OPTIMAL RESOURCE ALLOCATION FOR NON-REGENERATIVE MULTIWAY RELAYING WITH RATE SPLITTING

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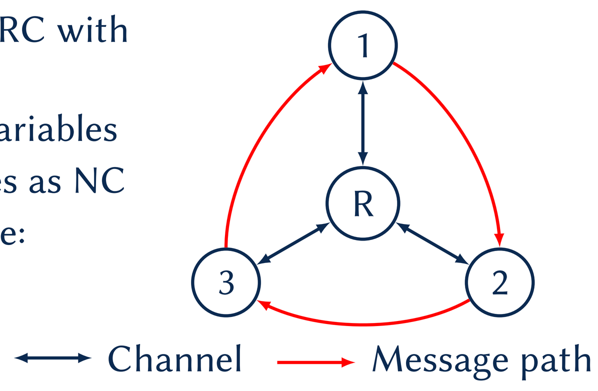
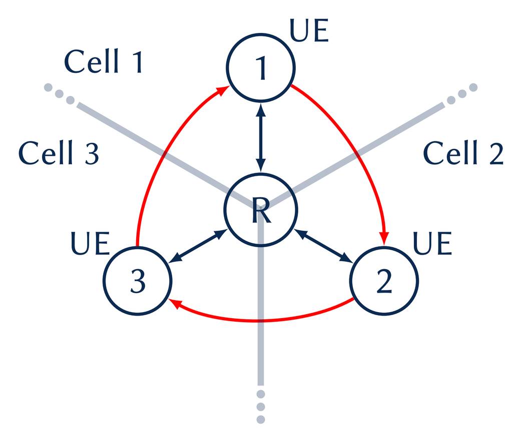
Summary

- Global optimal resource allocation for a 3-user Gaussian MWRC with SND, AF relaying, and rate splitting.
- Non-convex optimization problem but only few non-convex variables
- SoA (e.g. canonical monotonic optimization): treat all variables as NC
- Resource allocation framework that exploits problem structure:
  - improved performance
  - numerically stable and guaranteed convergence
  - feasible solution even if terminated prematurely
- Numerical evaluation of rate splitting vs. "true" SND [1] vs. "traditional" SND vs. IAN

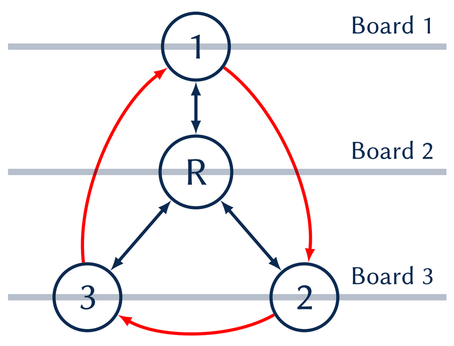


Motivation

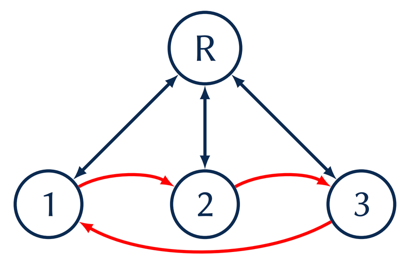
- Heterogeneous dense small-cell networks:



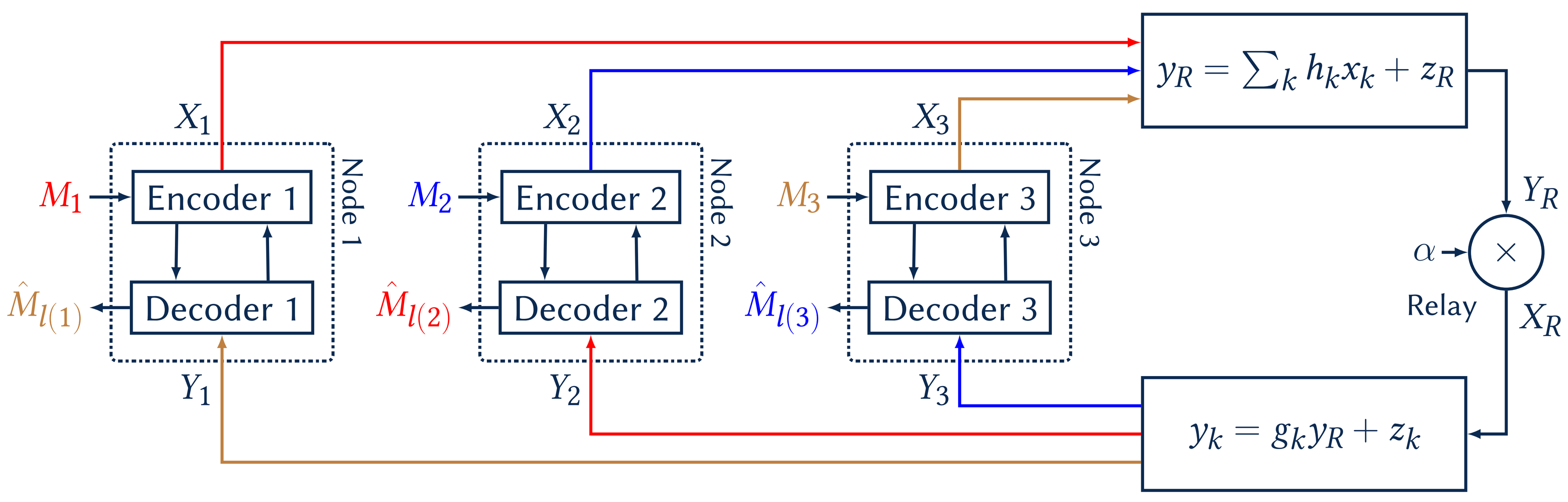
- Wireless board-to-board communication in highly adaptive computing:



- Industry 4.0:
- Satellite Communications:



System Model & Achievable Rate Regions



Block diagram of the 3-user Gaussian MWRC with multiple unicast transmissions and amplify-and-forward relaying.

- Gaussian channels: Tx power $P_k \leq \bar{P}_k$, noise $Z_k \sim \mathcal{CN}(0, N_k)$, SNR $S_k = \frac{P_k}{N_r} \leq \bar{S}_k = \frac{\bar{P}_k}{N_R}$.
- Node k transmits msg M_k to $q(k)$ and receives $M_{l(k)}$. $l(k)$ is interfering with transmission of M_k.
- Relay amplification: For relay Tx power P_R choose $\alpha = \sqrt{P_R / \left(\sum_{k\in\mathcal{K}} |h_k|^2 P_k + N_R\right)}$.

Lemma (Rate Splitting [2])

A rate triple (R_1, R_2, R_3) is achievable for the Gaussian MWRC with AF relaying if, for all $k \in \mathcal{K}$,

$$R_k < B_k, \qquad R_k + R_{q(k)} < A_k + D_{q(k)},$$
$$R_\Sigma < A_k + C_{q(k)} + D_{l(k)}, \qquad R_k + R_\Sigma < A_k + C_{q(k)} + C_{l(k)} + D_k,$$

and, $R_\Sigma < C_1 + C_2 + C_3$,

with
$$A_k = \log\left(1 + \frac{|h_k|^2 S_k^p}{\gamma_k(\boldsymbol{S})}\right), \qquad B_k = \log\left(1 + \frac{|h_k|^2 (S_k^p + S_k^c)}{\gamma_k(\boldsymbol{S})}\right),$$
$$C_k = \log\left(1 + \frac{|h_k|^2 S_k^p + |h_{l(k)}|^2 S_{l(k)}^c}{\gamma_k(\boldsymbol{S})}\right), \qquad D_k = \log\left(1 + \frac{|h_k|^2 (S_k^p + S_k^c) + |h_{l(k)}|^2 S_{l(k)}^c}{\gamma_k(\boldsymbol{S})}\right),$$

where $S_k^c + S_k^p \leq \bar{S}_k$ and $\gamma_k(\boldsymbol{S}) = 1 + |h_{l(k)}|^2 S_{l(k)}^p + \tilde{g}_{q(k)}^{-1} \left(1 + \sum_{i\in\mathcal{K}} |h_i|^2 (S_i^c + S_i^p)\right)$, with $\tilde{g}_k = |g_k|^2 \frac{\bar{P}_0}{N_k}$.

Lemma (Single Message)

A rate triple (R_1, R_2, R_3) is achievable for the Gaussian MWRC with AF relaying if, for all $k \in \mathcal{K}$,

$$R_k \leq \log\left(1 + \frac{|h_k|^2 S_k}{\gamma_k(\boldsymbol{S})}\right) \quad \text{or} \quad \begin{aligned} R_k &\leq \log\left(1 + \frac{|h_k|^2 S_k}{\delta_k(\boldsymbol{S})}\right) \\ R_k + R_{l(k)} &\leq \log\left(1 + \frac{|h_k|^2 S_k + |h_{l(k)}|^2 S_{l(k)}}{\delta_k(\boldsymbol{S})}\right) \end{aligned}$$

where $S_k \leq \bar{S}_k$, $\gamma_k(\boldsymbol{S})$ and $\tilde{g}_k$ as above, and $\delta_k(\boldsymbol{S}) = 1 + \tilde{g}_{q(k)}^{-1}\left(1 + \sum_{i\in\mathcal{K}} |h_i|^2 S_i\right)$.

Problem Statement

Problem (R)

$$\max_{\boldsymbol{R},\boldsymbol{S}} \sum_{k\in\mathcal{K}} w_k R_k \quad \text{s.t.} \quad \boldsymbol{R} \in \mathcal{R}(\boldsymbol{S}), \quad \boldsymbol{R} \geq \underline{\boldsymbol{R}}, \quad \boldsymbol{S} \in [\boldsymbol{0}, \bar{\boldsymbol{S}}]$$

- $\boldsymbol{w} \in \mathbb{R}^3_{\geq 0} \setminus \{\boldsymbol{0}\}$, $\underline{\boldsymbol{R}} \geq 0$, $\bar{\boldsymbol{S}} > 0$
- $\mathcal{R}(\boldsymbol{S})$ achievable rate region: Non-convex in $\boldsymbol{S}$, linear in $\boldsymbol{R}$
- RHS of $\mathcal{R}(\boldsymbol{S})$:

$$\log\left(1 + \frac{\boldsymbol{a}^T \boldsymbol{S}}{\boldsymbol{b}^T \boldsymbol{S} + c}\right) = \log((\boldsymbol{a} + \boldsymbol{b})^T \boldsymbol{S} + c) - \log(\boldsymbol{b}^T \boldsymbol{S} + c)$$

→ Difference of Increasing & Concave functions

Goal: Exploit problem structure & branch only over non-convex variables

Robust Global Optimization [3]

Problem (P)

$$\max_{\boldsymbol{x}\in[\boldsymbol{a},\boldsymbol{b}]} f(\boldsymbol{x}) \quad \text{s.t.} \quad g_i(\boldsymbol{x}) \leq 0, \quad i = 1, 2, \dots, m$$

- g_i, $i = 1, 2, \dots, m$: Non-convex functions
- Usual approach: Solve ε-relaxed problem
- This approach has numerical problems:
  - Convergence in finite iterations not guaranteed
  - Might give incorrect solution far away from optimum

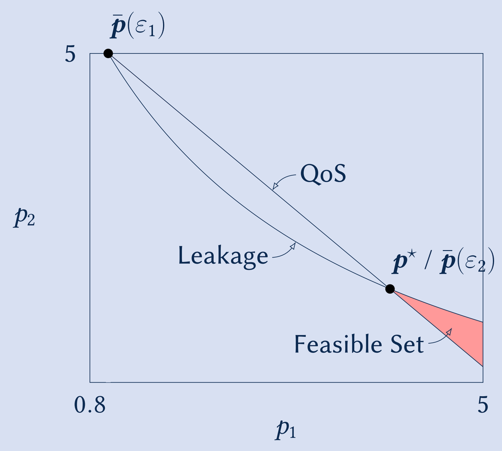
Example (Numerical Problems of ε-Approximate Solutions)

Consider a MAC with QoS and individual eavesdropper information leakage constraints:

$$\min_{p_1, p_2} p_1$$
$$\text{s.t.} \quad \log(1 + |h_1|^2 p_1 + |h_2|^2 p_2) \geq Q + \varepsilon \quad \text{(QoS)}$$
$$\log(1 + |g_1|^2 p_1) + \log(1 + |g_2|^2 p_2) \leq L + \varepsilon \quad \text{(Leakage)}$$
$$p_1 \leq P_1, \quad p_2 \leq P_2$$

Numerical example:

- $|h_i|^2 = 10$, $|g_1|^2 = \frac{1}{2}$, $|g_2|^2 = 1$, $Q = \log(61)$, $L = \log(8.99)$
- True optimal solution: $\boldsymbol{p}^* = (4.00665, 1.99335)$
- ε-approximate solution $\bar{\boldsymbol{p}}(\varepsilon)$:
  - $\varepsilon_1 = 10^{-3}$: $\bar{\boldsymbol{p}}(\varepsilon_1) = (0.995843, 5)$
  - $\varepsilon_2 = 10^{-4}$: $\bar{\boldsymbol{p}}(\varepsilon_2) = (4.00541, 1.99417)$



Solution: ε-essential feasibility

A solution of (P) is said to be *essential* (ε, η)-*optimal* if it satisfies

$$f(\boldsymbol{x}^*) + \eta \geq \sup\{f(\boldsymbol{x}) | \boldsymbol{x} \in [\boldsymbol{a}, \boldsymbol{b}], \forall i: g_i(\boldsymbol{x}) \leq -\varepsilon\}, \qquad \text{for some } \eta > 0.$$

- $\varepsilon, \eta \to 0$: essential (ε, η)-optimal solution is a nonisolated feasible point which is optimal
- SIT: Sequence of feasibility problems

$$\min_{\boldsymbol{x}\in[\boldsymbol{a},\boldsymbol{b}]} \max_{i=1,\dots,m} g_i(\boldsymbol{x}) \quad \text{s.t.} \quad f(\boldsymbol{x}) \geq \gamma \qquad (Q_\gamma)$$

- $\forall \varepsilon > 0: \quad \min(Q_\gamma) > -\varepsilon \;\Rightarrow\; \max(P_\varepsilon) < \gamma$
- Efficient solution with Branch-and-Bound if $f(\boldsymbol{x})$ is concave

Successive Incumbent Transcending (SIT):

- Initialize $\gamma \leq f(\boldsymbol{x})\ \forall \boldsymbol{x} \in \mathcal{F}$.
- Find nonisolated feasible solution $\boldsymbol{x}$ satisfying $f(\boldsymbol{x}) \geq \gamma$ of (P) or establish that no such ε-essential feasible $\boldsymbol{x}$ exists and terminate.
- Update $\bar{\boldsymbol{x}} \leftarrow \boldsymbol{x}$ and $\gamma \leftarrow f(\bar{\boldsymbol{x}}) + \eta$. Repeat.
- Terminate: $\bar{\boldsymbol{x}}$ is an essential (ε, η)-optimal solution; else (P) is ε-essential infeasible.

Application to Resource Allocation Problems

Problem (R)

$$\max_{(\boldsymbol{x},\boldsymbol{\xi})\in\mathcal{C}} f(\boldsymbol{x}, \boldsymbol{\xi}) \quad \text{s.t.} \quad g_i^+(\boldsymbol{x}, \boldsymbol{\xi}) - g_i^-(\boldsymbol{x}) \leq 0, \quad i = 1, \dots, m$$

Dual Problem (Q)

$$\min_{(\boldsymbol{x},\boldsymbol{\xi})\in\mathcal{C}} \max_{i=1,\dots,m} \left(g_i^+(\boldsymbol{x}, \boldsymbol{\xi}) - g_i^-(\boldsymbol{x})\right) \quad \text{s.t.} \quad f(\boldsymbol{x}, \boldsymbol{\xi}) \geq \gamma$$

- Non-convex variables $\boldsymbol{x}$, convex variables $\boldsymbol{\xi}$; $-f$, g_i^+ convex; g_i^- convex & decreasing
- Dual Problem has convex feasible set → no isolated feasible points!
- Core Problem: Compute lower bound for (Q) over box $M = [\boldsymbol{p}, \boldsymbol{q}]$:

$$\min_{(\boldsymbol{x},\boldsymbol{\xi})\in\mathcal{C}} \max_{i=1,\dots,m} \left(g_i^+(\boldsymbol{x}, \boldsymbol{\xi}) - g_i^-(\boldsymbol{p})\right) \quad \text{s.t.} \quad f(\boldsymbol{x}, \boldsymbol{\xi}) \geq \gamma, \quad \boldsymbol{x} \in M$$

→ Convex optimization problem

Rate Splitting:

- Naive implementation: $\boldsymbol{\xi} = \boldsymbol{R}$, $\boldsymbol{x} = \boldsymbol{S} \to 6$ non-convex variables
- Non-convexity due to negative $\log(\gamma_k(\boldsymbol{S}))$ terms: substitute $y = \sum_{k\in\mathcal{K}} |h_k|^2 S_k^c \to 4$ NC vars

$$\max_{\boldsymbol{R},\boldsymbol{S},y} \boldsymbol{w}^T \boldsymbol{R}$$
$$\text{s.t.} \quad \boldsymbol{a}_i^T \boldsymbol{R} - L_i^+(\boldsymbol{S}, y) + L_i^-(\boldsymbol{S}^p, y) \leq 0, \quad i = 1, 2, \dots$$
$$y = \sum_{k\in\mathcal{K}} |h_k|^2 S_k^c$$
$$S_k^c + S_k^p \leq \bar{S}_k, \quad k \in \mathcal{K}, \qquad \boldsymbol{R} \geq \underline{\boldsymbol{R}}, \quad \boldsymbol{S} \geq 0$$

$$L_i^+(\boldsymbol{S}, y) = \sum_{j\in\mathcal{I}_i} \log(f_j(\boldsymbol{S}) + \gamma_j(\boldsymbol{S}^p, y))$$
$$L_i^-(\boldsymbol{S}^p, y) = \sum_{j\in\mathcal{I}_i} \log(\gamma_j(\boldsymbol{S}^p, y))$$

Single Message:

- Rate region: $\mathcal{R} = \bigcap_{k\in\mathcal{K}} (\mathcal{R}_{k,\text{IAN}} \cup \mathcal{R}_{k,\text{SND}}) = \bigcup_{\boldsymbol{d}\in\{\text{IAN},\text{SND}\}^{|\mathcal{K}|}} \bigcap_{k\in\mathcal{K}} \mathcal{R}_{k,d_k}$
- Optimization problem: $\sup_{\boldsymbol{x}\in\bigcup_i \mathcal{D}_i} f(\boldsymbol{x}) = \max_i \sup_{\boldsymbol{x}\in\mathcal{D}_i} f(\boldsymbol{x}) \to$ Solve 8 individual problems

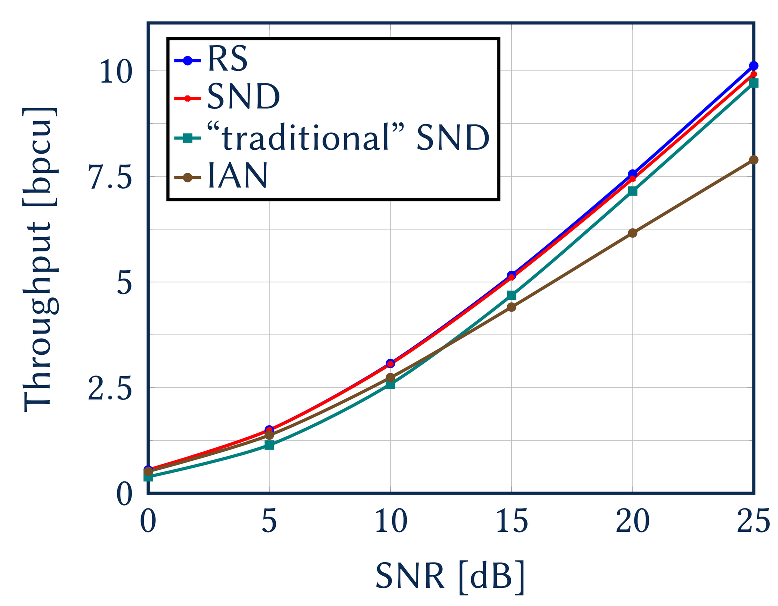
Numerical Evaluation

- SND dominates "traditional" SND and IAN: Gain solely due to per user decoder selection (10 dB: 18 % / 0.48 bpcu & 12 % / 0.32 bpcu)
- Average gain of Rate Splitting small (0.2 bpcu @ 25 dB)
- For some channels: Up to 0.5 bpcu @ 10 dB

Future work:

- Energy efficiency
- Improve bounding

→ Journal version in the making



MWRC with AF relaying. Averaged over 800 i.i.d. channel realizations.

References

[1] B. Bandemer, A. El Gamal, and Y.-H. Kim, "Optimal achievable rates for interference networks with random codes", *IEEE Trans. Inf. Theory*, vol. 61, no. 12, pp. 6536–6549, Oct. 2015.

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[3] H. Tuy, "$\mathcal{D}(\mathcal{C})$-optimization and robust global optimization", *J. Global Optim.*, vol. 47, no. 3, pp. 485–501, Oct. 2009.

DFG Deutsche Forschungsgemeinschaft

HAEC