Statistical Analysis of EH Battery State under Noisy Energy Arrivals

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Abstract - **It has been recently shown that optimum transmission policies for energy harvesting (EH) fundamentally depends on whether or not the battery capacity is considerably larger than the average amount of energy arrivals. In other words, adequately determining the battery capacity is central for determining how much rate is sacrificed by a given transmission policy, which in turn requires knowledge on the distribution of the stored energy. In this paper, we therefore contribute with the derivation of a statistical model for the stored energy of a EH node subjected to a noisy, Gaussian excitation, which models actual piezoelectric EH circuits. Comparisons between the analytically-derived distribution and simulation results illustrate the accuracy of the model. As a side result, we provide evidence that the expressions can be used to analytically optimize the raw rate, or the bits/Joule efficiency of an EH node, measured as the ratio between the average rate over the average available energy.**

I. System Model for EH Node

We assumed that an energy harvesting (EH) node is equipped with a piezoelectric[1] harvesting circuit and a battery with sufficiently high capacity.

A. Energy Arrival Model

- **Ambient vibration** can be modeled as white Gaussian noise following the probability density function (PDF) because of the principle of maximum entropy and the central limit theorem,

$$n \sim p_{\mathcal{N}}(n;\mu_\varepsilon,\sigma_\varepsilon) = \frac{1}{\sqrt{2\pi\sigma_\varepsilon}} e^{-\frac{(n-\mu_\varepsilon)^2}{2\sigma_\varepsilon}}. \tag{1}$$

n : The amplitude of the vibration. μ_ε and σ_ε : Mean and the variance, respectively.

- **The energy associated** with a Gaussian random stimuli of amplitude n, with μ_ε and σ_ε, is proportional to n^2 which in turn follows a non-central chi-square distribution whose PDF is given by

$$r \sim p_{\chi^2}(r;\mu_\varepsilon,\sigma_\varepsilon) = \frac{e^{-\frac{(\mu_\varepsilon+\sqrt{r})^2}{2\sigma_\varepsilon^2}}}{2\sqrt{2\pi}\sqrt{r}\sigma_\varepsilon}\left(e^{\frac{2\mu_\varepsilon\sqrt{r}}{\sigma_\varepsilon^2}}+1\right), \tag{2}$$

$r \triangleq n^2$, $0 < r < \infty$.

- The piezoelectric energy harvesters also consume some energy in order to operate [1]. Only the amount above a threshold required to power the harvesting process itself can be collected. We assume that the threshold ε_0.

Considering the circuit power consumption (CPC), the statistics of **the energy stimuli sufficiently strong to enable harvesting** is given by

$$g(\varepsilon;\sigma_\varepsilon,\mu_\varepsilon,\varepsilon_0) = \frac{p_{\chi^2}(\varepsilon;\mu_\varepsilon,\sigma_\varepsilon)}{\int_{\varepsilon_0}^{\infty} p_{\chi^2}(r;\mu_\varepsilon,\sigma_\varepsilon)dr} = \frac{2\times p_{\chi^2}(\varepsilon;\mu_\varepsilon,\sigma_\varepsilon)}{\text{erfc}\left(\frac{\sqrt{\varepsilon_0}-\mu_\varepsilon}{\sqrt{2}\sigma_\varepsilon}\right)+\text{erfc}\left(\frac{\sqrt{\varepsilon_0}+\mu_\varepsilon}{\sqrt{2}\sigma_\varepsilon}\right)}, \quad \text{for } \varepsilon_0 < \varepsilon < \infty. \tag{3}$$

- **The average amount of harvestable energy** under the given model is

$$\mathbb{E}[\varepsilon] = \int_{\varepsilon_0}^{\infty} \varepsilon \times g(\varepsilon;\mu_\varepsilon,\sigma_\varepsilon,\varepsilon_0)d\varepsilon - \varepsilon_0 = \frac{(\sqrt{2}\sigma_\varepsilon)^2}{\sqrt{\pi}} \times \frac{\delta^+ e^{-(\delta^-)^2}+\delta^- e^{-(\delta^+)^2}}{\text{erfc}(\delta^-)+\text{erfc}(\delta^+)} + \mu_\varepsilon^2+\sigma_\varepsilon^2-\varepsilon_0 \tag{4}$$

where $\delta^+ \triangleq \frac{\sqrt{\varepsilon_0}+\mu_\varepsilon}{\sqrt{2}\sigma_\varepsilon}$ and $\delta^- \triangleq \frac{\sqrt{\varepsilon_0}-\mu_\varepsilon}{\sqrt{2}\sigma_\varepsilon}$ are implicitly defined.

Distribution of the Amount of Harvested Energy and the Inter-Arrival Time

The k-th **instantaneous amount of harvested energy** at the beginning of a k-th epoch,

$$\varepsilon_k \sim p(x;\lambda_{\varepsilon:\mathrm{E}}) \triangleq \lambda_{\varepsilon:\mathrm{E}} e^{-\lambda_{\varepsilon:\mathrm{E}}x}; \tag{5}$$

where $\lambda_{\varepsilon:\mathrm{E}} \triangleq \frac{1}{\mathbb{E}[\varepsilon]}$.

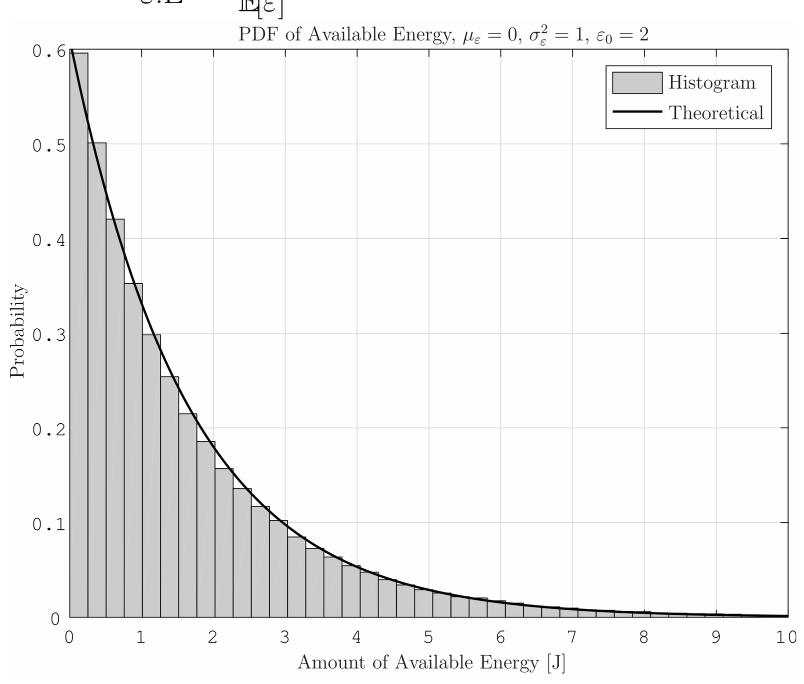

Fig. 1. Histogram and theory of PDF of amount of harvested energy

The k-th **inter-arrival time** (between the k-th and $(k+1)$-th epoch) [3],

$$t_k \sim p(x;\lambda_{\varepsilon:\mathrm{T}}) \triangleq \lambda_{\varepsilon:\mathrm{T}} e^{-\lambda_{\varepsilon:\mathrm{T}}x}; \tag{6}$$

$\lambda_{\varepsilon:\mathrm{T}}$: the inter-arrival rate of the distribution.

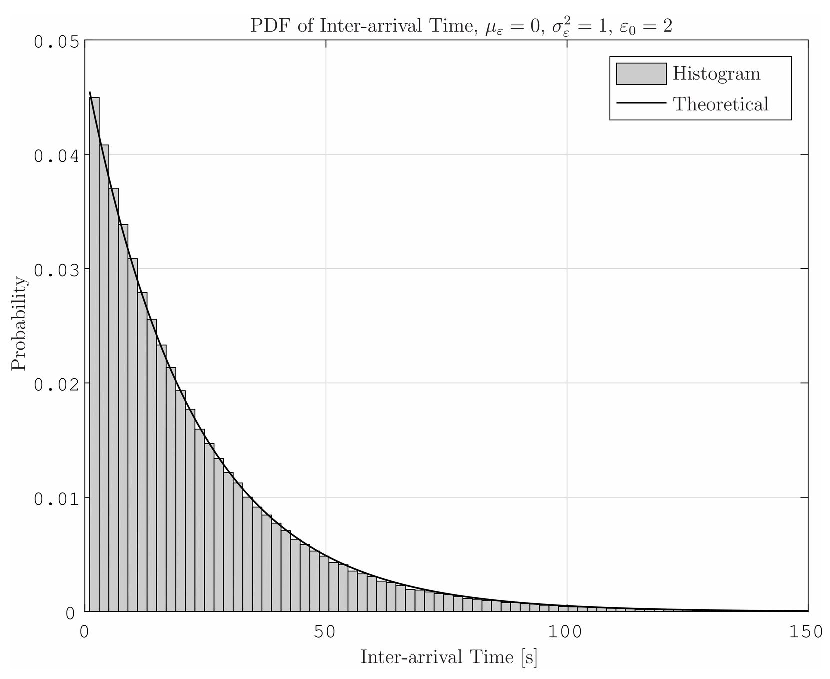

Fig. 2. Histogram and theory of PDF of inter-arrival time

B. Energy Harvesting, Consumption and Storing and Transmission

- **Harvesting and Transmission Process** of a EH node is illustrated in Figure 3.

1) An EH node obtains energy harvested from the excitation of ambient vibration. Epoch starts.
2) The harvested energy ε_1 is stored into battery. Then the amount of stored energy in the battery is B_1.
3) The EH node consumes energy ξ_1 from B_1 for transmission. Then, consumption time t_1 is a constant transmission time is T or a time until the EH node obtains next-harvested energy.
4) This epoch ends and moves to next epoch.

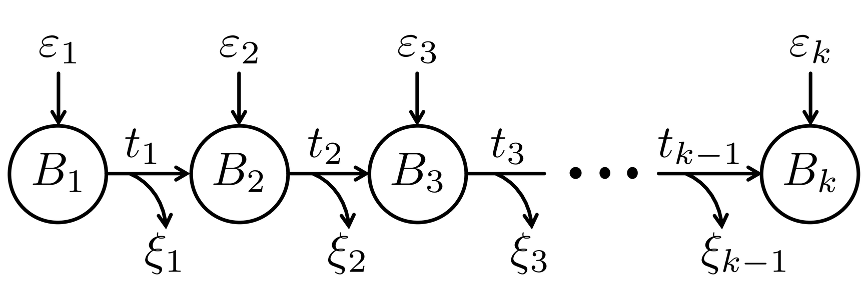

Fig. 3. Processes of harvesting, stored and consumed energy of an EH node

[1]We focus here on the piezoelectric harvesting case due to the existence of supporting literature. The analysis is, however, expected to work for other energy sources as well, as many of those are also noisy.

- The instantaneous and average **amount of stored energy** available for transmission at the beginning of the k-th epoch are respectively given by

$$B_k = \varepsilon_k + B_{k-1}\left(1-\frac{\min(T,t_{k-1})}{T}\right), \tag{7a}$$

$$\mathbb{E}[B_k] = \frac{1}{\lambda_{\varepsilon:\mathrm{E}}} + \mathbb{E}[B_{k-1}]\left(1-\frac{1-e^{-\lambda_{\varepsilon:\mathrm{T}}T}}{\lambda_{\varepsilon:\mathrm{T}}T}\right). \tag{7b}$$

- The **maximum achievable rate** for k-th epoch [4],

$$R_k = \log_2\left(1+\rho\frac{B_k}{T}\right)\frac{\min(T,t_k)}{t_k}. \tag{8a}$$

$$\bar{R} = \mathbb{E}\left[\log_2\left(1+\rho\frac{B_k}{T}\right)\right] \times \left(1-e^{-\lambda_{\varepsilon:\mathrm{T}}T}+\lambda_{\varepsilon:\mathrm{T}}TE_i(\lambda_{\varepsilon:\mathrm{T}}T)\right) \tag{8b}$$

ρ : the signal-to-noise-ratio (SNR).

II. Distribution of Stored Energy

- The **state of the battery** at each epoch,

$$B_1 = \varepsilon_1$$
$$B_2 = \varepsilon_2 + \alpha_1 B_1 = \varepsilon_2 + \alpha_1\varepsilon_1$$
$$B_3 = \varepsilon_3 + \alpha_2 B_2 = \varepsilon_3 + \alpha_2\varepsilon_2 + \alpha_1\varepsilon_1$$
$$\vdots$$
$$B_k = \varepsilon_k + \alpha_{k-1}B_{k-1} = \varepsilon_k + \overbrace{\sum_{l=1}^{k-1}\left(\prod_{i=l}^{k-1}\alpha_i\cdot\varepsilon_l\right)}^{\triangleq\gamma_k} \tag{9}$$

α_k $(0 \leqslant \alpha_k \leqslant 1)$: Left unused energy rate,

$$\alpha_k \triangleq 1-\frac{\min(T,t_k)}{T}, \tag{10a}$$

$$\mathbb{E}[\alpha_{k-1}] = 1-\frac{1-\exp(-\lambda_{\varepsilon:\mathrm{T}}T)}{\lambda_{\varepsilon:\mathrm{T}}T}, \tag{10b}$$

γ_k : The **amount of residual energy** in the battery at the beginning of the k-th epoch.

B_k can be well approximated as a gamma random variable by the Welch-Satterthwaite approximation for the hypoexponential distribution [5], [6].

- The 1st and 2nd **moment of the battery state** $\mu_n \triangleq \mathbb{E}[B_k^n]$, we have

$$\mu_1 = \frac{\lambda_{\varepsilon:\mathrm{T}}T}{\lambda_{\varepsilon:\mathrm{E}}(1-\exp(-\lambda_{\varepsilon:\mathrm{T}}T))} \tag{11a}$$

$$\mu_2 = \frac{(\lambda_{\varepsilon:\mathrm{T}}T)^2\mu_1}{\lambda_{\varepsilon:\mathrm{E}}(\exp(-\lambda_{\varepsilon:\mathrm{T}}T)+\lambda_{\varepsilon:\mathrm{T}}T-1)}. \tag{11b}$$

Distribution of Stored Energy in An Infinite Battery

The distribution of stored energy in an infinite battery at the k-th epoch is approximated the Gamma distribution, which is given by

$$B_k \sim p_\Gamma(B_k,\phi,\theta) = \frac{1}{\Gamma(\phi)\theta^\phi}B_k^{(\phi-1)}\exp\left(-\frac{B_k}{\theta}\right) \tag{12}$$

where the shape and scale parameters ϕ and θ are respectively,

$$\theta = \frac{\mu_2-\mu_1^2}{\mu_1} \quad \text{and} \quad \phi = \frac{\mu_1^2}{\mu_2-\mu_1^2}. \tag{13}$$

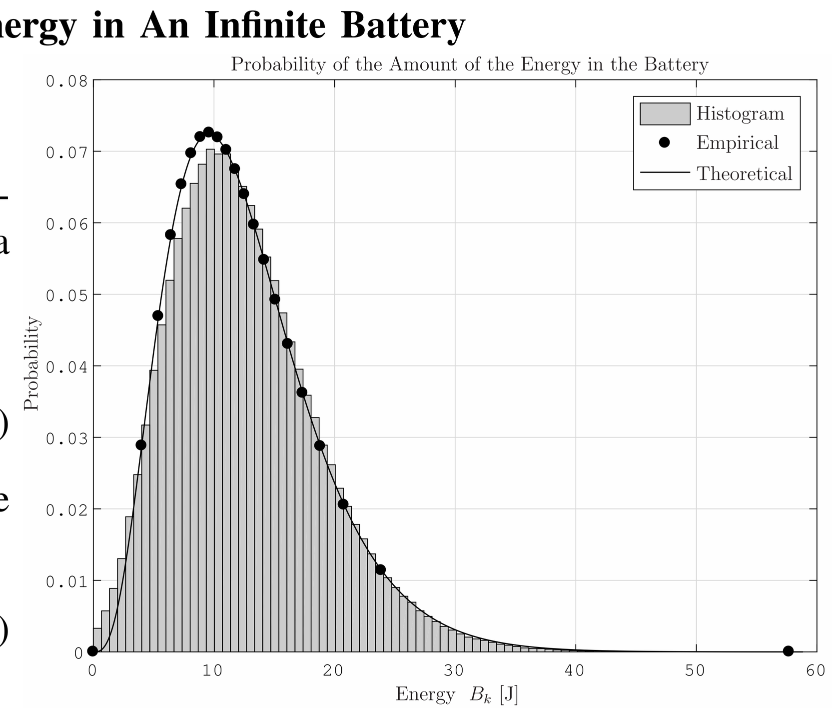

Fig. 4. The distribution of energy stored in a infinite battery, Empirical vs theoretically approximated PDFs.

III. Application

- The maximum achievable rate can be rewritten and this is a concave function of T,

$$\bar{R} = \int_0^\infty \log_2\left(1+\rho\frac{B_k}{T}\right)p_\Gamma(B_k,\phi,\theta)\,dB_k \times \left[1-e^{-\lambda_{\varepsilon:\mathrm{T}}T}+\lambda_{\varepsilon:\mathrm{T}}TE_i(\lambda_{\varepsilon:\mathrm{T}}T)\right]. \tag{14}$$

The optimization of the transmission period T can be easily achieved numerically with efficient methods.

- Analyzing $\bar{R}/\bar{B}$ is contributed to optimization problem for the bits/Joule efficiency of EH systems.

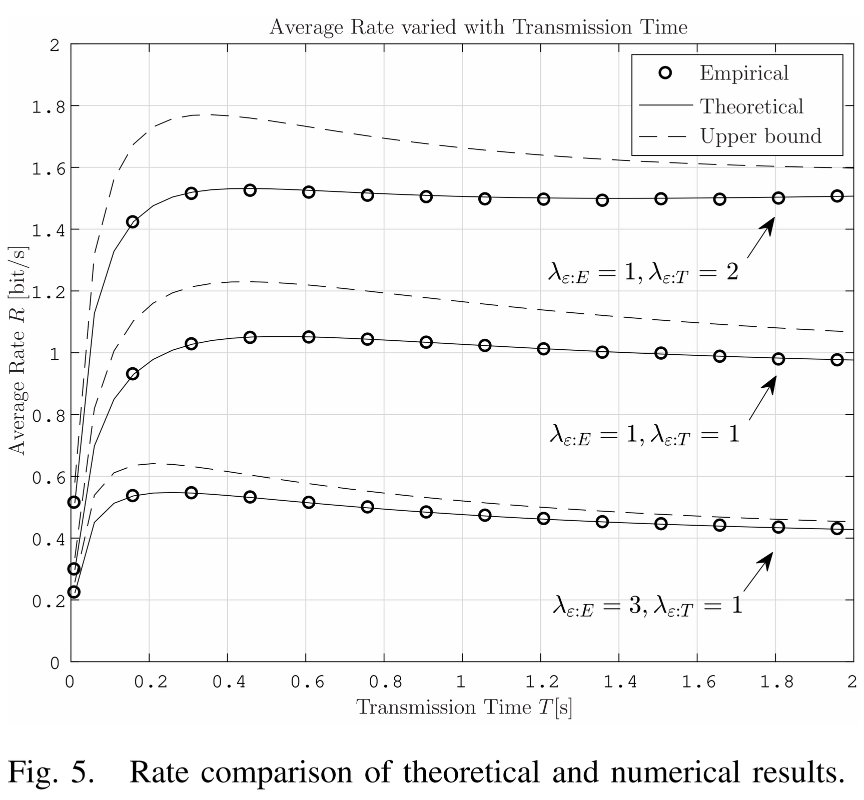

Fig. 5. Rate comparison of theoretical and numerical results.

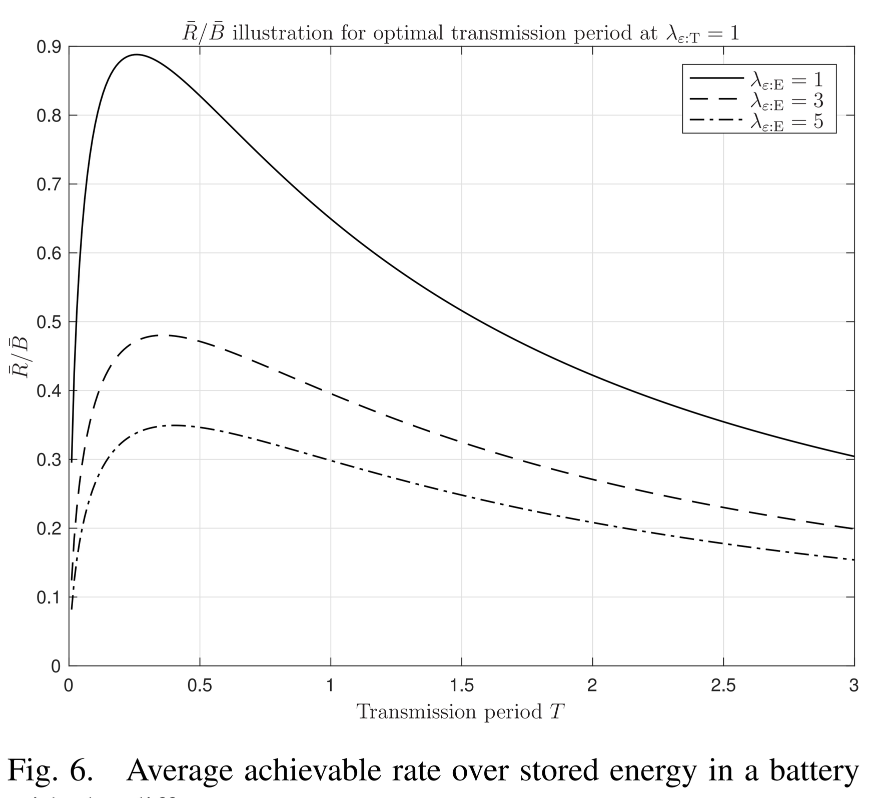

Fig. 6. Average achievable rate over stored energy in a battery with the different parameters.

IV. Conclusion

We derived a model for the energy stored by an energy harvesting node under noisy stimulus, simplifying the latter as a Gamma distribution, with formulas for the required parameters for a greedy transmission policy are given as a function of the average amount of energy and average inter-arrival time.

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