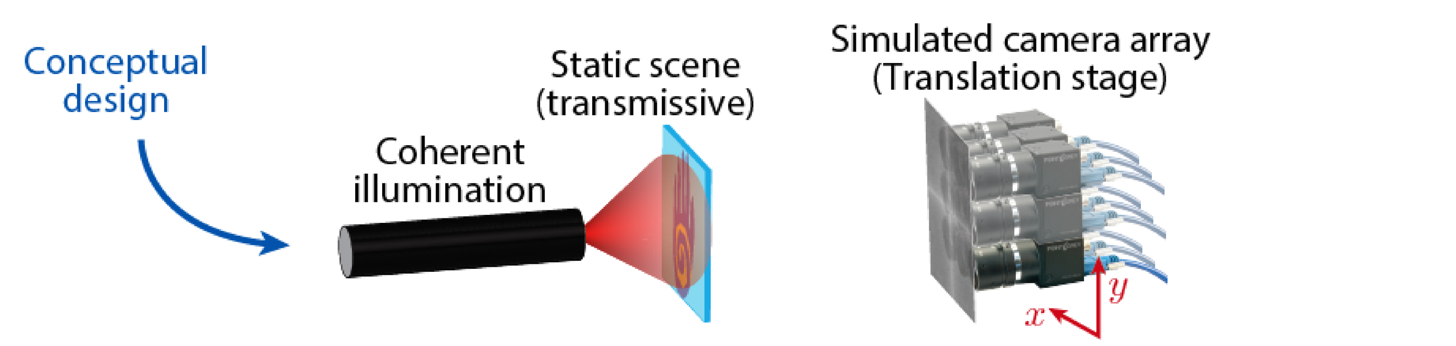

Figure 1: Conceptual design for a fourier ptychography system. From [HASMHCV16]

Data Acquisition Setup and Under-sampling Strategies

Recover video matrix $\mathbf{X} := [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_q]$, $\mathbf{X} \in \mathbb{R}^{n \times q}$, using measurement operator $\mathcal{A}_{i,k}$, where $i \in [1, \ldots, N]$ (camera index) and $k \in [1, \ldots, q]$ (video frame index) from measurements $\mathbf{y}_{i,k}$,

$$\mathbf{y}_{i,k} = |\mathcal{A}_{i,k}(\mathbf{x}_k)|$$
$$\mathcal{A}_{i,k}(\cdot) = \mathcal{M}_{i,k}\mathcal{F}^{-1}\mathcal{P}_i \circ \mathcal{F}(\cdot)$$

We assume that the rank of the true matrix $\mathbf{X}^*$ is no greater than r.

Pixel-wise Uniform Random Under-sampling:
Mask all pixels, with mask entries 1 with probability f, 0 otherwise:

$$\mathcal{M}_{i,k}(\mathbf{v})_j = \begin{cases} 0 & u^i_j > f, \\ v_j & u^i_j < f, \quad v \in \mathbb{C}^n \end{cases}$$

Uniform Random Camera Under-sampling:
Select all pixels corresponding to some camera with probability f:

$$\mathcal{M}_{i,k}(\mathbf{v}) = \begin{cases} \mathbf{0} & u_i > f, \\ \mathbf{v} & u_i < f, \quad v \in \mathbb{C}^n \end{cases}$$

Flow of Mearsurement

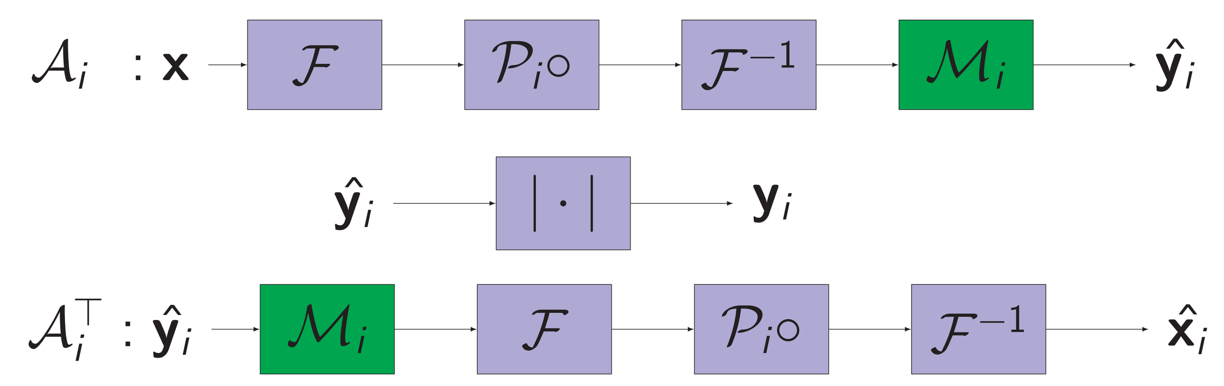

Figure 2: Sequence of operations defined by $\mathcal{A}_i$. Green box is extra sub-sampling step.

Reconstruction Algorithm: LR-Ptych

"Slowly changing" video assumption: first few (r) singular values of $\mathbf{X}^*$ are much greater than remaining. Then, we recover $\mathbf{X}$ as the solution to the non-convex optimization problem:

$$\underset{\mathbf{X}}{\text{argmin}} \sum_{k=1}^{q} \sum_{i=1}^{N} \|\mathbf{y}_{i,k} - |\mathcal{A}_{i,k}(\mathbf{x}_k)|\|_2^2, \quad \text{s.t.} \quad \text{rank}(\mathbf{X}) = r$$

Solution methodology:

- Adapt *low-rank phase retrieval* (LRPR) algorithm [VNE17].
- Rank-r matrix $\mathbf{X}^*$ can be written as $\mathbf{X}^* = \mathbf{U}_{n\times r}\mathbf{B}_{r\times q}$, where $\mathbf{U}, \mathbf{V}$ have mutually orthonormal columns.

LR-Ptych

Initialization:

- $\mathbf{x}_k^0 \leftarrow \sqrt{\frac{1}{N}\sum_{i=1}^N \mathbf{y}_{i,k}^2}$ for $k = 1, 2, \ldots, q$
- $[\mathbf{U}^0, \mathbf{S}^0, \mathbf{V}^0] \leftarrow SVD((\mathbf{X}^0))$
- $\mathbf{b}_k^0 \leftarrow (\mathbf{S}^0\mathbf{V}^{0\top})_k$

Descent:
Use $\mathbf{U}^0$ and $\mathbf{b}_k^0$ as initialization

- $\mathbf{C}_k^t \leftarrow \mathrm{diag}(phase(\mathcal{A}_k(\mathbf{U}^{t-1}\mathbf{b}_k^{t-1})))$, $k = 1, 2, \ldots, q$
- $\mathbf{U}^{tmp} \leftarrow \text{argmin}_{\tilde{\mathbf{U}}} \sum_k \left\|\mathbf{C}_k^t\mathbf{y}_k - \mathcal{A}_k(\tilde{\mathbf{U}}\mathbf{b}_k^{t-1})\right\|^2$
- $\mathbf{U}^t \leftarrow QR(\mathbf{U}^{tmp})$
- $\mathbf{b}_k^t \leftarrow \text{argmin}_{\tilde{\mathbf{b}}_k} \left\|\mathbf{C}_k^t\mathbf{y}_k - \mathcal{A}_k(\mathbf{U}^t\tilde{\mathbf{b}}_k)\right\|^2$, $k = 1, 2, \ldots, q$

Output: $\mathbf{x}_k^* = \mathbf{U}^T\mathbf{b}_k^T$.

Modified LR-Ptych (Mod-LR-Ptych) (Submitted to ICIP'18)

If real video is not low-rank, do "model-correction":

$$\hat{\mathbf{X}} := \tilde{\mathbf{X}} + \text{argmin}_{\mathbf{E}} \sum_{k=1}^{q}\sum_{i=1}^{N} \|\mathbf{y}_{i,k} - |\mathcal{A}_{i,k}(\mathbf{x}_k + \mathbf{e}_k)|\|_2^2$$

where $\mathbf{E} = [\mathbf{e_1}, \mathbf{e_2}, \ldots \mathbf{e_q}]$, $E \in \mathbb{R}^{n\times q}$ is the modeling error.

Results

Pixel-wise random under-sampling:

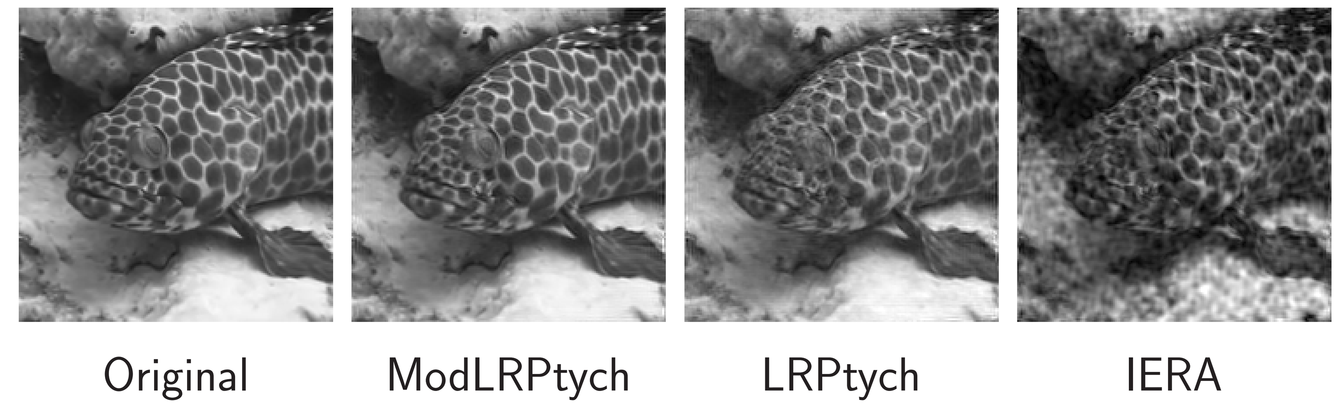

Fig1: Visual comparison for video "fish" frame 66 under sample ratio of 0.5

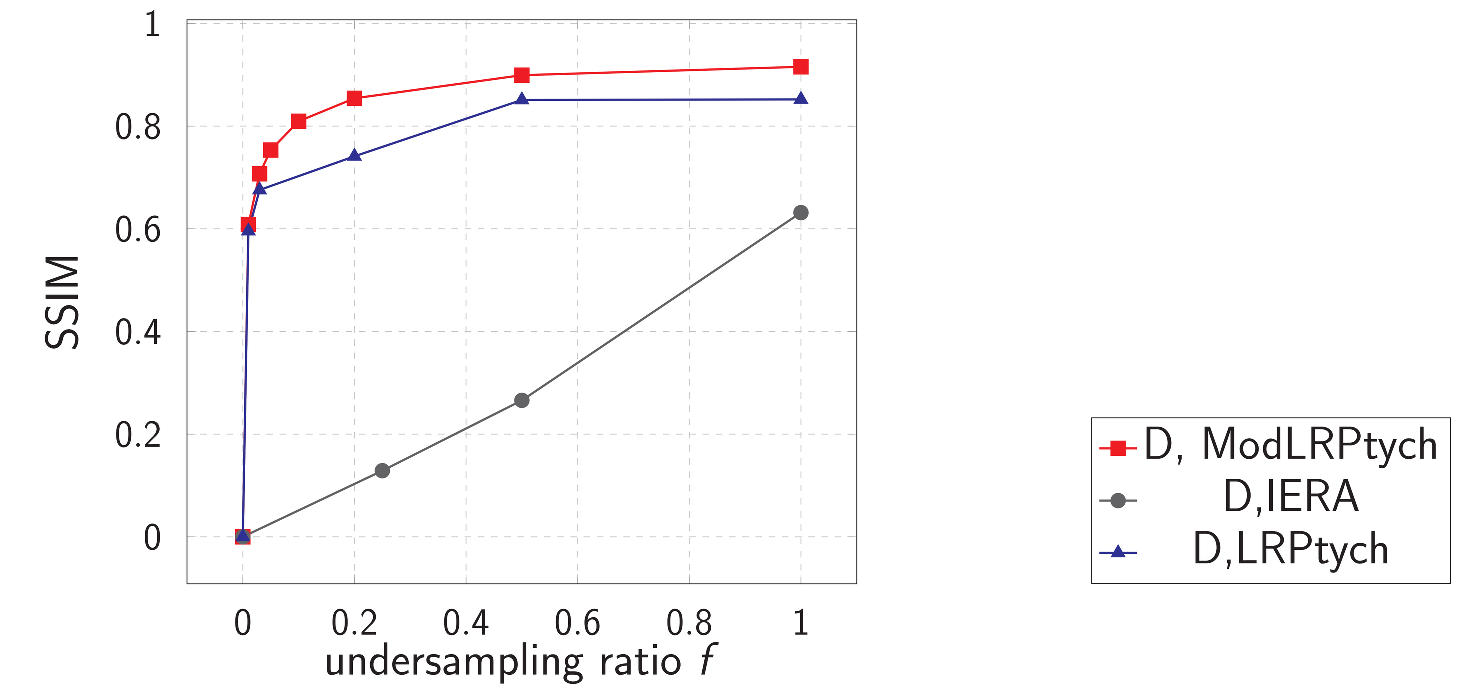

Fig2: Comparison on different under sample ration for video "dog"

Random camera under-sampling:

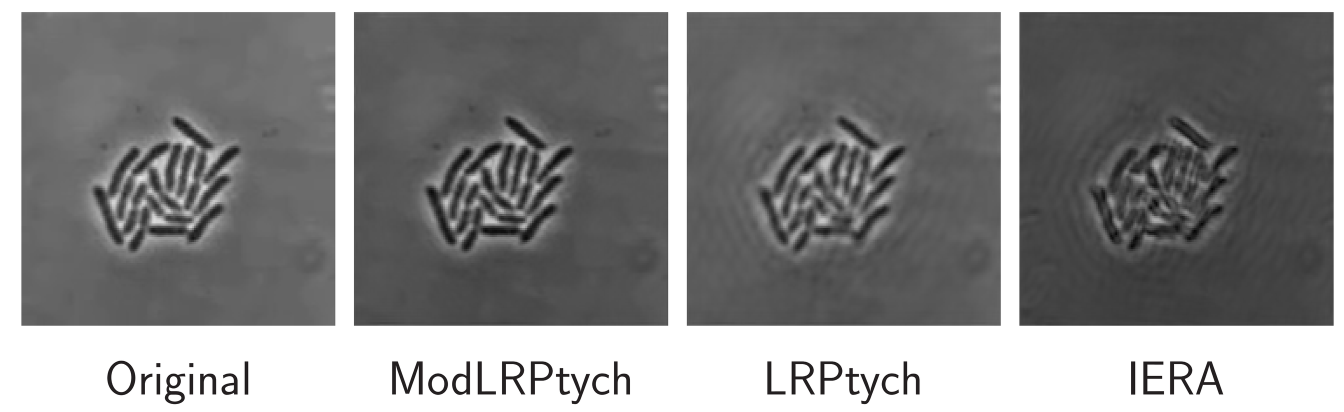

Fig3: Visual comparison for video "bacteria" frame 66 with 0.5 camera was used.

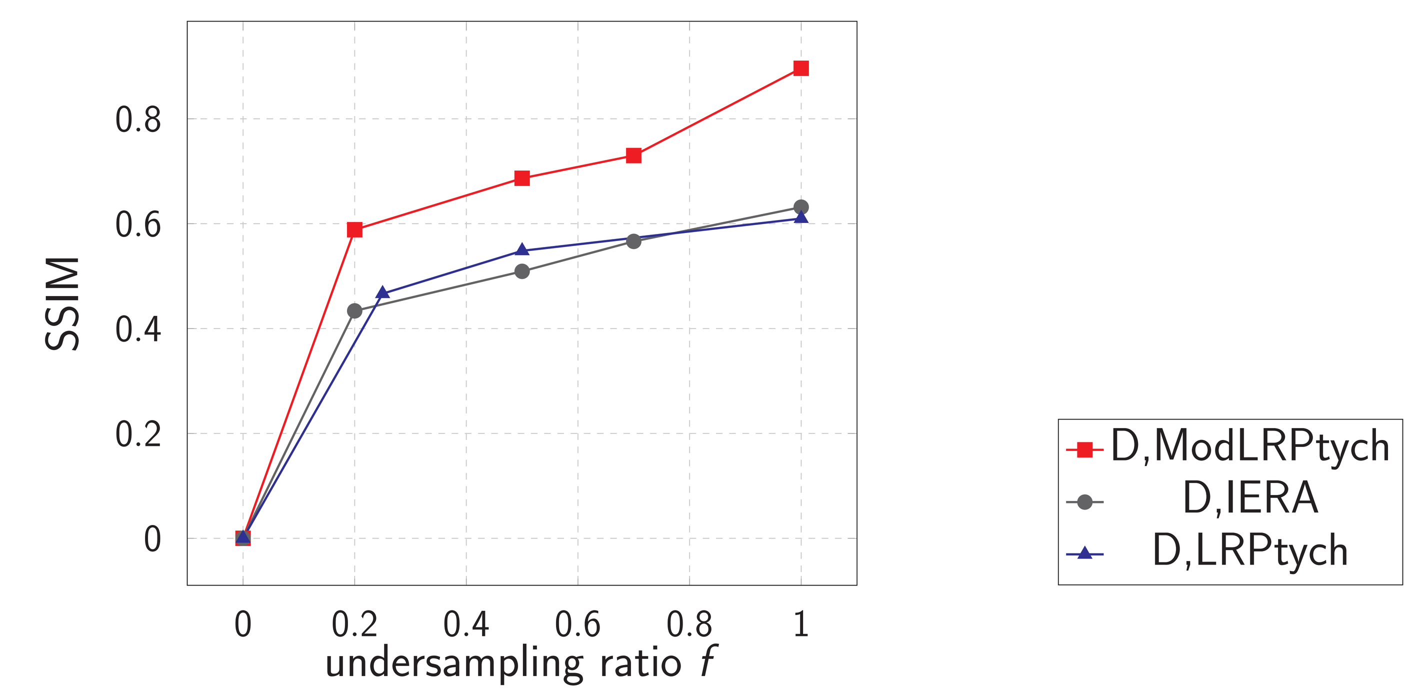

Fig4: Comparison on different camera usage ration for video "dog"

References

[VNE17] N. Vaswani, S. Nayer and Y. Eldar. "Low Rank Phase Retrieval", IEEE Trans. Signal Processing, 2017.
[HASMHCV16] J. Holloway, M.S. Asif, M.K. Sharma, N. Matsuda, R. Horstmeyer, O. Cossairt, A. Veeraraghavan, "Toward long-distance subdiffraction imaging using coherent camera arrays." IEEE Trans. on Computational Imaging 2016.