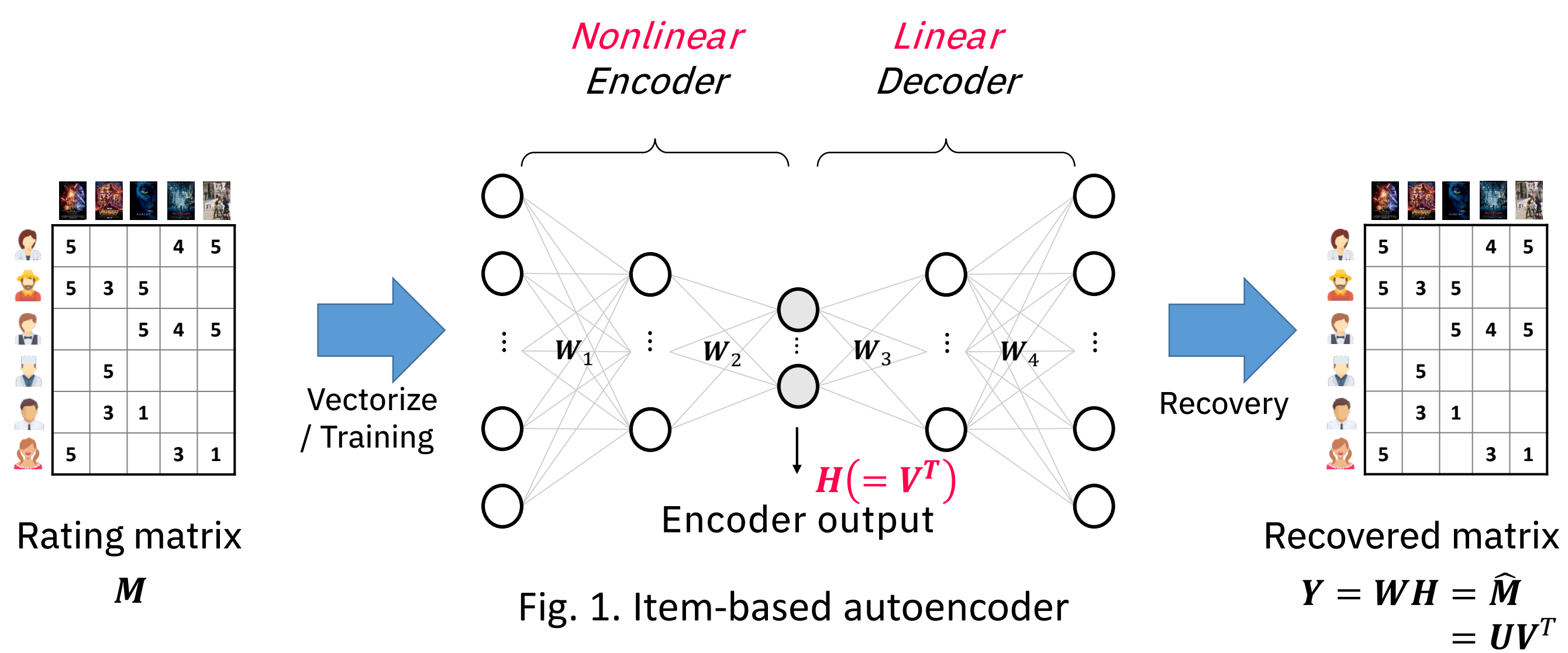


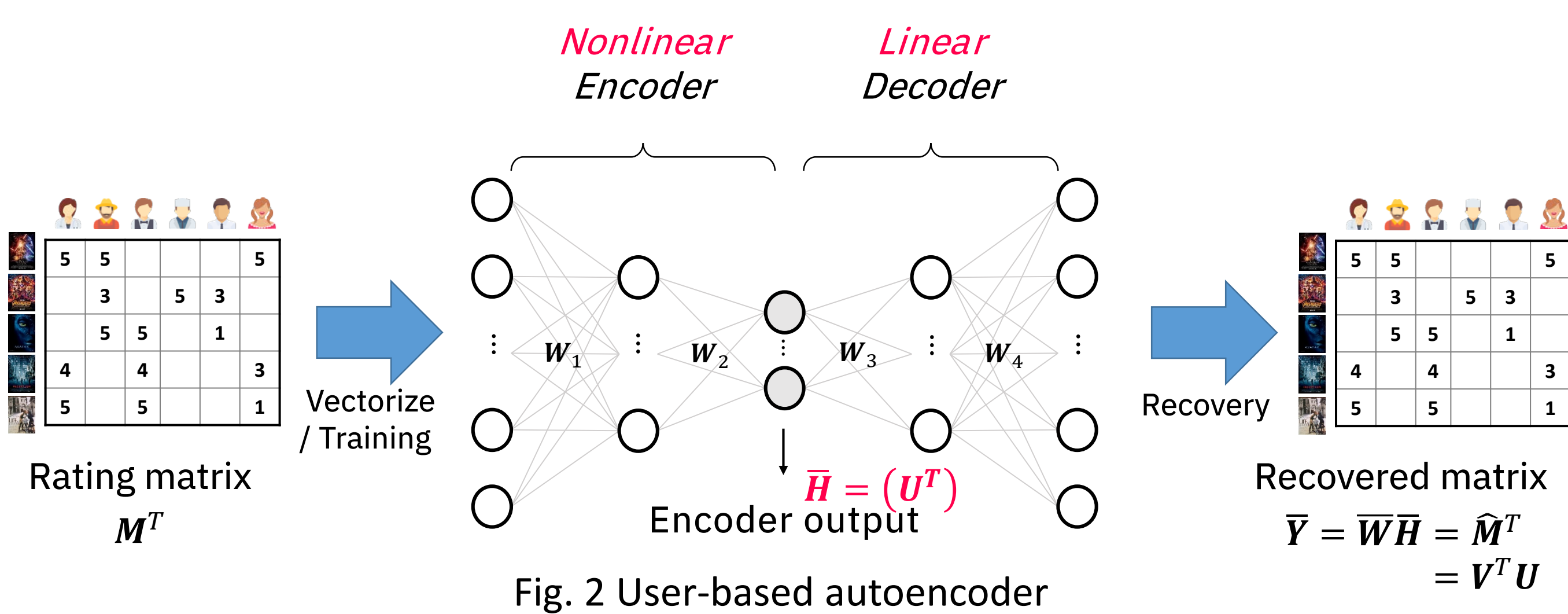
Overview

Autoencoders (AEs)

- Autoencoders (AEs) for matrix completion (MC) and collaborative filtering (CF)
- **Item-based autoencoder (I-AE)**



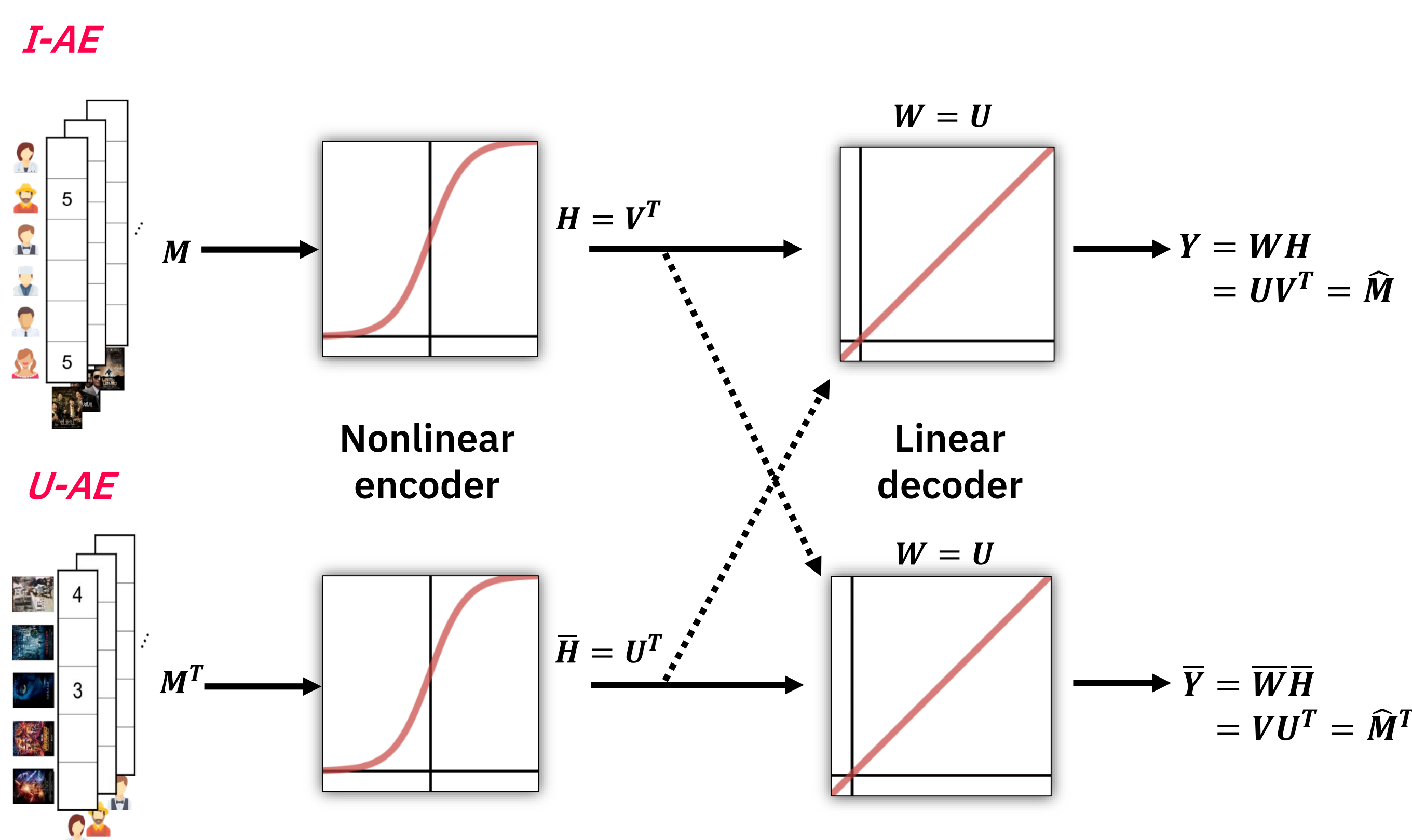
- **User-based autoencoder (U-AE)**



Sequential estimation property of AEs

- For I-AE, the encoder first estimates the item feature matrix $V^T (= H)$ and then the decoder estimates user feature matrix $U (= W)$ as a function of $V^T (= H)$.
- Similarly, for U-AE, $U^T (= \bar{H})$ is estimated first and then $V (= \bar{W})$ is obtained.
- This *sequential* estimation of feature matrices can cause some performance degradation, because one of the estimated feature matrices always depends on the other.

Alternating autoencoders



- After each training epoch, AAE evaluates the RMSE of the current AE, and *alternates* with the other AE if the RMSE becomes less than the RMSE of the previous AE.

Alternating autoencoders

Algorithm 1 Alternating autoencoders.

- 1: **Input:** Incomplete matrix M , error tolerance δ and maximum training epoch k_{\max} .
- 2: **Initialize** \bar{H}_0 and all weighting matrices of AEs, $k = 0$, $\tau_0 = \infty$ and $k_{al} = 0$.
- 3: **repeat**
- 4: $k = k + 1$
- 5: Update H_k of I-AE, while fixing $W_k = \bar{H}_{k_{al}}^T$, via back-propagation.
- 6: Evaluate e_k .
- 7: **until** $e_k < \tau_{k-1}$
- 8: **Set** $\tau_k = e_k - \delta$ and $k_{al} = k$.
- 9: **repeat**
- 10: $k = k + 1$
- 11: Update \bar{H}_k of U-AE, while fixing $\bar{W}_k = H_{k_{al}}^T$, via back-propagation.
- 12: Evaluate e_k .
- 13: **until** $e_k < \tau_k$
- 14: **Set** $\tau_k = e_k - \delta$ and $k_{al} = k$
- 15: **Go to** step 3 and repeat the process until $k = k_{\max}$.
- 16: **Output:** Estimated low-rank matrix $\hat{M} = \bar{H}_{k_{\max}}^T H_{k_{\max}}$

Confirming sequential estimation property

Observation 1

- After convergence, the matrices Y, W and H for I-AE satisfy $\text{rank}(H) = \text{rank}(W) = \text{rank}(Y)$
- This observation is confirmed by showing that $\text{col}(Y) = \text{col}(W)$, and $\text{col}(Y^T) = \text{col}(H^T)$ via computer simulation.

Observation 2 (Matrix factorization)

$$Y = \hat{M}, W = U, \text{ and } H = V^T$$

- This is a consequence of Observation 1. It is also shown via simulation that each row of $W (= U)$ can be represented in terms of the pseudo-inverse of a submatrix of $H (= V^T)$

U-AE can be confirmed in a similar manner.

Experiments

Simulations with synthetic data

- Generate 500×500 matrices of rank 10, denoted as M_0
- Obtain incomplete matrices M by choosing $|M|$ entries of M_0
- Reconstruction rate: $\Pr(\text{reconstruction error} \leq 10^{-4})$

Simulations with practical data

- MovieLens-100k (943 users & 1,682 movies) MovieLens-1M (6,040 users & 3,952 movies)
- Train : Test = 9 : 1

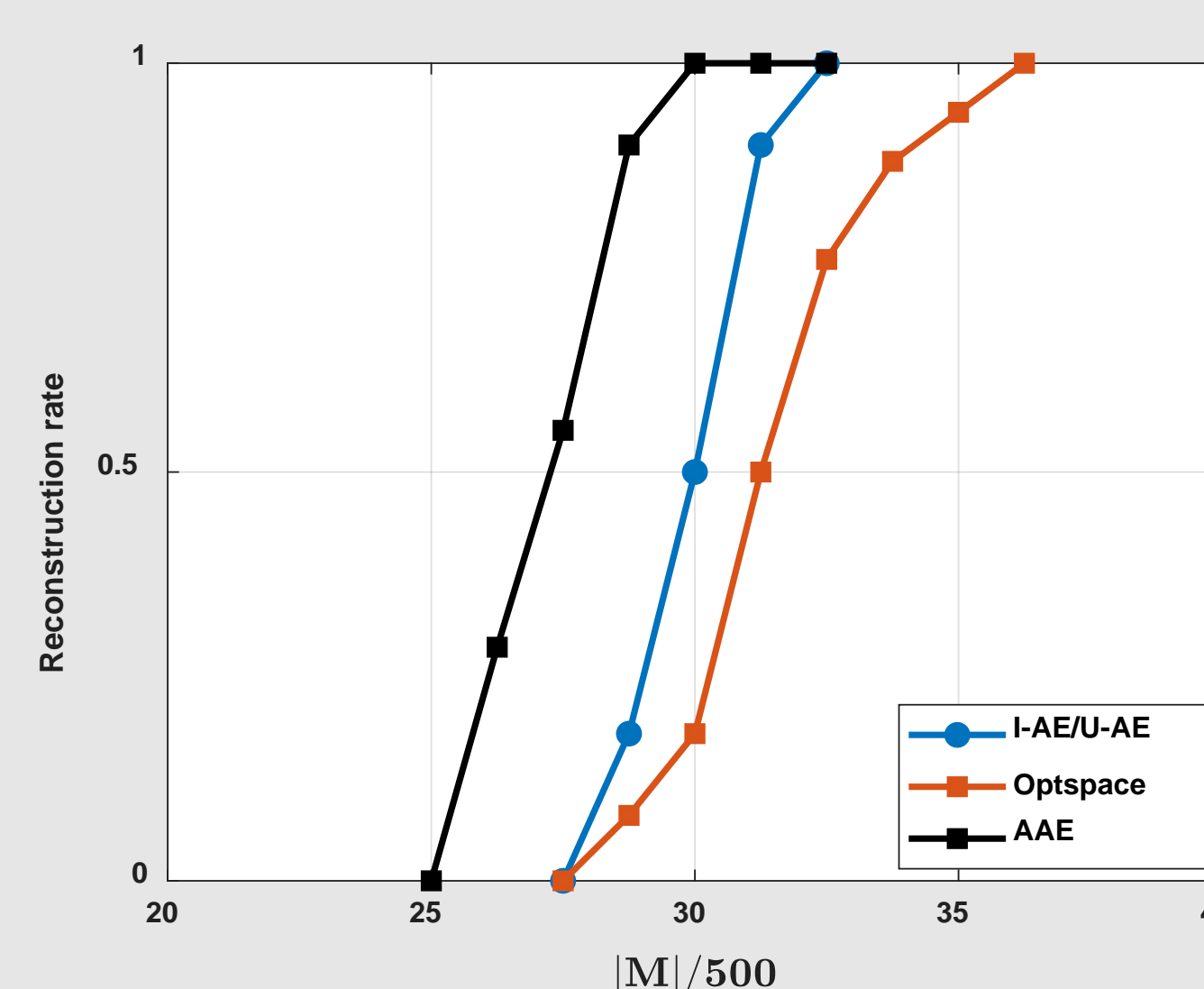


Fig. 4. Reconstruction rate

	RMSE	ML-100k	ML-1M
Optspace		0.911	0.873
ALS-WR		0.913	0.843
LLORMA		0.898	0.833
CF-NADE		-	0.829
I-Autorec		-	0.831
U-AE		0.905	0.841
I-AE		0.884	0.829
AAE		0.877	0.826

Table 1. RMSE comparison